

Measurement of Generalized Form Factors near the Pion Threshold In high momentum transfer square

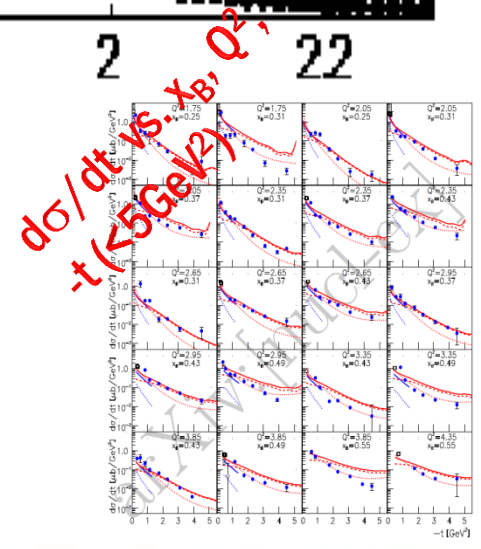
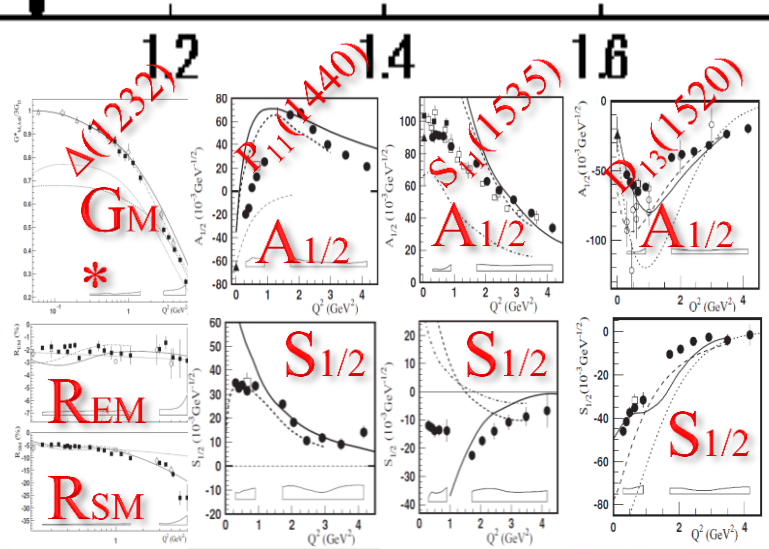
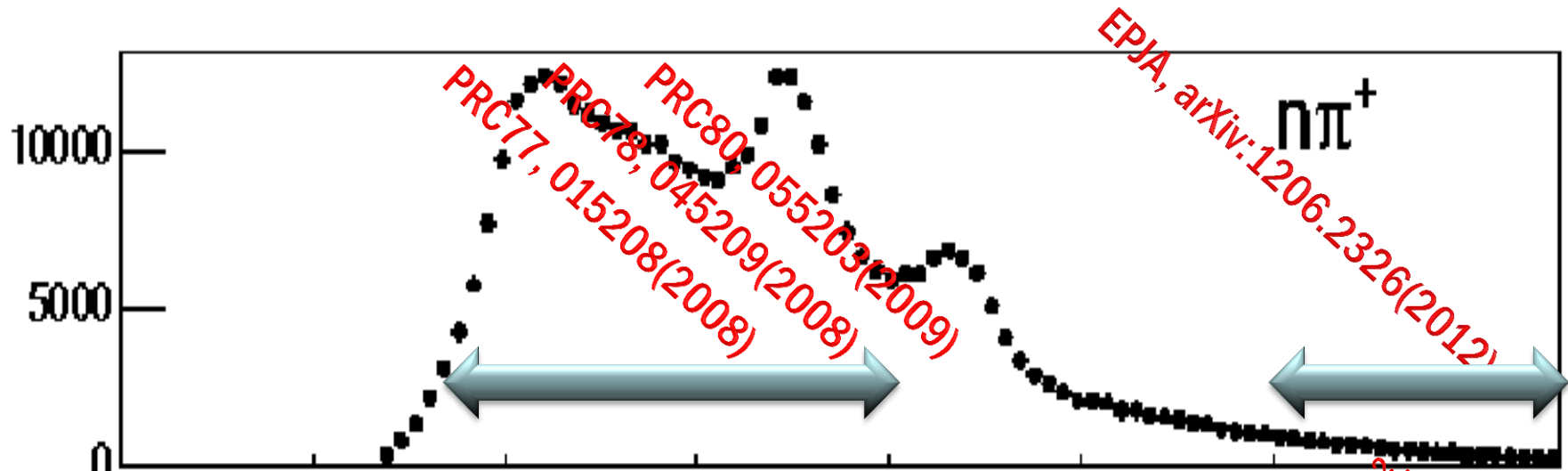
Aug. 13 - 15, 2012
 γ^*NN^* Workshop at USC



Exclusive single positively charged pion electroproduction off the proton

$Q^2 < 5.0 \text{ GeV}^2$

from CLAS

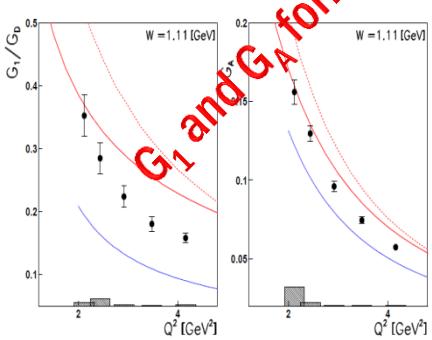
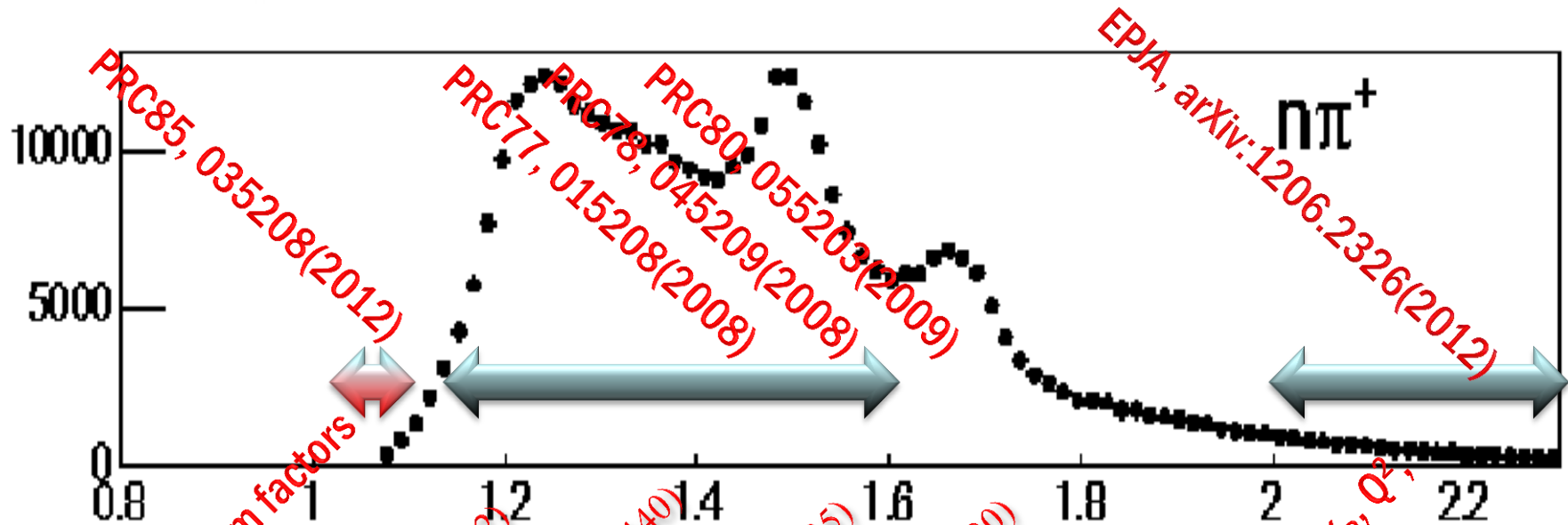




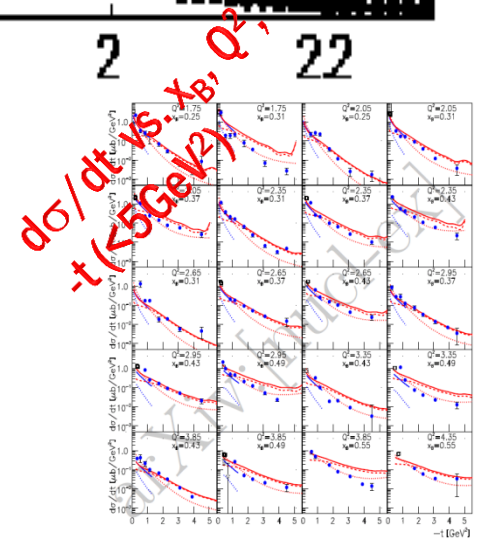
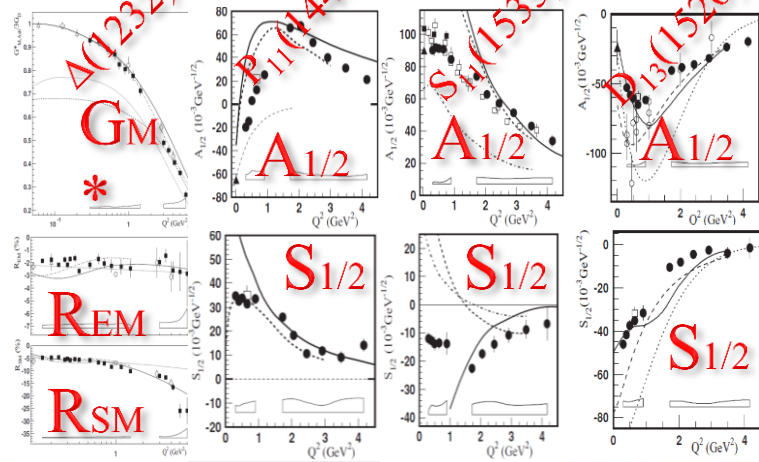
Exclusive single positively charged pion electroproduction off the proton

