

van Hove singularities and vortex motion in superconductors

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When vortices move in a type-II superconductor under the action of an electric current, they interact with the crystal lattice generating acoustic waves. This results in a radiation-friction component to the vortex dissipation, which peaks at values of vortex velocity where there are phonons in the Brillouin zone of the crystal with matching phase and group velocities (the van Hove condition). Thus sharp peaks are expected in the current-voltage characteristic resulting from an enhancement of acoustic power generated by moving vortices.

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I. INTRODUCTION

The interaction of sound waves with vortices in type-II superconductors is a field of active study: The attenuation of transverse sound waves through a vortex lattice has been analyzed experimentally in Ref. 1; the generation of ultrasonic waves in the mixed state was observed experimentally;² sound attenuation by helicon-vortex modes was considered in Ref. 3; sound-vortex interaction and the acoustic Faraday effect has been studied in Ref. 4; the generation of sound waves by moving vortices was considered theoretically.⁵ Since vortex motion is driven by an electric current, any peculiarity of the vortex radiation friction due to emission of acoustic waves influences the current-voltage characteristic, and will imprint a nonlinear feature on the *I-V* curve.

The current-voltage characteristic of a type-II superconductor might be expected to be linear at currents greatly exceeding the depinning critical value but lower than the depairing current. Nevertheless such a free-flux-flow (FFF) regime does not extend as far as one might expect because of several physical mechanisms that introduce nonlinearity. One of them is the creation of a nonequilibrium state of the electrons (even with the perfect removal of the Joule heat from the lattice) due to the finite rate of energy relaxation between electrons and lattice. As Larkin and Ovchinnikov⁶ have shown, such a state, created by a large constant electric field, results in enhancement of superconductivity like action of microwaves.⁷ The increase of the order parameter is equivalent to a reduction of the vortex core size which, in turn, leads to a deviation of the *I-V* curve from linearity as has been observed experimentally.^{8–10} At low temperatures a mechanism related to a reduction of the order parameter leads to a somewhat similar but distinct nonlinearity.¹¹

Another mechanism leading to deviations in the *I-V*, is the radiation of sound waves by moving vortices. When a vortex moves under the action of a transport electric current, an electric field is produced in its vicinity.^{12,13} This electric field acts on the crystal lattice resulting in the generation of sound waves. If the vortex velocity exceeds the speed of sound in the crystal, each vortex produces a shock wave or Cherenkov cone. In a thin film in perpendicular magnetic field, the situation is two dimensional and the cone reduces to two lines. The additional dissipation due to motion of the

crystal lattice is proportional to the product of the crystal velocity, which is localized on the shock wave, and the force on the crystal, localized at the vortex positions. When the shock wave coincides with certain directions within the vortex lattice with a maximum number of vortices falling inside the shock wave, there is an enhancement in the dissipation.¹⁴ Since the angle of the shock wave is determined by vortex velocity (electric field), maxima in dissipation occur at certain values of the electric field, producing a series of maxima in the *I-V* curve.

A calculation of the Cherenkov radiation, when the particle (vortex) velocity \vec{v} is given, requires making a summation of the expression $[\omega(\vec{k}) - \vec{v}\vec{k}]^{-1}$ over all waves with wave vectors \vec{k} . $\omega(\vec{k})$ is the sound-wave spectrum.¹⁵ To some extent, this is opposite to the calculation of the Landau damping of waves, when the wave vector \vec{k} is given and one must make a summation over all particle velocities \vec{v} . Nevertheless, in both cases the important condition is a coincidence of the velocity \vec{v} and the phase velocity of the wave $\omega(\vec{k}) = \vec{v}\vec{k}$. If vortices are arranged in a perfect lattice the wave vector \vec{k} should be substituted by a vector \vec{b} of the reciprocal vortex lattice. The resonance condition between the Cherenkov cone and vortices¹⁴ corresponds to the condition $\omega(\vec{b}) = \vec{v}\vec{b}$ with $\omega(\vec{k}) \approx sk$, since typical reciprocal vectors b of the vortex lattice are smaller than the size of the Brillouin zone of the crystal lattice (s is the sound velocity).