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from CLAS



G_1 and G_A form factors



$\frac{d^2\sigma}{dt^2}$ vs. $X_B' Q^2$

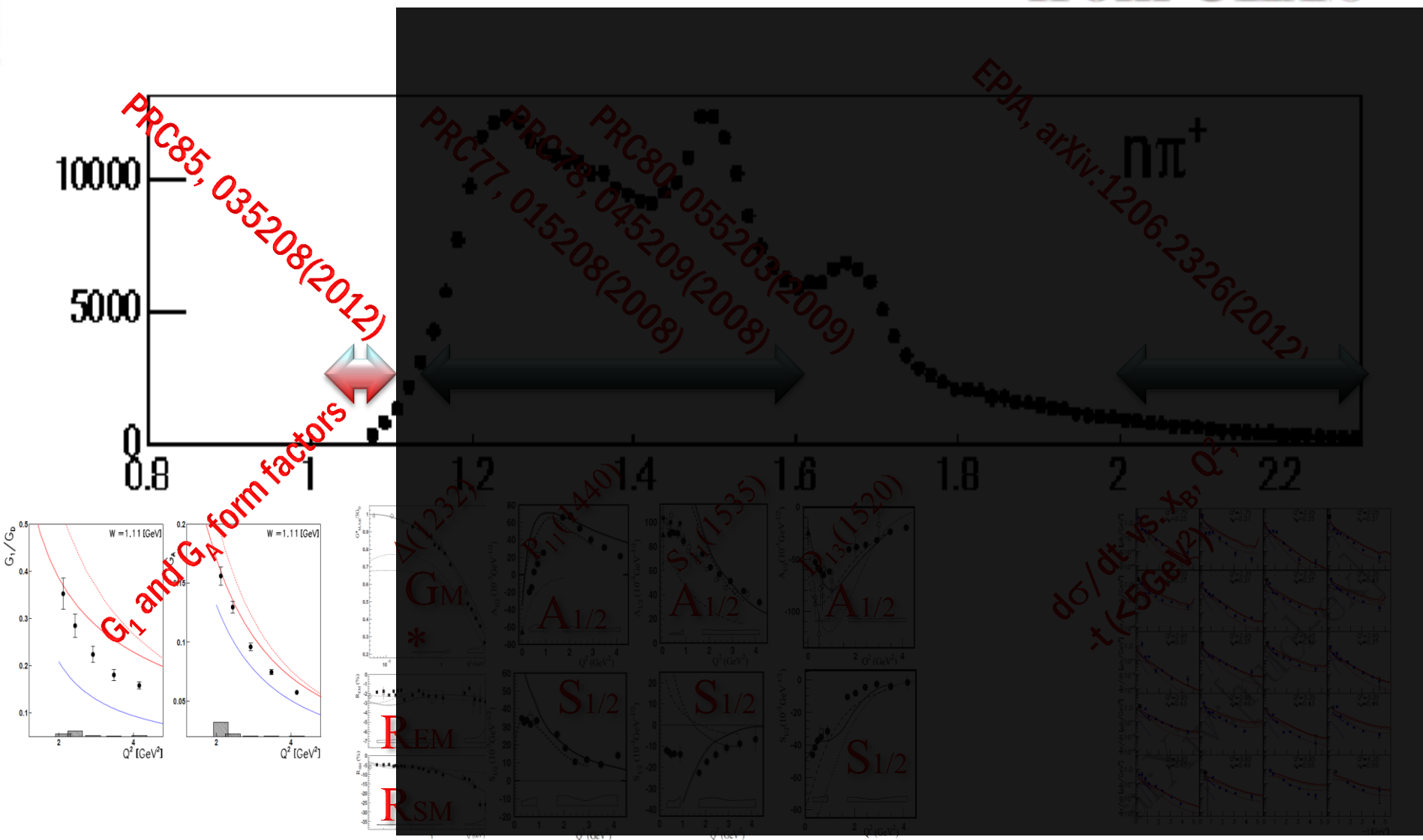




Exclusive single positively charged pion electroproduction off the proton

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from CLAS





- **Historically, threshold pion in the photo- and electroproduction is the very old subject that has been receiving continuous attention from both experiment and theory sides for many years.**
- **Pion mass vanishing approximation in Chiral Symmetry allows us to make an exact prediction for threshold cross section known as LET**
- **The LET established the connection between charged pion electroproduction and axial form factor in nucleon.**
- **Therefore, It is very interesting to extracting Axial Form Factor which is dominated by S- wave transverse multipole E_{0+} in LCSR**





Perspective of soft pion in terms of Q^2 at threshold

$Q^2=0 \text{ GeV}^2$

Low-Energy Theorem (LET) for $Q^2=0$

1954
Kroll-Ruderman

Restriction to the charged pion

Chiral symmetry + current algebra for electroproduction

1960s
Nambu, Laurie, Schrauner

$Q^2 \ll \Lambda/m_\pi \sim 1 \text{ GeV}^2$

Re-derived LETs

1970s
Vainshtein, Zakharov

Current algebra + PCAC

Chiral perturbation theory

1990s
Scherer, Koch

$Q^2 \sim 1 - 10 \text{ GeV}^2$

???

$Q^2 \gg \Lambda/m_\pi$

pQCD factorization methods

Brodsky, Lepage, Efremov, Radyunshkin, Poblitsa, Polyakov, Strikman, et al





LCSR (Light Cone Sum Rule)

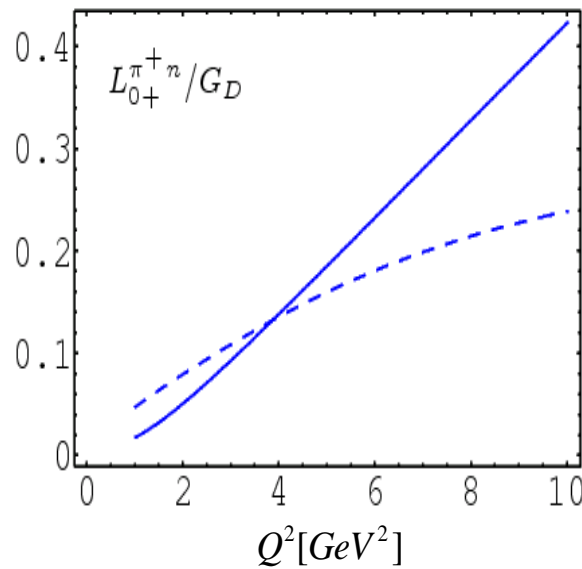
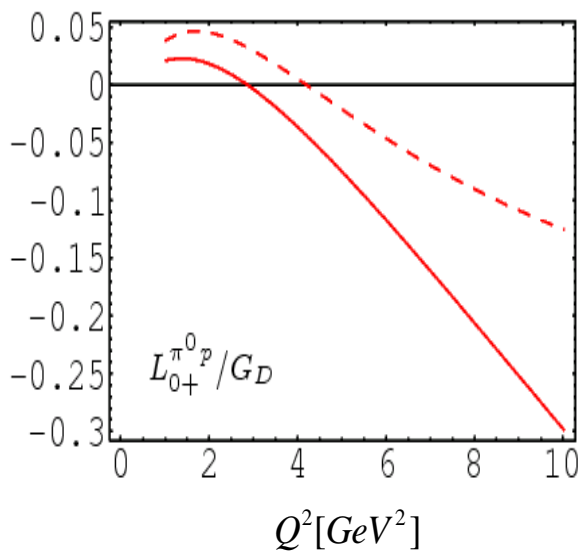
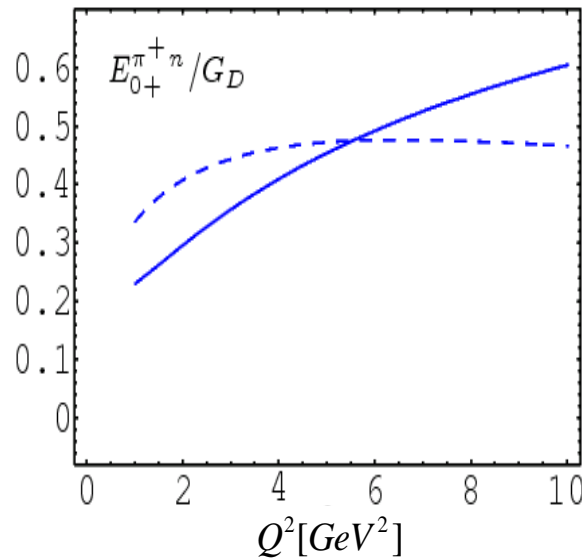
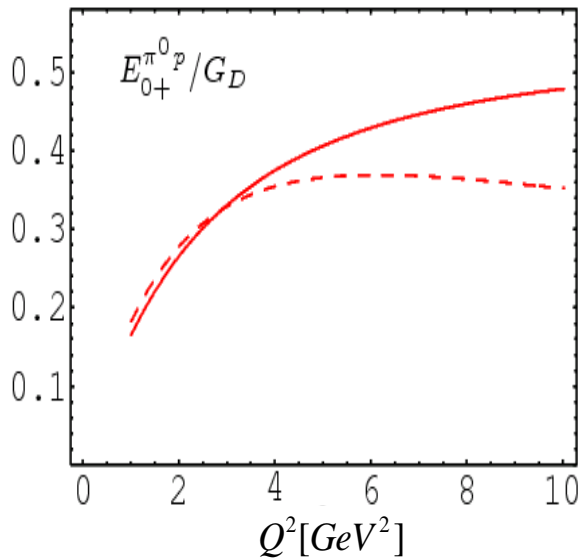
$$\langle N(P')\pi(k)|j_{\mu}^{\text{em}}(0)|p(P)\rangle = -\frac{i}{f_{\pi}}\bar{N}(P')\gamma_5\left\{(\gamma_{\mu}q^2 - q_{\mu}\not{q})\frac{1}{m_N^2}G_1^{\pi N}(Q^2) - \frac{i\sigma_{\mu\nu}q^{\nu}}{2m_N}G_2^{\pi N}(Q^2)\right\}p(P) + \frac{ic_{\pi}g_A}{2f_{\pi}[(P'+k)^2 - m_N^2]}\bar{N}(P')\not{k}\gamma_5(P'+m_N)\left\{F_1^p(Q^2)\left(\gamma_{\mu} - \frac{q_{\mu}\not{q}}{q^2}\right) + \frac{i\sigma_{\mu\nu}q^{\nu}}{2m_N}F_2^p(Q^2)\right\}p(P)$$

- S-wave: generalized form factors from LCSR ($G_1^{\pi N}$ and $G_2^{\pi N}$)
- P-wave: pion emission from final state nucleon

- **Constructed relating the amplitude for the radiative decay of Σ^+ ($p\gamma$) to properties of the QCD vacuum in alternating magnetic field.**
- **An advantage of study because soft contribution to hadron form factor can be calculated in terms of DA's that enter pQCD calculation without other non-perturbative parameters.**
- **New technique : the expansion of the standard QCD sum rule approach to hadron properties in alternating external fields.**



Prediction - LCSR



$$G_D(Q^2) = 1 / \left(1 + \frac{Q^2}{\mu_0^2}\right)^2$$

$$\mu_0^2 = 0.71 \text{ GeV}^2$$

symbol index

Dashed Lines :
pure LCSR

Solid Lines :
LCSR using
experimental
EM form factor
as input

V. M. Braun et al.,
Phys. Rev. D
77:034016, 2008.



Differential Cross-section

$$\frac{d^4\sigma}{dQ^2 dW d\Omega_\pi^*} = |J|\Gamma_v \frac{d^2\sigma_u}{d\Omega_\pi^*}$$

$$|J|\Gamma_v = \frac{\alpha}{2\pi^2 Q^2} \frac{(W^2 - M_p^2) E_f}{2M_p E_i (1 - \epsilon)}$$

$$\epsilon = \left[1 + 2 \left(1 + \frac{v^2}{Q^2} \right) \tan^2 \frac{\theta_e}{2} \right]^{-1}$$

$$\frac{d^2\sigma_u}{d\Omega_\pi^*} = \underbrace{\sigma_T + \epsilon\sigma_L}_{\uparrow} + \underbrace{\epsilon\sigma_{TT} \cos 2\phi_\pi^*}_{\uparrow} + \sqrt{2\epsilon(1+\epsilon)} \underbrace{\sigma_{LT} \cos \phi_\pi^*}_{\uparrow}$$