Another kind of resonance effect can be expected at big reciprocal vectors \vec{b} , which satisfy the condition of phase velocity $\omega(\vec{b}) = \vec{v}\vec{b}$ and group velocity $\partial\omega(\vec{b})/\partial\vec{b} = \vec{v}$. This condition can be met, in the vicinity of zero-wave vector, at some finite points \vec{k}_0 of the Brillouin zone of the crystal lattice and at certain values of the velocity \vec{v} . Close to these points $(\omega(\vec{k}) - \vec{v}\vec{k})^{-1} \sim 1/(\vec{k} - \vec{k}_0)^2$ and the two dimensional k integration results in a singularity. At the scale of the Brillouin zone one can neglect the discreetness of \vec{b} and treat it as a continuous vector \vec{k} . The matching condition $\partial\omega(\vec{k})/\partial\vec{k} = \vec{v}$ occurs at velocities v less than s [a conventional van Hove singularity satisfies the condition $\partial\omega(\vec{k})/\partial\vec{k} = 0$].

As can be seen, there are two effects that produce reso-

nant features in the I - V curve. The first is due to the emission of shock waves at supersonic vortex velocities¹⁴ and the second is by an enhancement of vortex radiation friction at velocities satisfying the van Hove condition. In the latter case a vortex does not produce a Cherenkov cone, but changes the character of the lattice velocity field in its vicinity (from an exponential decay to a power law as a function of the distance from the vortex) when the van Hove condition is satisfied. At high vortex velocities the moving lattice is relatively perfect due to dynamic crystallization.^{16–18} Whereas the Cherenkov-resonance phenomenon is sensitive to vortex lattice perfection, the van Hove structure in the I - V curve is not. Mathematically speaking the Cherenkov effect requires a discrete summation over \vec{b} , but the van Hove effect can have a continuous \vec{k} integration. The goal of this work is to study the nature of resonant features in the I - V curve of a superconductor arising from the interaction between van Hove singularities and moving vortices.

II. DYNAMICS OF THE CRYSTAL LATTICE

When vortices move in a superconductor with velocity \vec{v} under the action of a transport current, they act on the crystal lattice producing a displacement $\vec{u}(\vec{r}_n, t)$, where \vec{r}_n is the lattice site position. The lattice displacement obeys the equation

$$\left[\frac{\partial^2}{\partial t^2} + \gamma \frac{\partial}{\partial t} + \omega^2(\vec{k}) \right] \vec{u}(\vec{k}, t) = \frac{a^2}{\rho_0} \sum_n \vec{f}(\vec{r}_n - \vec{v}t) \exp(-i\vec{k}\vec{r}_n). \quad (1)$$

Here $\vec{f}(\vec{r})$ is the force on the lattice by vortices, the vector \vec{k} belongs to the two-dimensional Brillouin zone, which is perpendicular to the vortex direction. a^2 is the unit cell area of this zone and ρ_0 is the crystal mass density. The coefficient of sound attenuation can be written approximately as $\gamma \approx \alpha sk$, where $\alpha = s/v_F$ and $s = (\partial\omega/\partial k)_{k=0}$ is the sound velocity.¹⁹

The solution of Eq. (1) can be written in the form

$$\vec{u}(\vec{r}_n, t) = \frac{1}{\rho_0} \int \frac{d^2 k}{(2\pi)^2} \frac{\vec{f}(\vec{k}) \exp[i\vec{k}(\vec{r}_n - \vec{v}t)]}{\omega^2(\vec{k}) - (\vec{v}\vec{k})^2 - i\gamma\vec{v}\vec{k}}. \quad (2)$$

In Eq. (2) $\vec{f}(\vec{k})$ is the Fourier transformation of the force $\vec{f}(\vec{r})$ and the integration is extended over all \vec{k} space. The phonon frequency $\omega(\vec{k})$ corresponds to repeated zones, i.e., $\omega(\vec{k})$ is a periodic function in the reciprocal space.