Differential Cross-section

$$\sigma_T \rightarrow G_1^{\pi N}, G_M^2$$

$$\sigma_{TT} \rightarrow 0$$

No D-wave contribution

$$\sigma_L \rightarrow G_2^{\pi N}, G_E^2$$

$$\sigma_{LT} \rightarrow \text{Re } G_1^{\pi N}, \text{Re } G_2^{\pi N}, G_E, G_M$$

$$\sigma'_{LT} \rightarrow \text{Im } G_1^{\pi N}, \text{Im } G_2^{\pi N}, G_E, G_M$$



Legendre moments vs. Form Factors

V. Braun PRD 77(2008)

$$D_0^{T+L} = \frac{1}{f_\pi^2} \left[\frac{4\vec{k}_i^2 Q^2}{m_N^2} |G_1^{n\pi^+}|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_N^2} Q^2 m_N^2 G_M^{n^2} \right. \\ \left. + \epsilon \left(\vec{k}_i^2 |G_2^{n\pi^+}|^2 + \frac{4c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_N^2} m_N^4 G_E^{n^2} \right) \right]$$

$$D_1^{T+L} = \frac{1}{f_\pi^2} \frac{4c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} \\ \times [Q^2 G_M^n \text{Re}(G_1^{n\pi^+}) - \epsilon m_N^2 G_E^n \text{Re}(G_2^{n\pi^+})]$$

$$D_0^{LT} = -\frac{1}{f_\pi^2} \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} \\ \times Q m_N [G_M^n \text{Re}(G_2^{n\pi^+}) + 4G_E^n \text{Re}(G_1^{n\pi^+})]$$

G_M^n and G_E^n
Sachs form factors

$c_\pi = \sqrt{2}$
isospin factor,

$f_\pi = 93$ MeV
pion decay constant

$g_A = 1.267$
axial coupling



Legendre moments vs. Form Factors

V. Braun PRD 77(2008)

assumption $m_\pi \sim 0$ $G_E^n \sim 0$

$$D_0^{T+L} = \frac{1}{f_\pi^2} \left[\frac{4\vec{k}_i^2 Q^2}{m_N^2} |G_1^{n\pi^+}|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_N^2} Q^2 m_N^2 G_M^n + \epsilon (\vec{k}_i^2 |G_2^{n\pi^+}|^2) \right]$$

$$D_1^{T+L} = \frac{1}{f_\pi^2} \frac{4c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} [Q^2 G_M^n \text{Re}(G_1^{n\pi^+})]$$

$$D_0^{LT} = -\frac{1}{f_\pi^2} \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} Q m_N [G_M^n \text{Re}(G_2^{n\pi^+})]$$

G_M^n and G_E^n
Sachs form factors

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Multipoles vs. F. F. for $n\pi^+$ channel

$$G_1^{n\pi^+} \quad E_{0+}^{n\pi^+}$$

$$\frac{E_{0+}^{n\pi^+}}{G_D} = \frac{\sqrt{4\pi\alpha_{em}}}{8\pi} \frac{Q^2 \sqrt{Q^2 + 4m_p^2}}{m_p^3 f_\pi} \frac{G_1^{n\pi^+}}{G_D}$$

$$G_D = 1/(1 + Q^2/\mu_0)^2, \mu_0 = 0.71$$

$$\frac{Q^2}{m_N^2} G_1^{n\pi^+} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{Q^2 + 2m_N^2} G_M^n + \frac{1}{\sqrt{2}} G_A$$

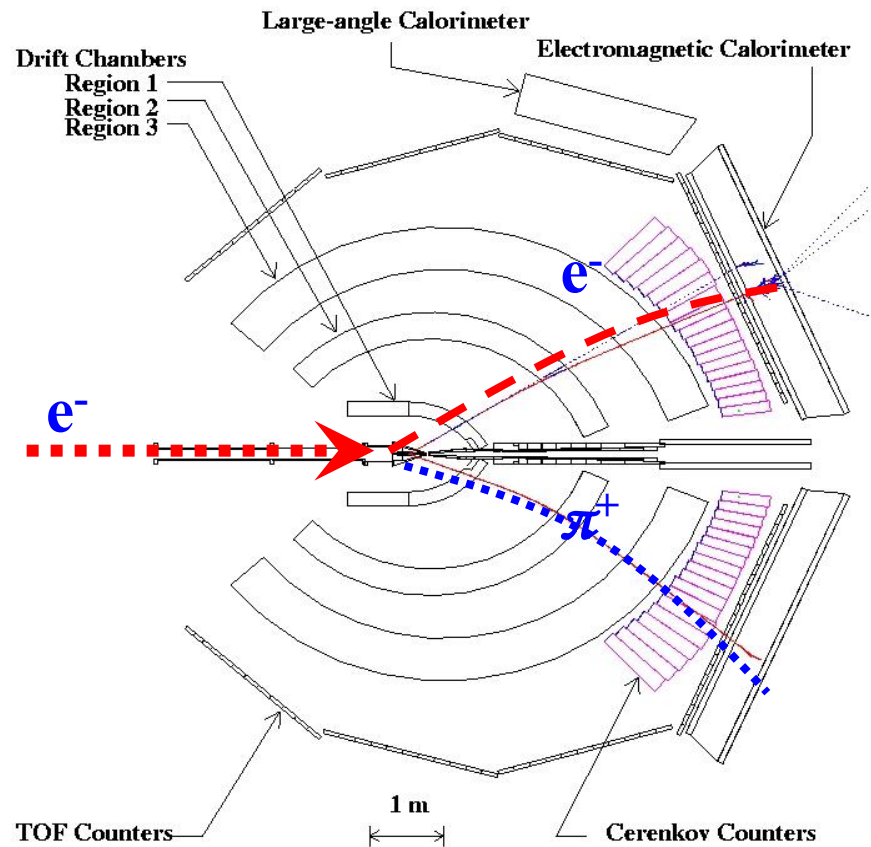
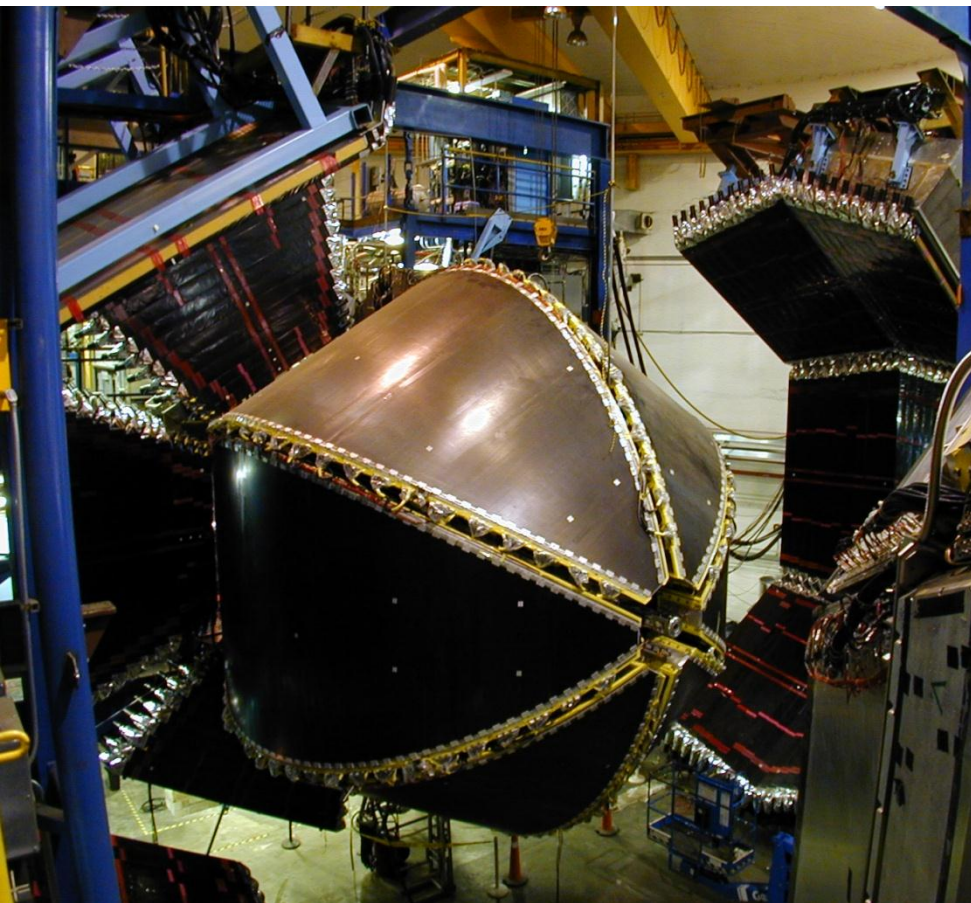
$$G_2^{n\pi^+} = \frac{2\sqrt{2}g_A m_N^2}{Q^2 + 2m_N^2} G_E^n$$

$$G_E^n \sim 0$$

$G_2^{n\pi^+}$ is negligible.