III. DISSIPATION

The power dissipation density, produced by motion of the crystal lattice,

$$W = a^2 \sum_n \vec{f}(\vec{r}_n - \vec{v}t) \frac{\partial \vec{u}(\vec{r}_n, t)}{\partial t} \quad (3)$$

can be written in the form

$$W = \frac{1}{\rho_0} \text{Im} \int \frac{d^2 k}{(2\pi)^2} \frac{\vec{v}\vec{k}|\vec{f}(\vec{k})|^2}{\omega^2(\vec{k}) - (\vec{v}\vec{k})^2 - i\gamma\vec{v}\vec{k}}. \quad (4)$$

The force on the crystal lattice from vortices is

$$\vec{f}(\vec{r}) = \sum_i \vec{f}^{(0)}(\vec{r} - \vec{r}_i), \quad (5)$$

where $\vec{f}^{(0)}(\vec{r} - \vec{r}_i)$ is the force the individual vortex positioned at the point \vec{r}_i . In the Fourier representation

$$\vec{f}(\vec{k}) = \frac{(2\pi)^2}{a_0^2} \sum_b \vec{f}^{(0)}(\vec{b}) \delta(\vec{k} - \vec{b}), \quad (6)$$

where $a_0^2 = \Phi_0/B$ is the area of the unit cell of the vortex lattice, which is expressed through the magnetic induction B , and \vec{b} is the reciprocal vector of this lattice. Insertion of the relation (6) into Eq. (4) results in the following dissipation $w = W/S$ per unit area (S is the total area of the sample in the plane perpendicular to the vortex direction)

$$w = \frac{1}{\rho_0 a_0^4} \text{Im} \sum_b \frac{\vec{v}\vec{b}|\vec{f}^{(0)}(\vec{b})|^2}{\omega^2(\vec{b}) - (\vec{v}\vec{b})^2 - i\gamma\vec{v}\vec{b}}. \quad (7)$$

As in Ref. 14, one can approximate the force $\vec{f}^{(0)} = en\vec{E}$, where n is the electron density and \vec{E} is the electric field generated by a moving vortex. We ignore the Lorentz force, since it does not contribute to dissipation and we omit the drag force on the crystal lattice due to impurities.^{20–22} We also neglect the drag force on the crystal lattice due to pinning¹ since we are concerned with vortex motion in the free-flux-flow limit. The electric field decreases with distance as $1/r^2$ far from the vortex core together with some exponential contribution proportional to $\exp(-r/l_E)$, where l_E is the screening length of the longitudinal electric field (l_E is much less than the London penetration depth).^{23,13} At r less than l_E the exponential contribution becomes big and compensates the $1/r^2$ -singularity. This leads to a Fourier representation of $E(k)$, which can be written to leading approximation as $E(k) \approx (v\Phi_0/c)(1+k^2l_E^2)^{-1}$. It is not easy to accurately estimate l_E in high-temperature superconductors, so we will assume it to be of the order of the core size ξ as in the Bardeen-Stephen approximation.¹² As a result, one can write

$$|\vec{f}^{(0)}(\vec{b})|^2 \approx \left(\frac{env\Phi_0}{c} \right)^2 \frac{1}{(1+b^2\xi^2)^2}. \quad (8)$$

IV. CURRENT-VOLTAGE CHARACTERISTIC

The total dissipation

$$\vec{j}\vec{E} = \vec{j}_0\vec{E} + w \quad (9)$$

has an additional part w due to lattice displacement. \vec{j}_0 is the current density at the fixed \vec{E} due to the bare vortex dissipation.

Using the relation

$$v = c \frac{E}{B}, \quad (10)$$

the current-voltage characteristic can be cast in the form

$$j = j_0 + en \frac{\hbar}{M\xi} \Phi(\vec{v}), \quad (11)$$

where $M = \rho_0/n$ is the average ion mass and

$$\Phi(\vec{v}) = \frac{\pi\xi v}{a_0^2} \operatorname{Im} \sum_b \frac{\vec{v}\vec{k}}{[\omega^2(\vec{k}) - (\vec{v}\vec{k})^2 - i\gamma\vec{v}\vec{k}] (1 + b^2\xi^2)^2}. \quad (12)$$

The current \vec{j} is directed along \vec{E} and the velocity \vec{v} is perpendicular to \vec{E} . Equations (11) and (12) now generalize the analogous equations (23) and (16) of Ref. 14 to the case of an arbitrary phonon spectrum $\omega(\vec{k})$.