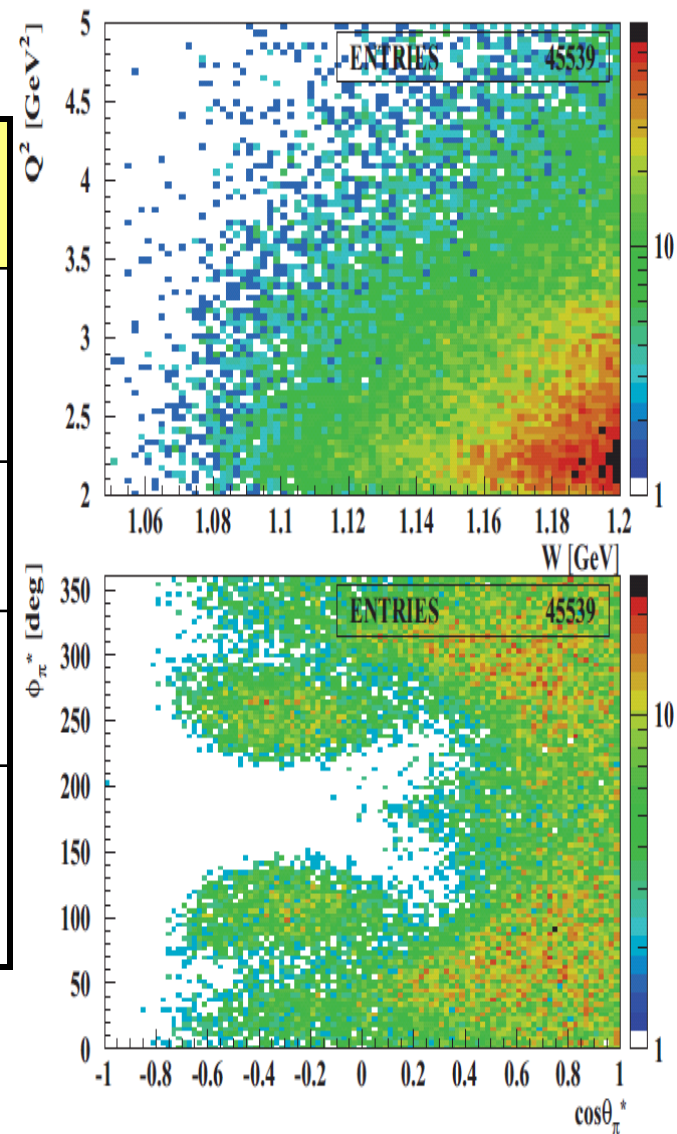
CLAS & Event display





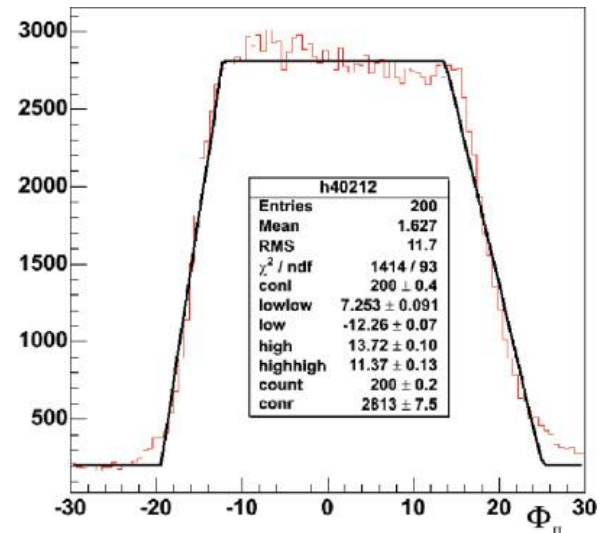
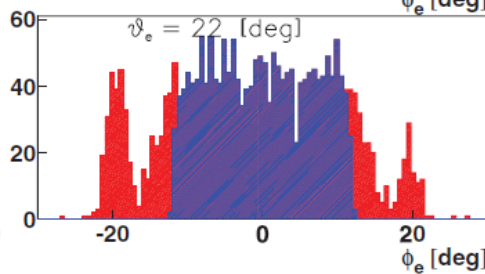
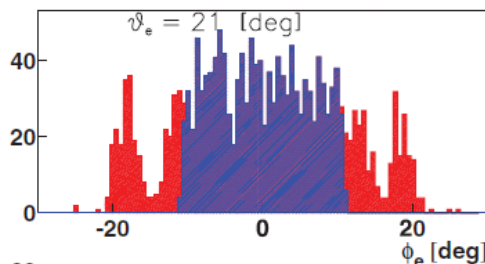
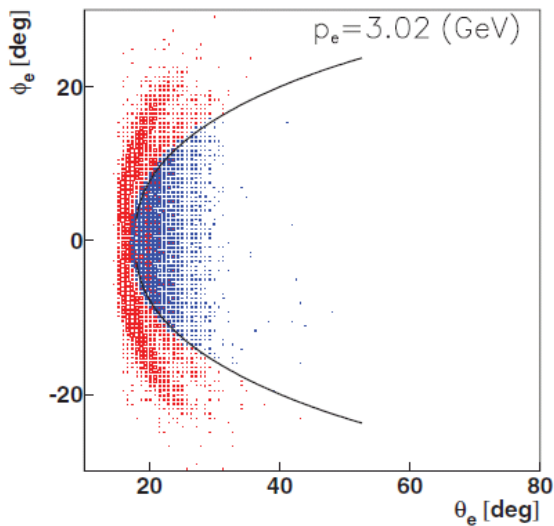
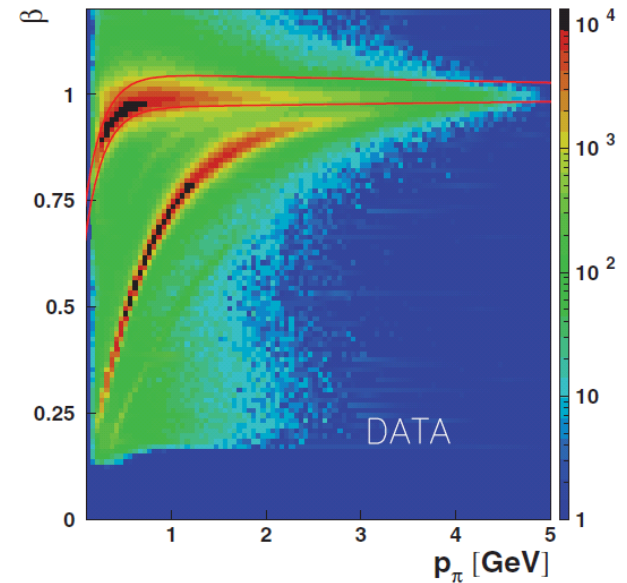
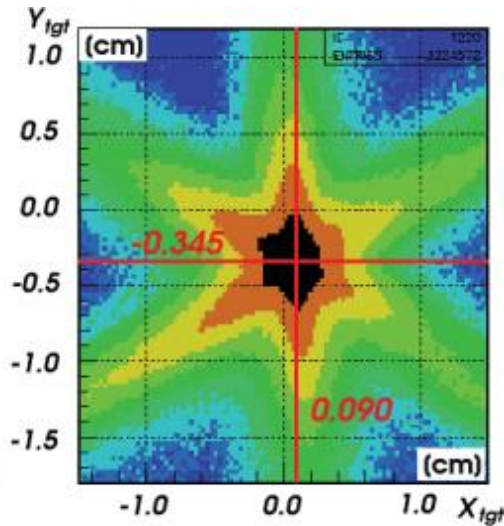
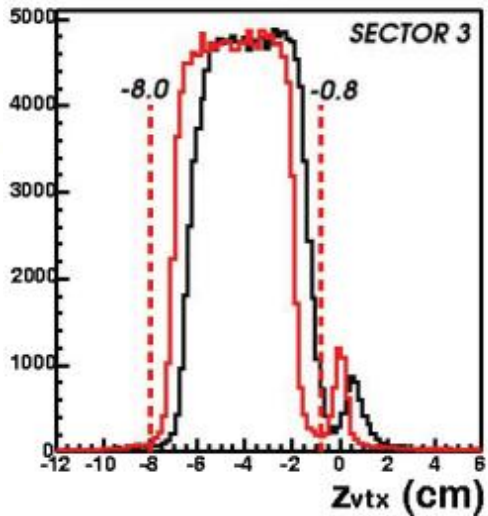
Kinematic regions

Variable	Unit	Range	# Bin	Width
Q^2	GeV_2^2	2.12 ~ 4.16	5	various
W	GeV	1.11 ~ 1.15	3	0.02
$\cos\theta_\pi^*$		-1.0 ~ 1.0	10	0.2
Φ_π^*	deg	0 ~ 360 0 ~ 360	12 6	$\cos\theta_\pi^* \geq -0.1$ $\cos\theta_\pi^* < -0.1$