V. VAN HOVE SINGULARITIES

A linear approximation of the phonon spectrum works well, when the vortex velocity v is close to the sound velocity s , as in the Cherenkov effect.¹⁴ The discrete structure of the vortex system [summation of discrete wave vectors in Eq. (12)] results in an oscillatory dissipation function of the vortex velocity as the Cherenkov cone coincides with certain directions in the vortex lattice. The Cherenkov dissipation was the subject of Ref. 14. In addition to this, peaks in dissipation can occur whenever the vortex velocity matches the group velocity of phonons close to a van Hove singularity. We now consider this situation in detail.

The main contribution to the sum in Eq. (12) comes from vectors \vec{b} , that are large and comparable to the size $1/a$ of the Brillouin zone of the crystal lattice. This enables us to go over from summation to integration in Eq. (12) and, as the attenuation γ is small, one can retain only the pole part in the denominator

$$\Phi(\vec{v}) = \frac{\pi}{2} \xi v \operatorname{Im} \int \frac{d^2k}{(2\pi)^2} \frac{1}{\Omega(\vec{k}) - i\gamma/2} \frac{1}{(1 + k^2\xi^2)^2}, \quad (13)$$

where

$$\Omega(\vec{k}) = \omega(k_x, k_y, 0) - \vec{v}\vec{k}, \quad \vec{k} = \{k_x, k_y\}. \quad (14)$$

The van Hove singularities of the function $\Omega(\vec{k})$ at points \vec{k}_0 are determined by the coincidence of \vec{v} and the group velocity

$$\frac{\partial \Omega(\vec{k})}{\partial \vec{k}} = 0 \quad (15)$$

and differ from positions of conventional van Hove singularities of the phonon spectrum given by the equation $\partial\omega(\vec{k})/\partial\vec{k}=0$. If in addition to the condition (15), \vec{v} equals the phase velocity, then we have

$$\Omega(\vec{k}) = 0 \quad (16)$$

and the integral in Eq. (14) becomes most singular since Ω is proportional to $(k - k_0)^2$. Now the condition of using integration instead of summation in Eq. (13) can be written explicitly. Close to the van Hove singularity $k_0 \sim 1/a$ one can estimate $\Omega \sim sa(k - k_0)^2 \sim \gamma$, where $\gamma \sim sk_0 s/v_F$. The close discreetness in k $2sa(k - k_0)\delta k \ll \gamma$, where $\delta k \sim a_0^{-1}$, is equivalent to the condition

$$\frac{a}{a_0} \ll \sqrt{\frac{s}{v_F}}. \quad (17)$$

Since a is typically a few Angstroms, $s/v_F \sim 10^{-2}$, and $a_0 \simeq 400$ Å at $B = 1$ T, the inequality (17) holds for the magnetic fields of a few Tesla typically used in experiments. For each vortex velocity \vec{v} the condition (15) determines the two-dimensional vector \vec{k}_0 . Then Eq. (16) can be considered to be a condition for $|\vec{v}|$ (for a fixed direction of \vec{v}) to meet the singularity of integration in Eq. (13), resulting in a singularity of the current-voltage characteristic (11).

VI. A MODEL PHONON SPECTRUM

To obtain a quantitative result, one must specify a detailed phonon spectrum. We defer considering the complicated phonon spectra of high- T_c materials to future investigation. Here we demonstrate the effect of the van Hove singularities using a simpler phonon spectrum

$$\omega(k_x, k_y, 0) = \frac{2s}{a} \sqrt{\sin^2 \frac{ak_x}{2} + \sin^2 \frac{ak_y}{2}}. \quad (18)$$