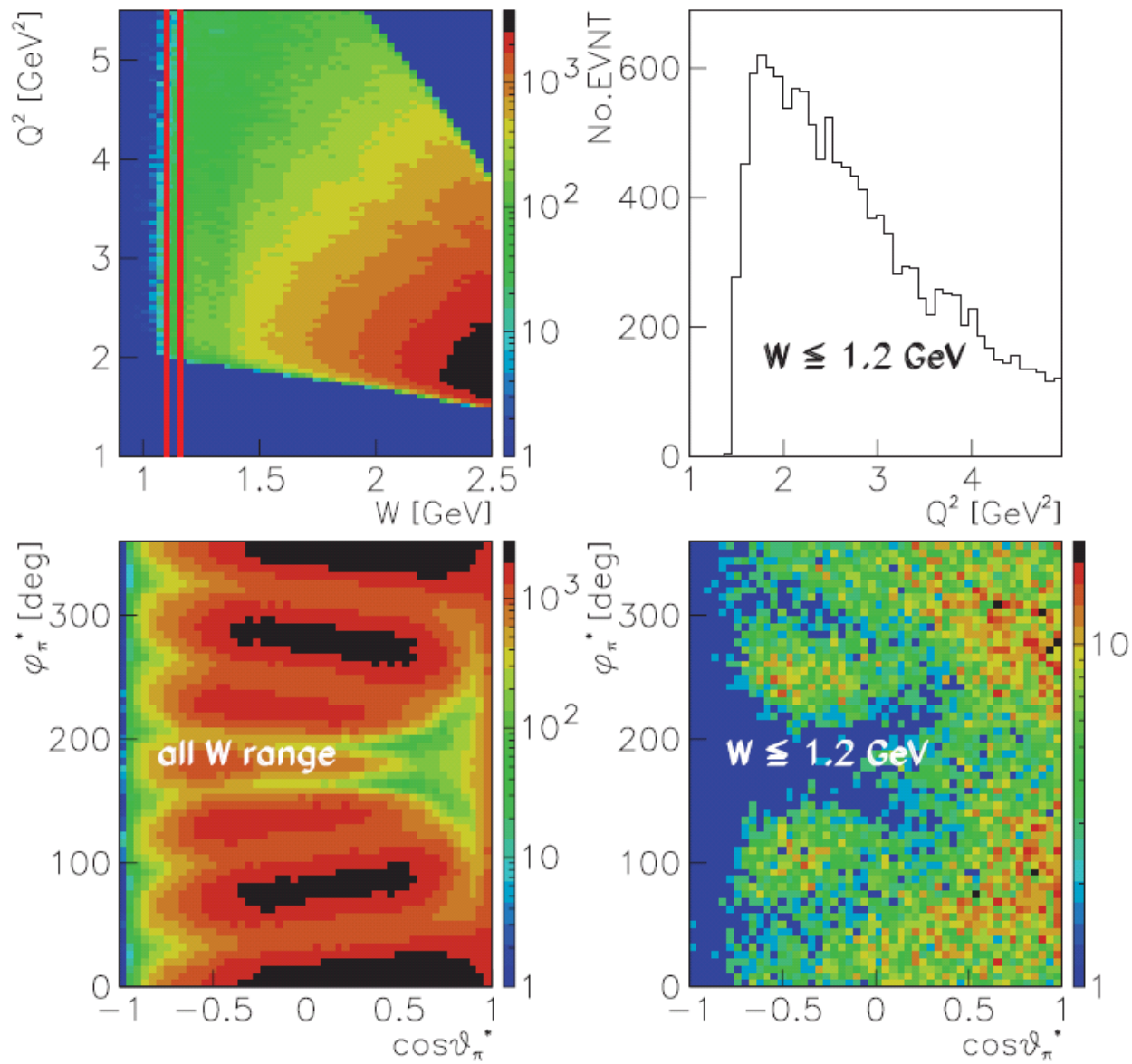


Snap shot of PID



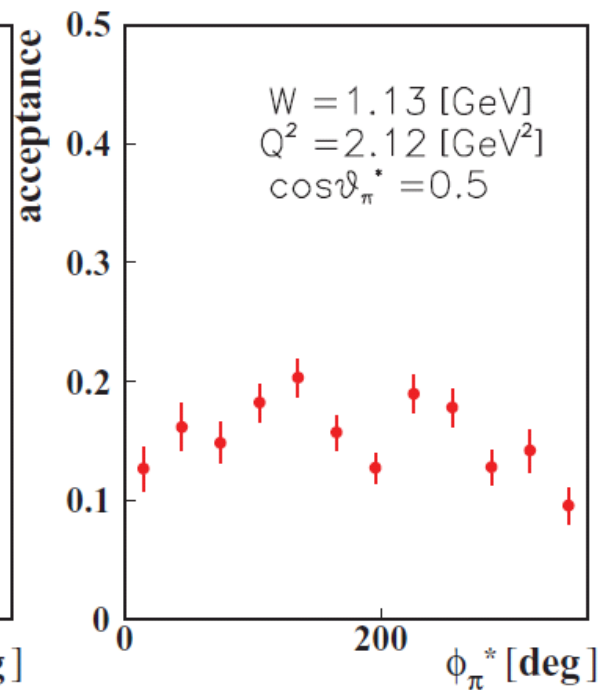
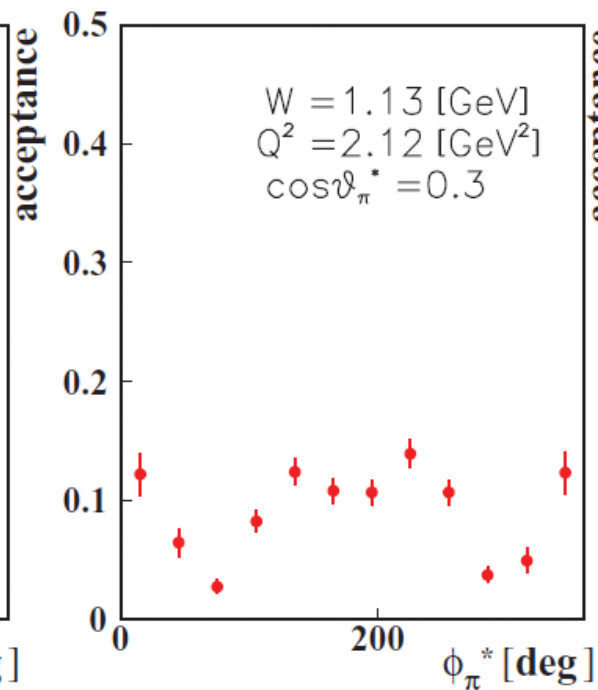
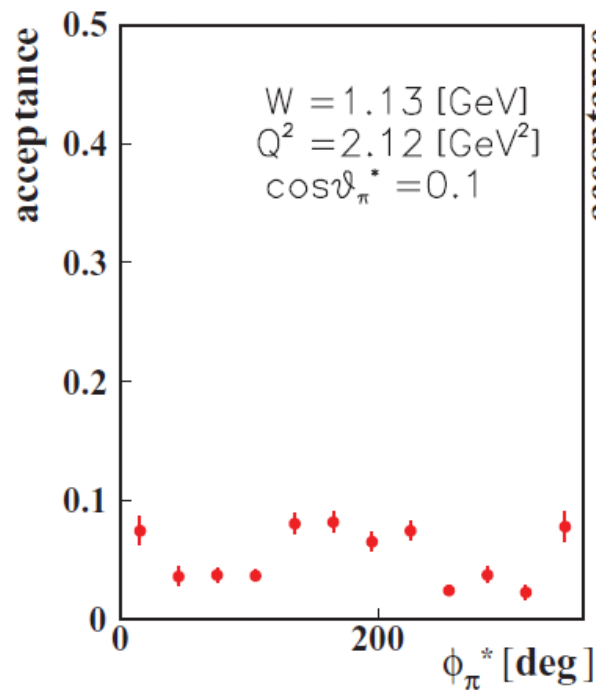


MC - GEANT3 Simulation



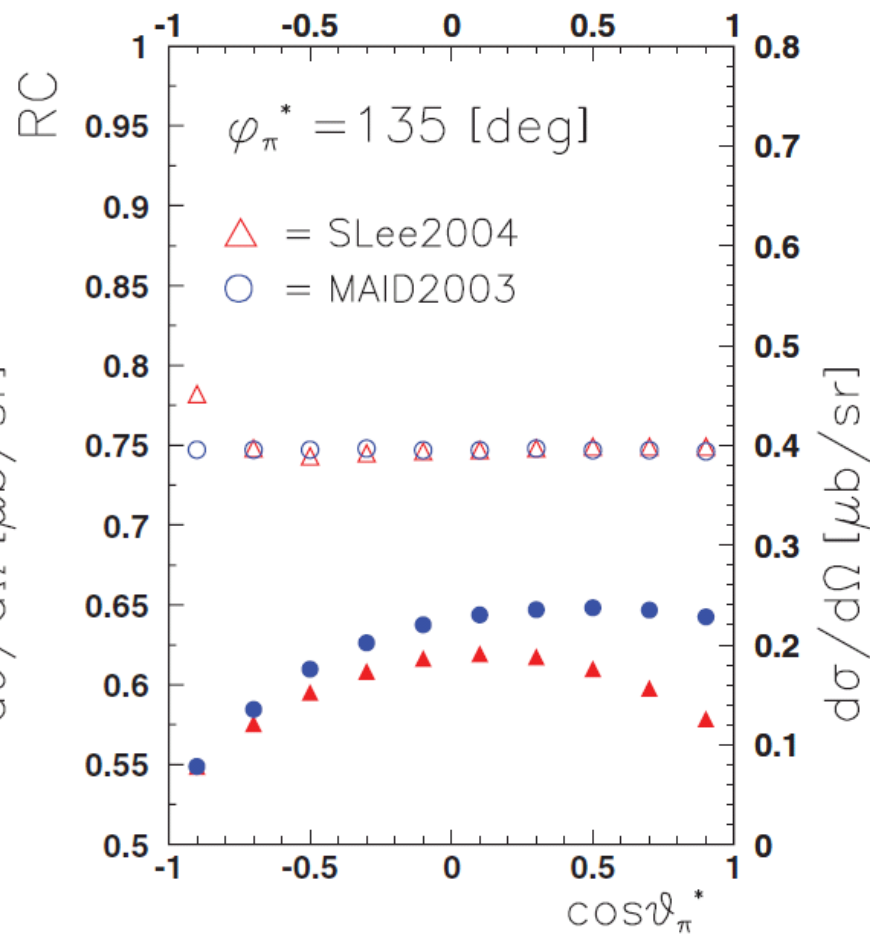
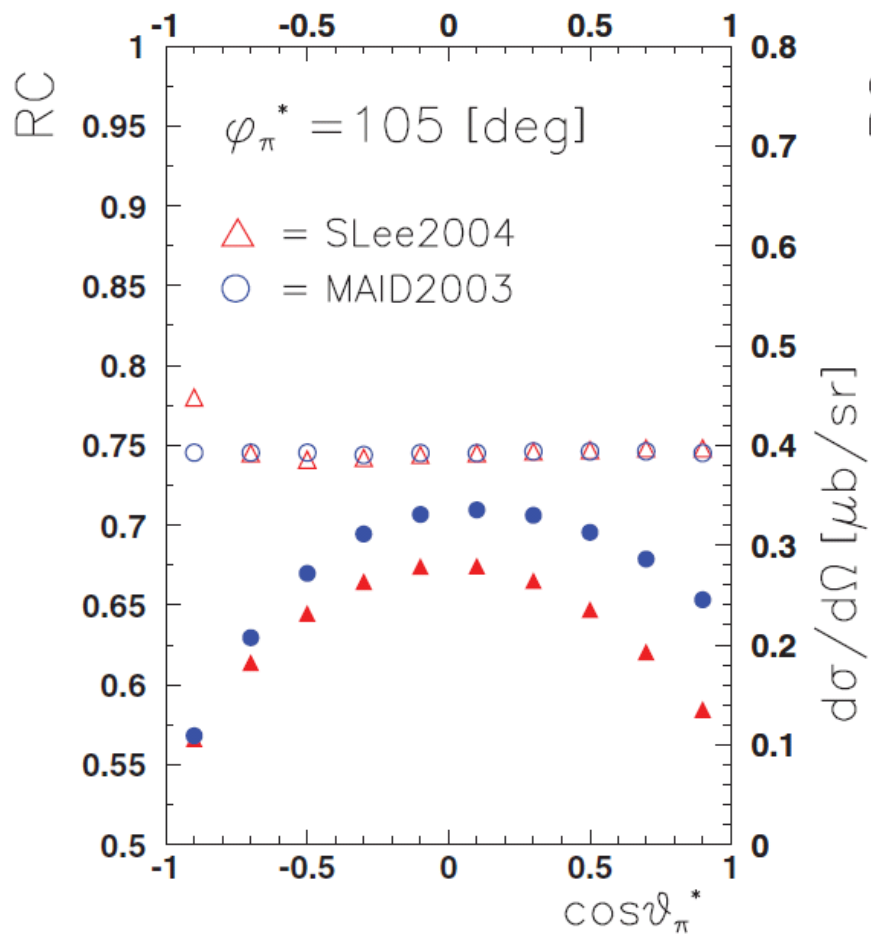


Acceptances – AAO_RAD





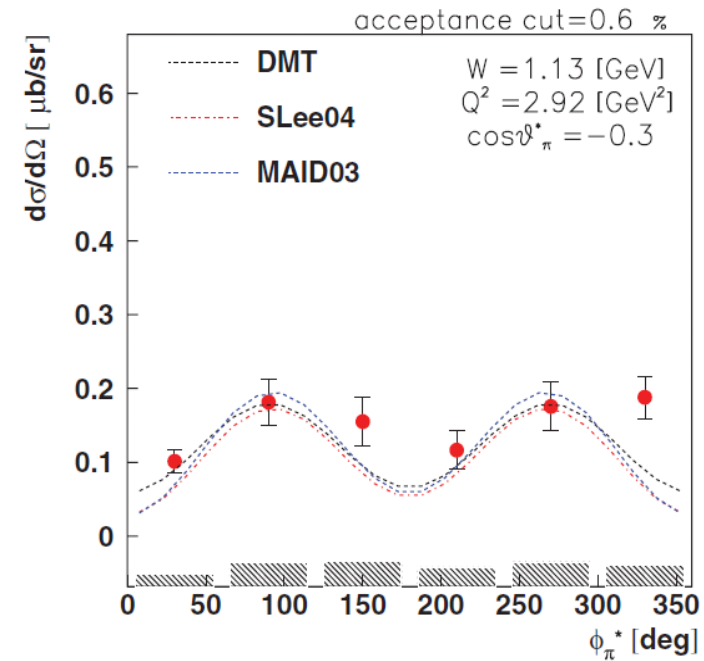
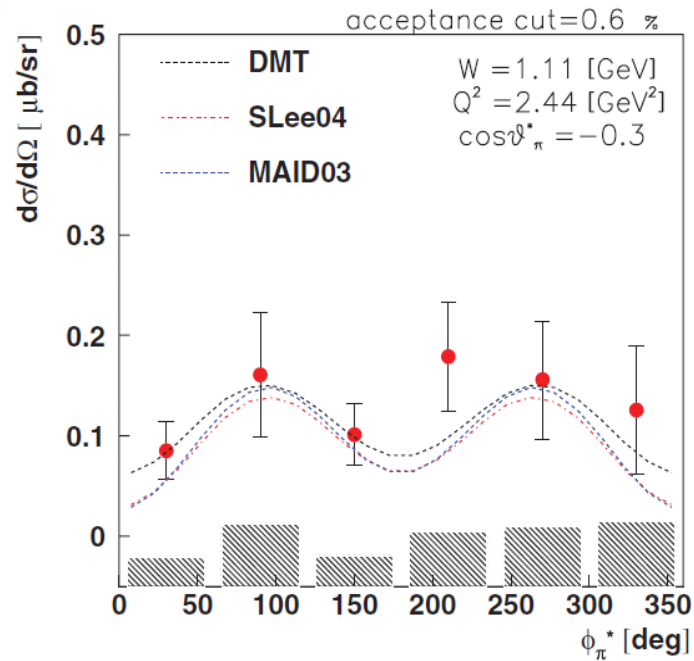
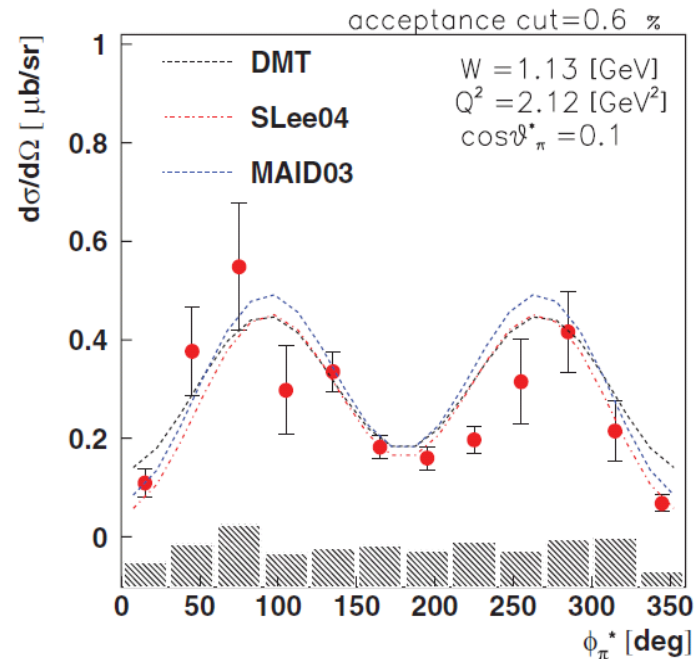
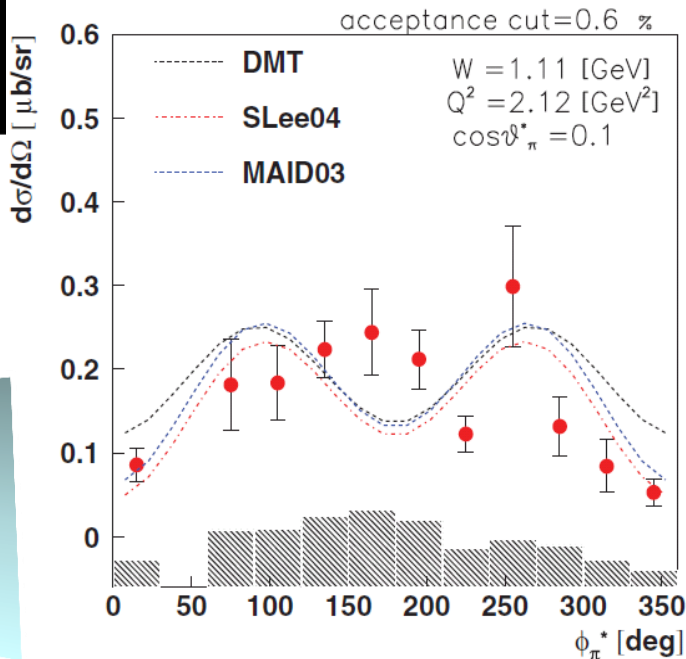
Radiative correction [EXCLURAD]





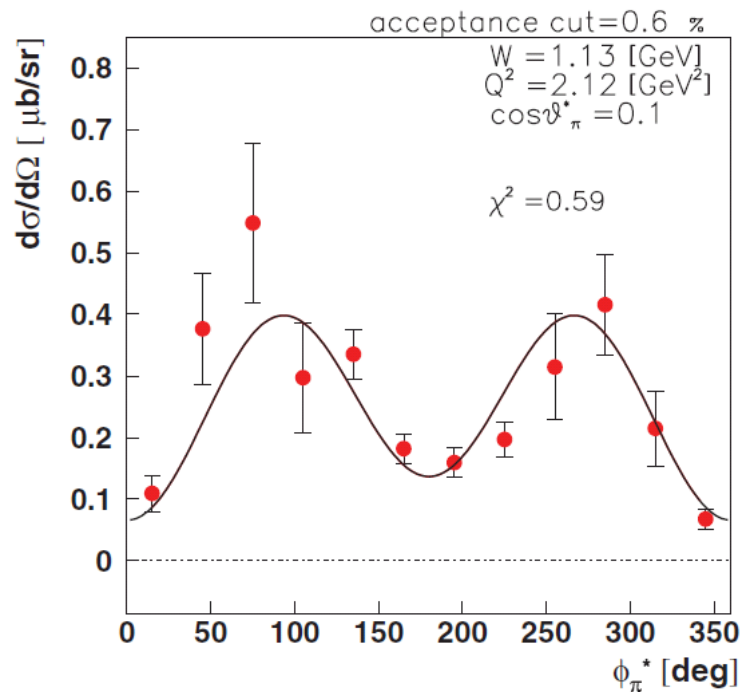
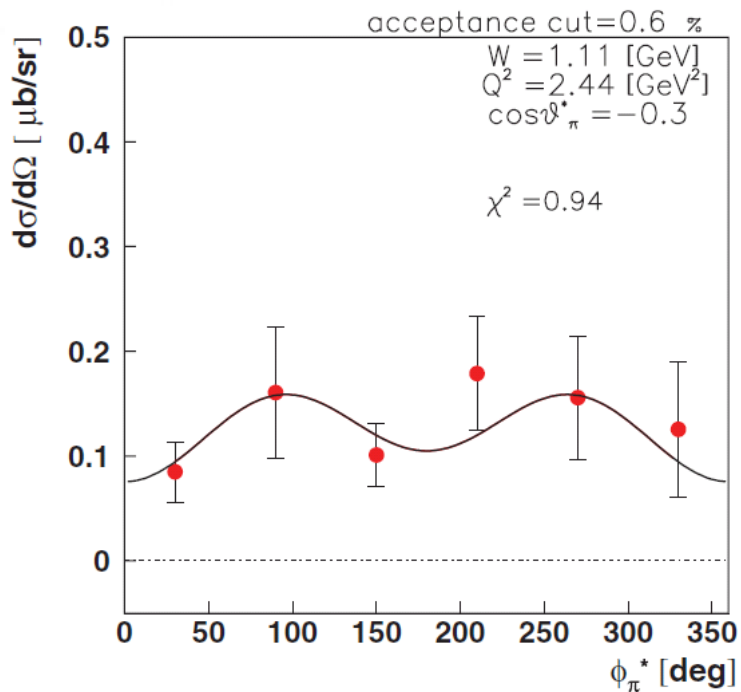
Differential Cross Section (ϕ^*)

ϕ^* – dependent cross section in term of $W, Q^2, \cos\theta^*$
Various physics models





A sample of fit





Structure Function

$$\sigma_T + \varepsilon\sigma_L$$

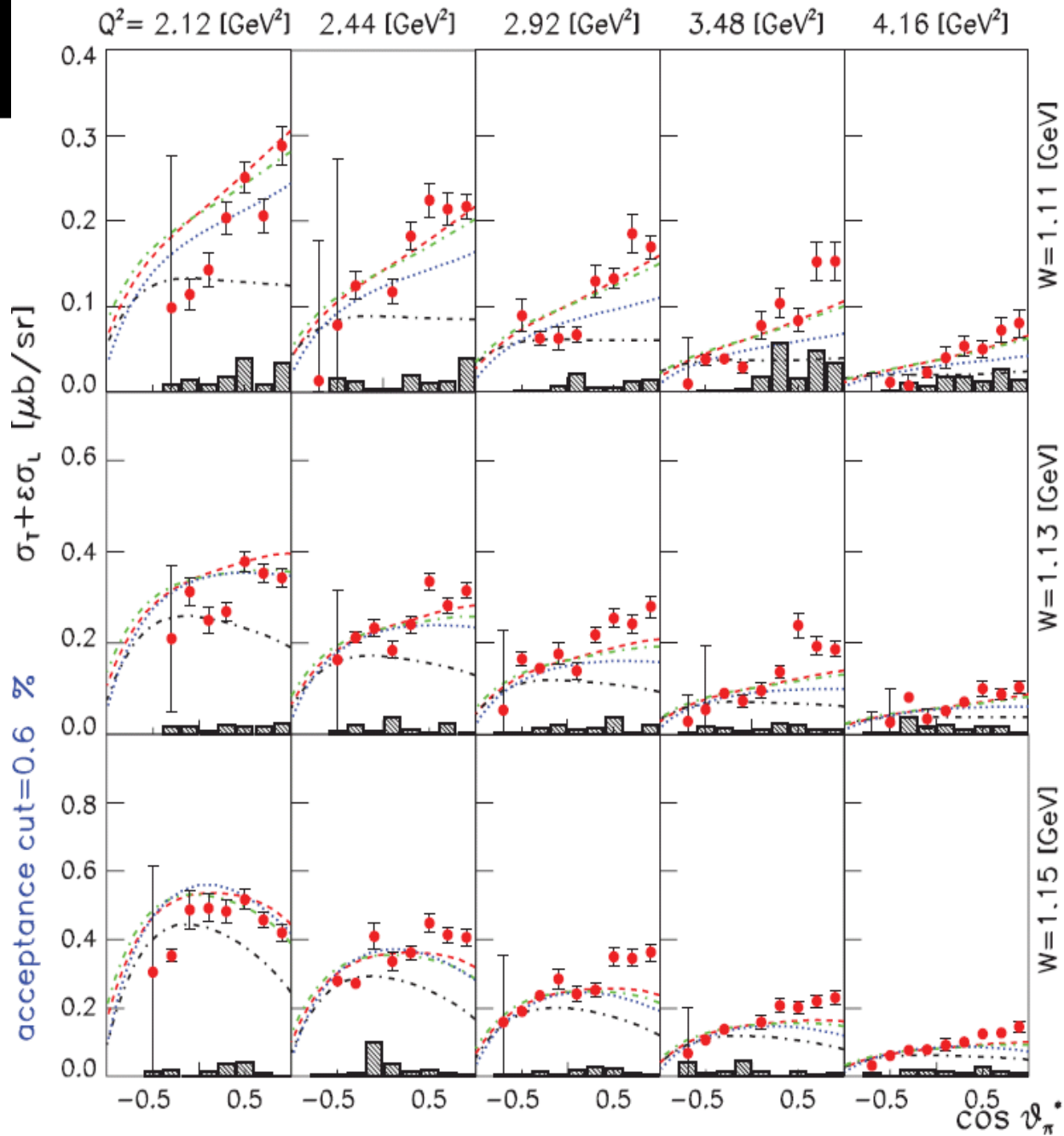
E_{0+} sensitive!