The Brillouin zone is plotted in Fig. 1, where $x = ak_x/2$, $y = ak_y/2$, the points a and b correspond to saddle points, and the local maximum is located at the point c . We consider further only the case of the velocity \vec{v} parallel to k_x . The saddle point a (and equivalent points) of the function $\omega(k_x, k_y, 0)$ generates the system of saddle points $\{x_{an}, \pi m\}$ on the $\{x, y\}$ plane, which are determined by the condition

$$x_{an} = \tan x_{an} \quad (19)$$

and close to these points the function Ω has the form

$$\Omega(x, y) = \frac{2s}{a} \sum_{n,m} \left[-|x_{an}| \frac{|v| - |v_{an}|}{s} + \frac{(y - \pi m)^2 - (x - x_{an})^2}{2|\sin x_{an}|} \right], \quad (20)$$

where v is supposed to be close to

$$v_{an} = s \cos x_{an} \operatorname{sgn}(\sin x_{an}). \quad (21)$$

As follows from Eq. (13),

$$\Phi(v) = \frac{\xi v}{2a^2} \int \frac{dxdy}{[1 + 4\xi^2(x^2 + y^2)/a^2]^2} \delta[\Omega(x, y)]. \quad (22)$$

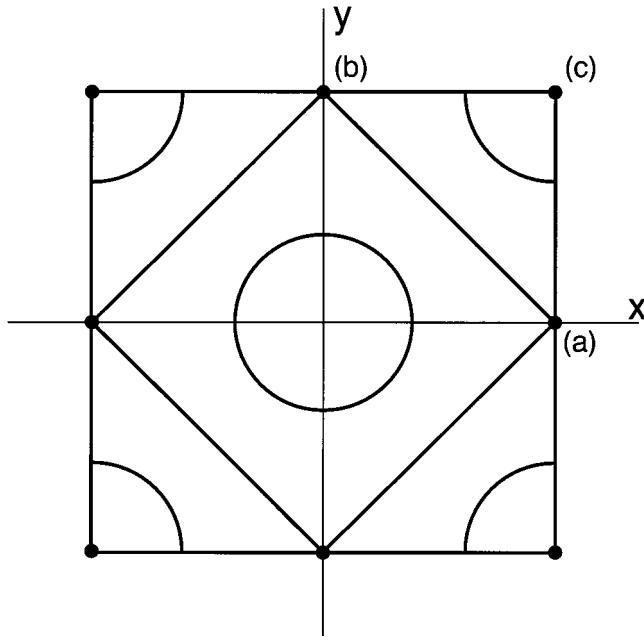


FIG. 1. The sketch of the Brillouin zone of a quadratic lattice, where $x=ak_x/2$ and $y=ak_y/2$. The lines denote constant value of the phonon frequency $\omega(k_x, k_y, 0)$. Vortices are aligned in the z direction.

The total singular part of $\Phi(v)$ can be written as a sum of contributions of points a , b , and c (and equivalent ones) of the Brillouin zone in Fig. 1

$$\Phi(v)=\Phi_a(v)+\Phi_b(v)+\Phi_c(v). \quad (23)$$

With use of the formula

$$\int dxdy \delta(x^2-y^2+A)=\ln \frac{1}{A}, \quad (24)$$

which holds with logarithmic accuracy since the integral diverges for big arguments, Eqs. (22) and (23) yield

$$\Phi_a(v)=\frac{\xi}{4a} \sum_{nm} \frac{|\sin 2x_{an}|}{[1+4\xi^2(x_{an}^2+\pi^2m^2)/a^2]^2} \ln \frac{s}{|v|-|v_{an}|}. \quad (25)$$

Since the interatomic distance is small compared to the coherence length (even in high- T_c superconductors) one can neglect the “one” in the denominator of Eq. (25). The m -summation can be done easily and Eq. (25) takes the form

$$\begin{aligned} \Phi_a(v)= & \frac{a^3}{16\xi^3} \sum_n |\sin 2x_{an}| \\ & \times \frac{\sinh 2x_{an}+2x_{an}}{(2x_{an})^3(\cosh 2x_{an}-1)} \ln \frac{s}{|v|-|v_{an}|}. \end{aligned} \quad (26)$$

Equation (26), together with Eqs. (19), (21), and (11), gives singular parts of the current-voltage characteristic generated by points of the type a (see Fig. 1) of the Brillouin zone. The

first two solutions of Eqs. (19) and (20) $x_{a1}\simeq 1.43\pi$, $v_{a1}\simeq 0.21s$ and $x_{a2}\simeq 2.46$, $v_{a2}\simeq 0.13s$ give the first two terms in $\Phi(v)$

$$\begin{aligned} \Phi_a(v)\simeq & 10^{-4} \frac{a^3}{\xi^3} \left(0.36 \ln \frac{s}{|v|-0.21s} \right. \\ & \left. + 0.042 \ln \frac{s}{|v|-0.13s} \right). \end{aligned} \quad (27)$$

The singular part of the function $\Omega(x,y)$ generated by points b and c (see Fig. 1) has the form

$$\begin{aligned} \Omega(x,y)= & \frac{2s}{a} \sum_{nm} \left[-|x_n| \frac{|v|-|v_n|}{s} - \frac{(x-x_n)^2}{2\sqrt{1+\sin^2 x_n}} \right. \\ & \left. + \frac{(y-\pi/2+\pi n)^2(3 \cos 2x_n - 1)}{4(1+\sin^2 x_n)^{3/2}} \right], \end{aligned} \quad (28)$$

where x_n and v_n satisfy the equations

$$3-\cos 2x_n=x_n \sin 2x_n \quad \text{and} \quad \frac{v_n}{s}=\frac{\sin 2x_n}{2\sqrt{1+\sin^2 x_n}}. \quad (29)$$

The saddle point b corresponds to $(3 \cos 2x_n - 1) > 1$ and for the point c this inequality is opposite. Repeating the same steps that resulted in Eq. (26) and using the formula

$$\int dxdy \delta(x^2+y^2-A)=\pi \theta(A), \quad (30)$$

one can obtain

$$\begin{aligned} \Phi_{b,c}(v)= & \frac{a^3}{16\xi^3} \sum_n |\sin 2x_n| \frac{\sinh 2x_n - 2x_n}{(2x_n)^3(\cosh 2x_n + 1)} \\ & \times \sqrt{\frac{3-\cos 2x_n}{|3 \cos 2x_n - 1|}} \begin{cases} \ln(s/|v-v_n|) & 1 < 3 \cos 2x_n \\ \pi \theta(v_n^2 - v^2) & 3 \cos 2x_n < 1 \end{cases} \end{aligned} \quad (31)$$

For the point b the first two solutions of Eqs. (29) are $x_1\simeq 1.11\pi$, $v_1\simeq 0.30s$, and $x_3\simeq 2.05\pi$, $v_3\simeq 0.22s$. For the point c $x_2\simeq 1.34\pi$, $v_2\simeq 0.32s$ and $x_4\simeq 2.41\pi$, $v_4\simeq 0.26s$. It follows from Eq. (31) that

$$\Phi_b(v)=10^{-4} \frac{a^3}{\xi^3} \left[1.45 \ln \frac{s}{|v|-0.36s} + 0.095 \ln \frac{s}{|v|-0.22s} \right] \quad (32)$$

and

$$\Phi_c(v)=10^{-4} \frac{a^3}{\xi^3} [3.17\theta(0.32s-|v|) + 0.30\theta(0.26s-|v|)]. \quad (33)$$

Equations (27), (32), (23), and (10) should be inserted into Eq. (11) for the current-voltage characteristic