MAID2003

(red bold dash) full multipoles
(green bold dash) without S_{0+}
(black dash-dot) without E_{0+}

MAID2007

(blue bold dot)





Structure Function

$$\sigma_{TT}$$

$$E_{0+}$$

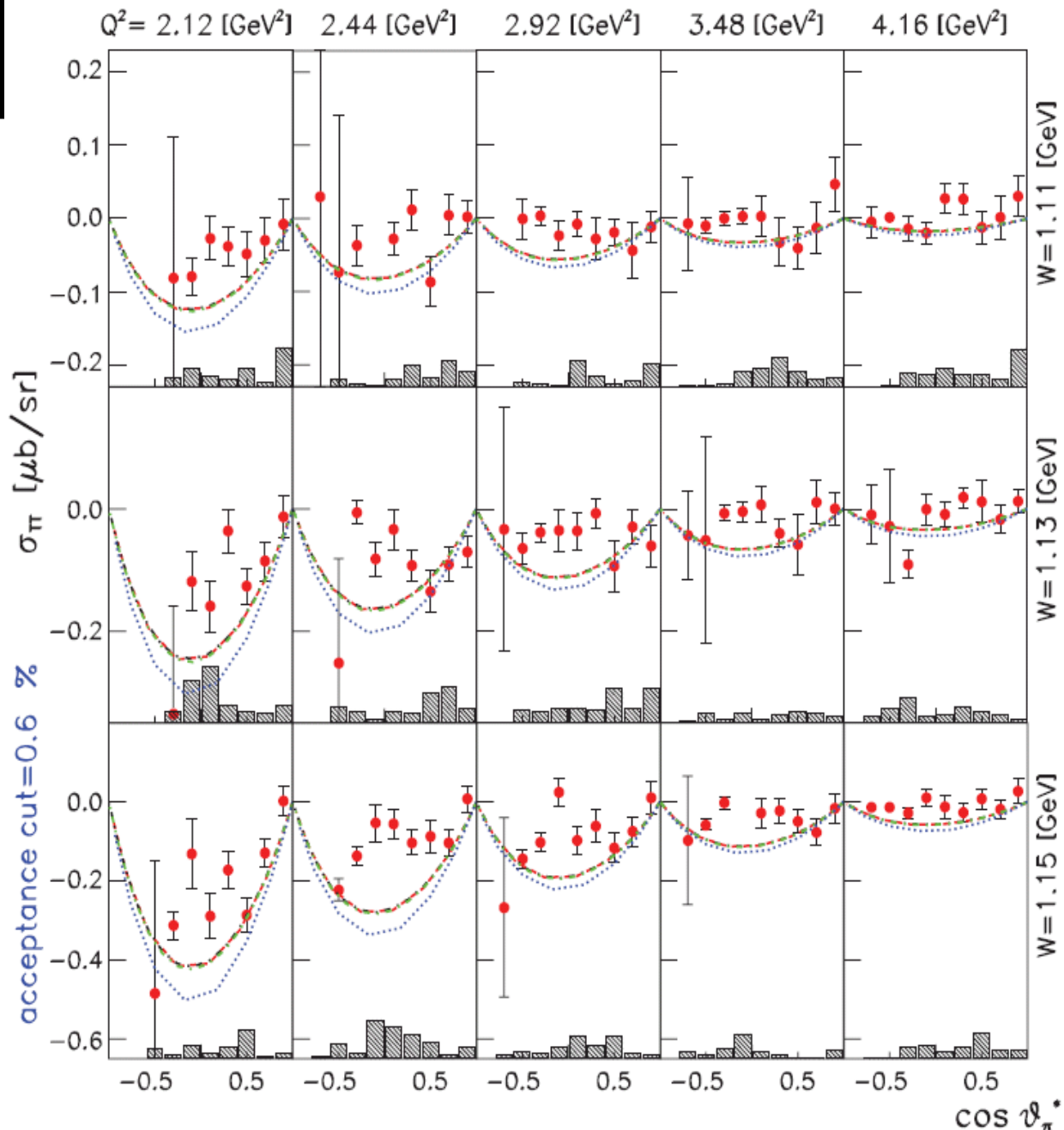
insensitive!

MAID2003

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(green bold dash) without S_{0+}
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MAID2007

(blue bold dot)



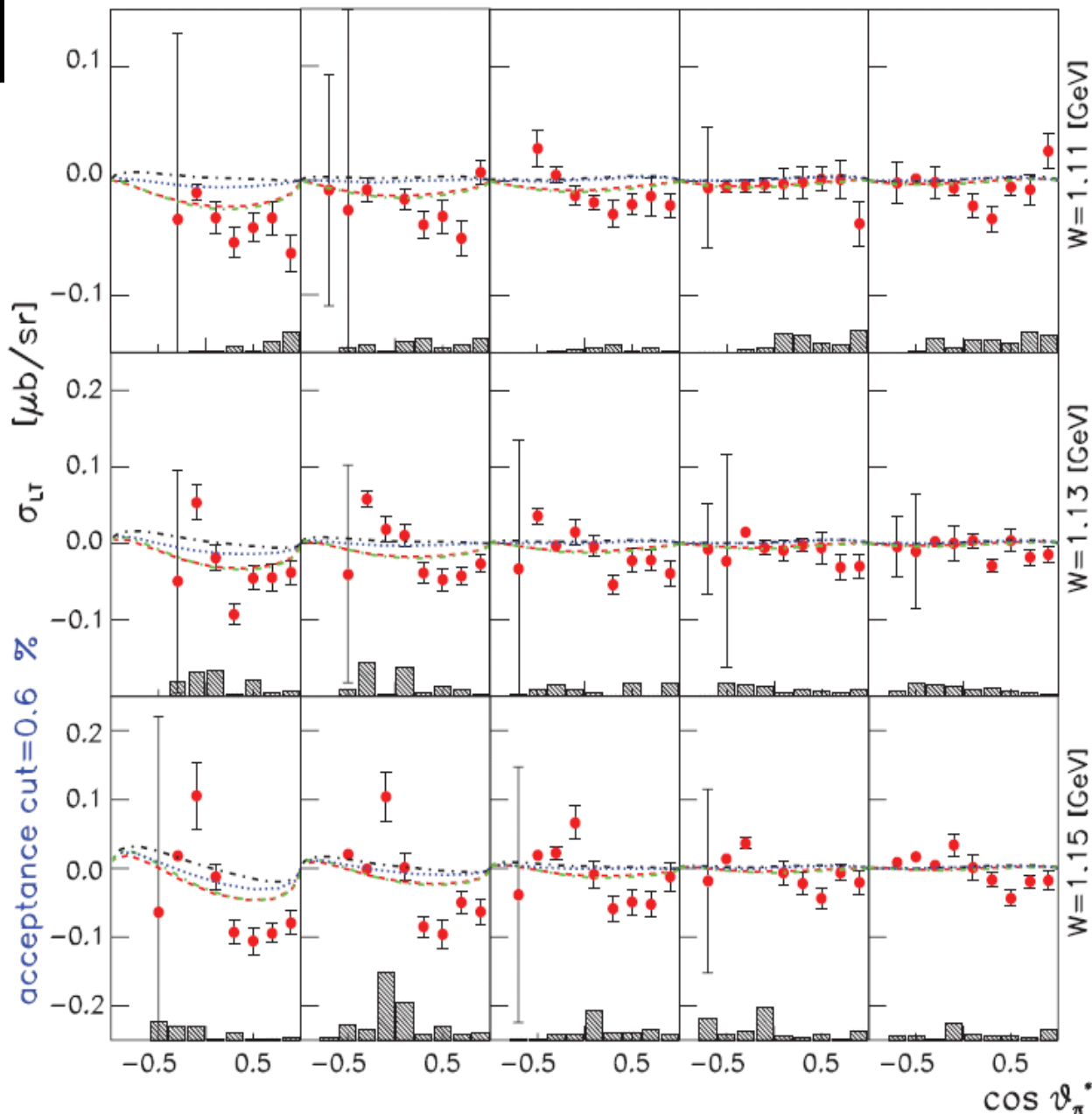


Structure Function

$$\sigma_{LT}$$

E_{0+} sensitive!

$Q^2 = 2.12$ [GeV²] 2.44 [GeV²] 2.92 [GeV²] 3.48 [GeV²] 4.16 [GeV²]



MAID2003

(red bold dash) full multipoles
(green bold dash) without S_{0+}
(black dash-dot) without E_{0+}

MAID2007

(blue bold dot)





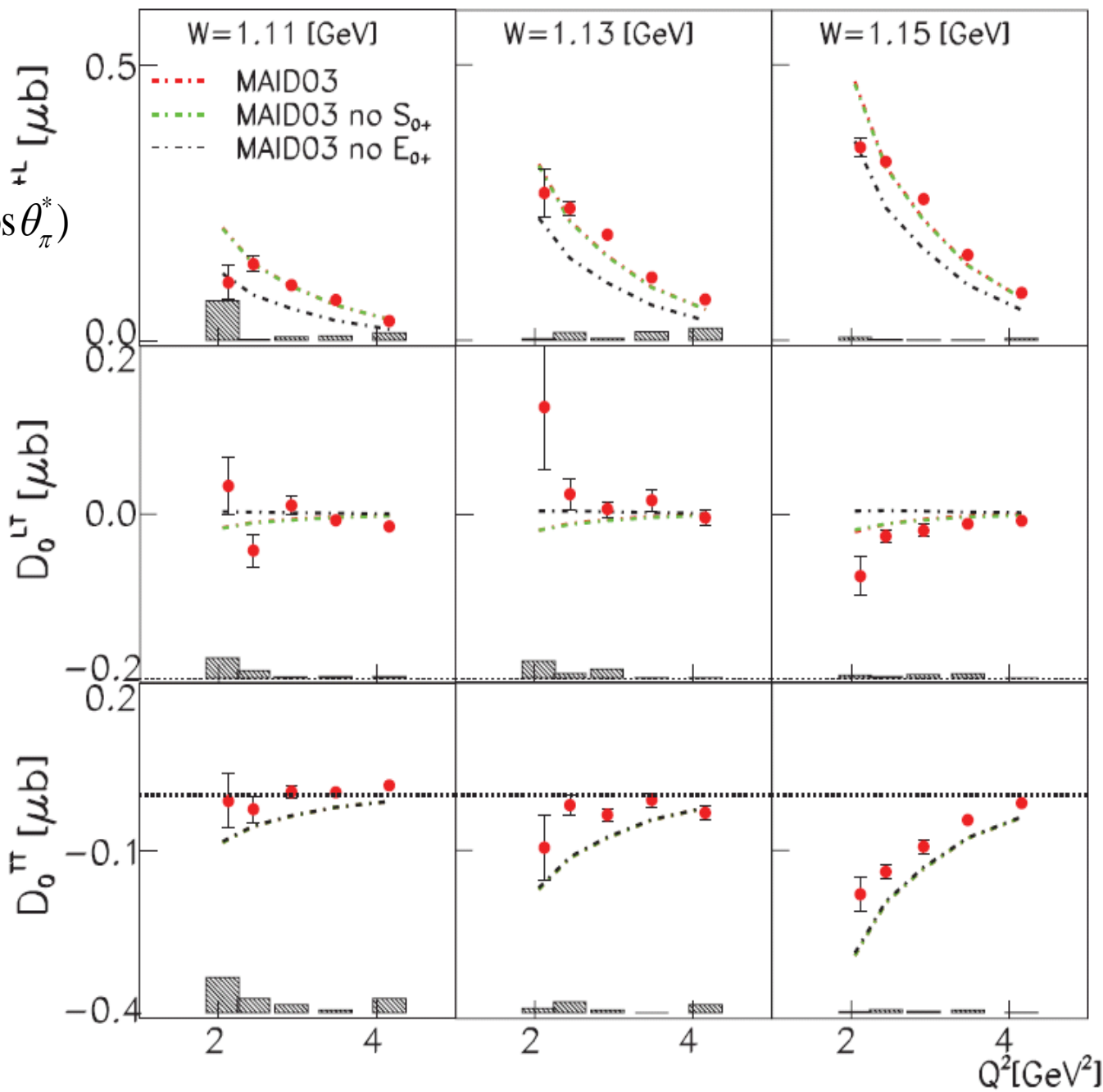
$$D_0^{T+L} + D_1^{T+L} P_1(\cos \theta_\pi^*)$$