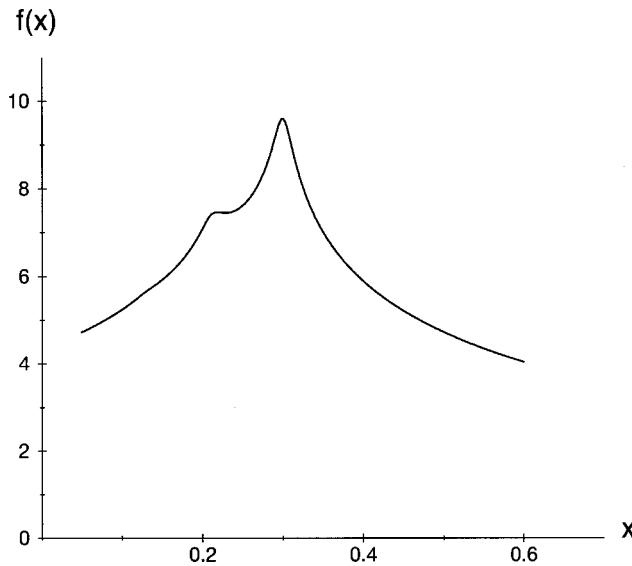


FIG. 2. The plot of the function $f(x)$, which determines the current-voltage characteristic according to Eq. (34), where $x = cE/sB$. The main peak corresponds to the van Hove singularity (b) at $x=0.30$ and the pre-peak at $x=0.21$ is due to the singularity (a). The singularity (c) at $x=0.32$ provides the rapid fall of the right-hand-side part of the curve.

$$j(E) = j_0(E) + en\left(\frac{\hbar}{M\xi}\right) 10^{-4} \left(\frac{a}{\xi}\right)^3 f\left(\frac{cE}{sB}\right), \quad (34)$$

where the function $f(v/s) = 10^4(\xi/a)^3\Phi(v)$ is plotted in Fig. 2, where the finite width of the functions $\ln x$ and $\theta(x)$ of the order of $x \sim 10^{-2}$ has been accounted for. Smearing out of singularities of $\Phi(v)$ is due to the finite sound attenuation γ in Eq. (14), which results in the finite width of singularities of the order of $(v - v_n)/s \sim s/v \sim 10^{-2}$. Imperfections in the vortex lattice do not lead to a broadening of the singularities in $\Phi(v)$. As one can see from Eqs. (27), (32), and (33), the van Hove singularities occur at velocities $0.32s$ and lower. Other terms in Eqs. (27), (32), and (33) result only in smaller corrections at lower velocities and can be neglected. At higher velocities $v > s$ there are Cherenkov singularities in the current-voltage characteristic due to a matching of the Cherenkov cone and directions of the vortex lattice.¹⁴ The Cherenkov peaks in the current density were estimated to be of the order of 10^7 A/cm². The van Hove singularities in the

I - V curve are significantly smaller and of the order of 10 A/cm², thereby requiring very high measurement sensitivity for their experimental observation. On the other hand, the van Hove effect is not sensitive to imperfections in the vortex lattice in contrast to the Cherenkov effect. As we have reported earlier,²⁴ by using a precision pulsed-current technique to measure the extended I - V response of $Y_xBa_{1-x}Cu_yO$ films, we have observed preliminary evidence of features in the I - V curves that could be related to the interaction of moving vortices with lattice vibrations. This matter is presently under further careful experimental investigation.

An alternative way to study peculiarities of vortex-sound interaction, instead of measurement of the I - V curve, is the direct registration of emitted sound like in Ref. 2. The power of the generated acoustic waves per unit volume w is given, according to Eq. (9), by the expression

$$w = Een\left(\frac{\hbar}{M\xi}\right) 10^{-4} \left(\frac{a}{\xi}\right)^3 f\left(\frac{cE}{sB}\right). \quad (35)$$

The maximum of the emitted acoustic power correspond approximately to the maximum position in Fig. 2.

VII. CONCLUSION

In conclusion, peculiarities in vortex radiation friction due to the emission of sound waves result in resonant effects in the I - V curve and an enhancement of the acoustic power radiated at certain voltages across a sample. These effects can be divided into two groups. The first one is the Cherenkov effect when the vortex velocity is close to the sound speed and vortices generate shock waves that match the vortex lattice resulting in resonances.¹⁴ The second one is the van Hove effect, when the emission of sound waves by vortices peaks at vortex velocities that match the phase and group velocities of phonons near van Hove singularities. The van Hove effect occurs at vortex velocities lower than the sound speed and is not accompanied by shock waves, but only the character of the velocity field of the lattice becomes different upon crossing the van Hove condition.

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