$$\sigma_T + \varepsilon_L \sigma_L = D_0^{T+L} + D_1^{T+L} P_1(\cos \theta_\pi^*)$$

$$\sigma_{LT} =$$

$$D_0^{LT} + D_1^{LT} P_1(\cos \theta_\pi^*)$$

$$\sigma_{TT} = D_0^{TT}$$



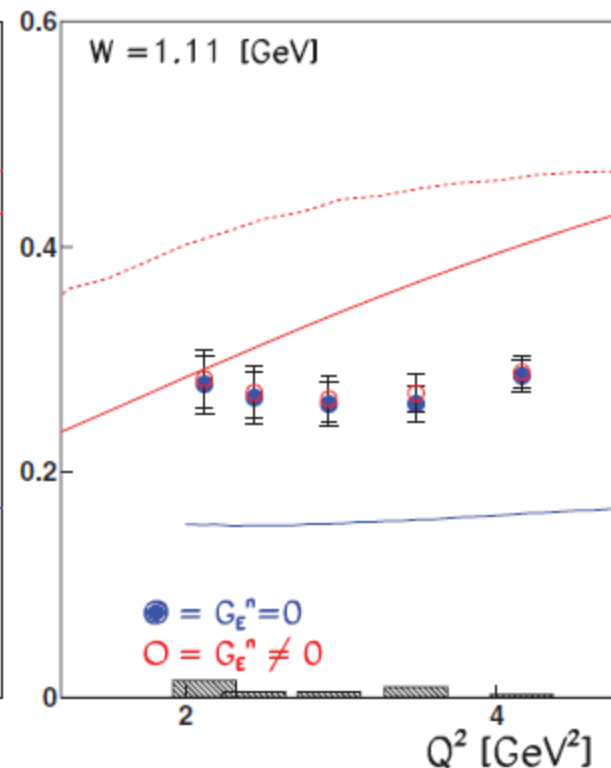
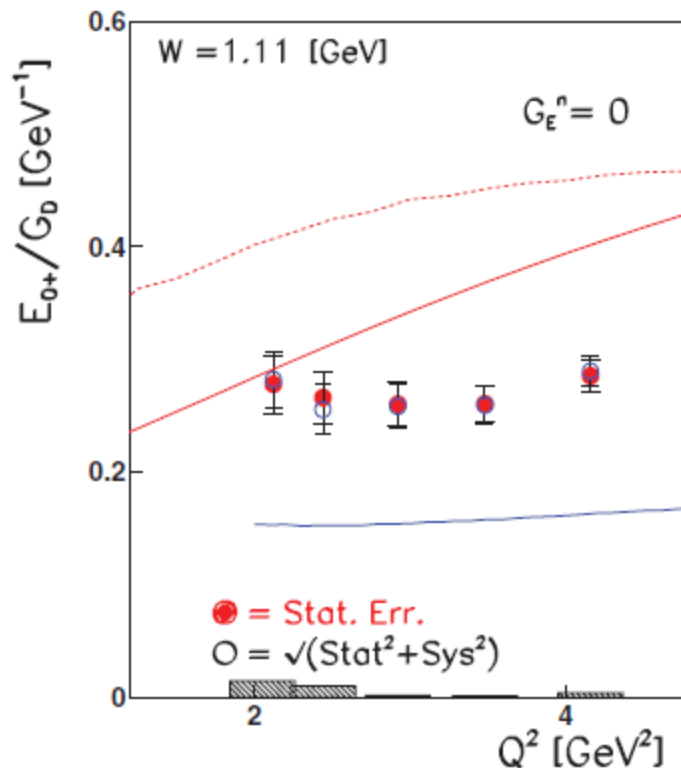


Multipole extraction

Q^2 dependence of the Normalized E_{0+} Multipole by dipole F. F.

Assumption of $\text{Im}(G_2)=y_2=0$
Assumption of $G_E^n=0$

Red lines : LCSR
solid line : pure calc.
dash line : exp. F. F. input
Blue line : MAID07, E_{0+}
Black MAID07 L_{0+}





Form factors and Multipole for $n\pi^+$ channel

$$G_1^{\pi N} = G_1^{\pi^+ n} \quad G_M = G_M^n \approx \mu_n G_D(Q^2)$$

$$G_2^{\pi N} = G_2^{\pi^+ n} \quad G_E = G_E^n \approx 0 \quad G_E = G_E^n \neq 0$$

P.E. Bosted
Phys. Rev. C 51 (1995)

S. Platchekov
Nucl.Phys. A 70 (1990)

J.J. Kelly
Phys. Rev. C 70 (2004)



Form factors and Multipole for $n\pi^+$ channel

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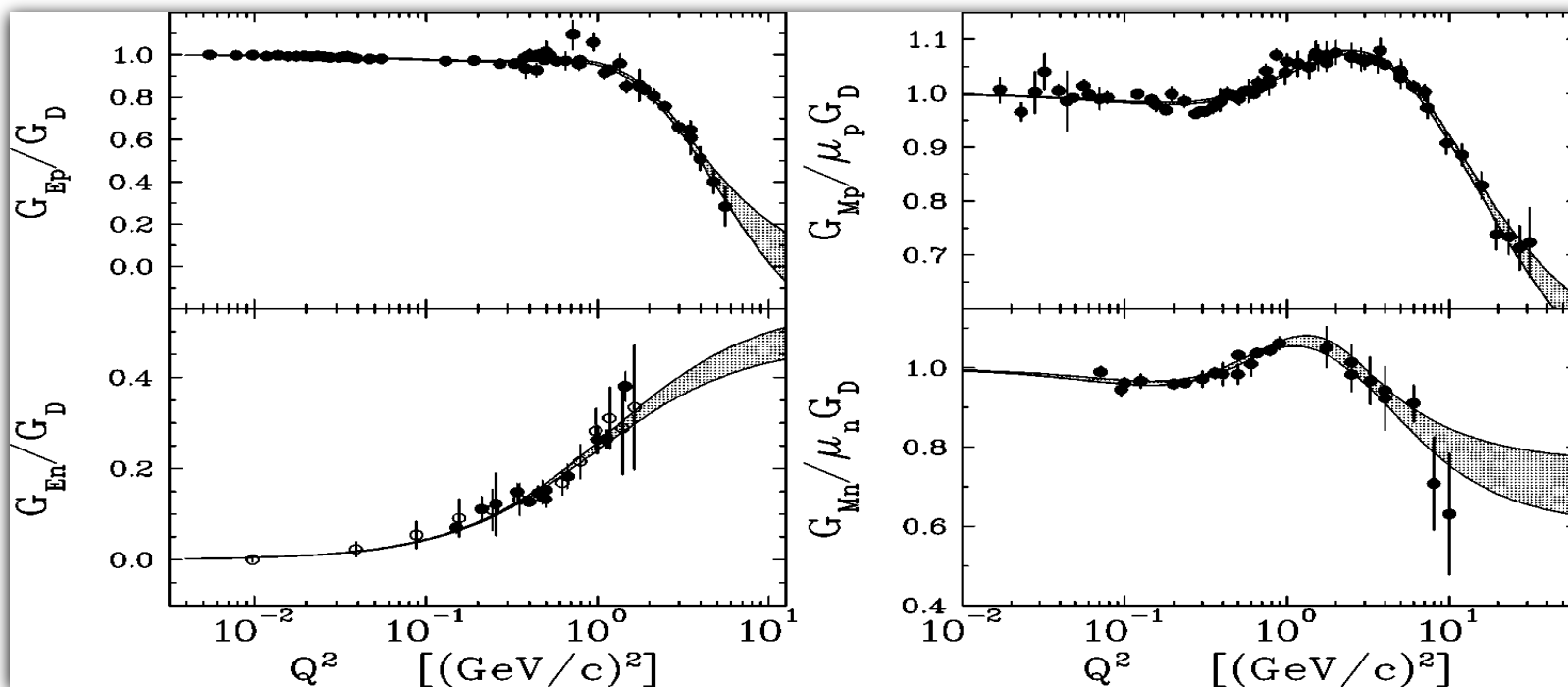
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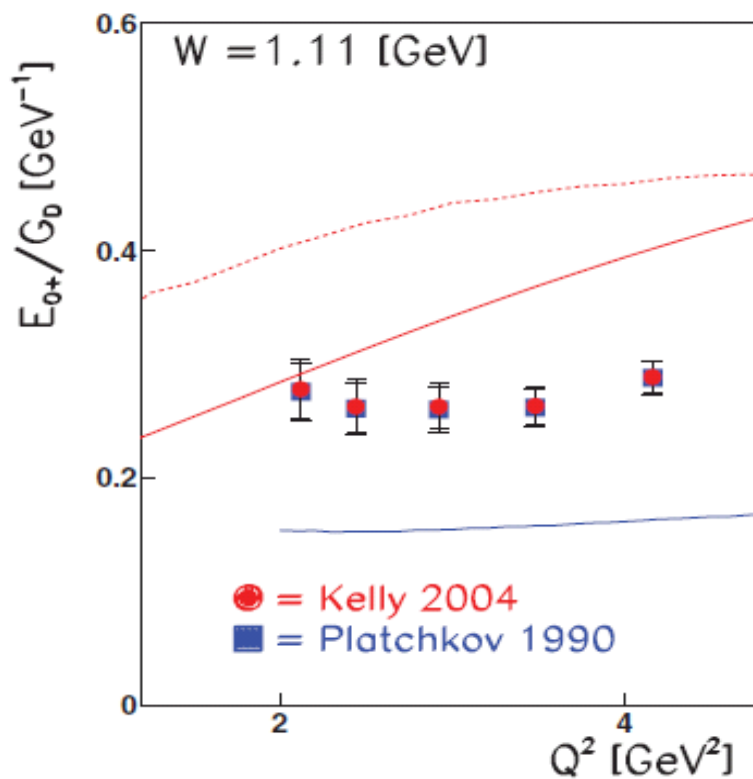
J. J. Kelly *et al.*, PRC 70:068202 (2004)





Form factors and Multipole for $n\pi^+$ channel

Red lines : LCSR
 solid line : pure calc.
 dash line : exp. F. F. input
 Blue line : MAID07, E_{0+}



● Blue : E_{0+} using
 J.J.Kelly form

○ Red : E_{0+} using
 S. Platchekov form



Form factors and Multipole for $n\pi^+$ channel

$$G_1^{\pi N} = G_1^{\pi^+ n}$$

$G_M = \text{CLAS DATA}$

J. Lachniet (2009)
Phys. Rev. Lett. 102

$$G_2^{\pi N} = G_2^{\pi^+ n}$$

$G_E = G_E^n \neq 0$

J.J. Kelly
Phys. Rev. C 70 (2004)



Form factors and Multipole for $n\pi^+$ channel

$$G_1^{\pi N} = G_1^{\pi^+ n}$$

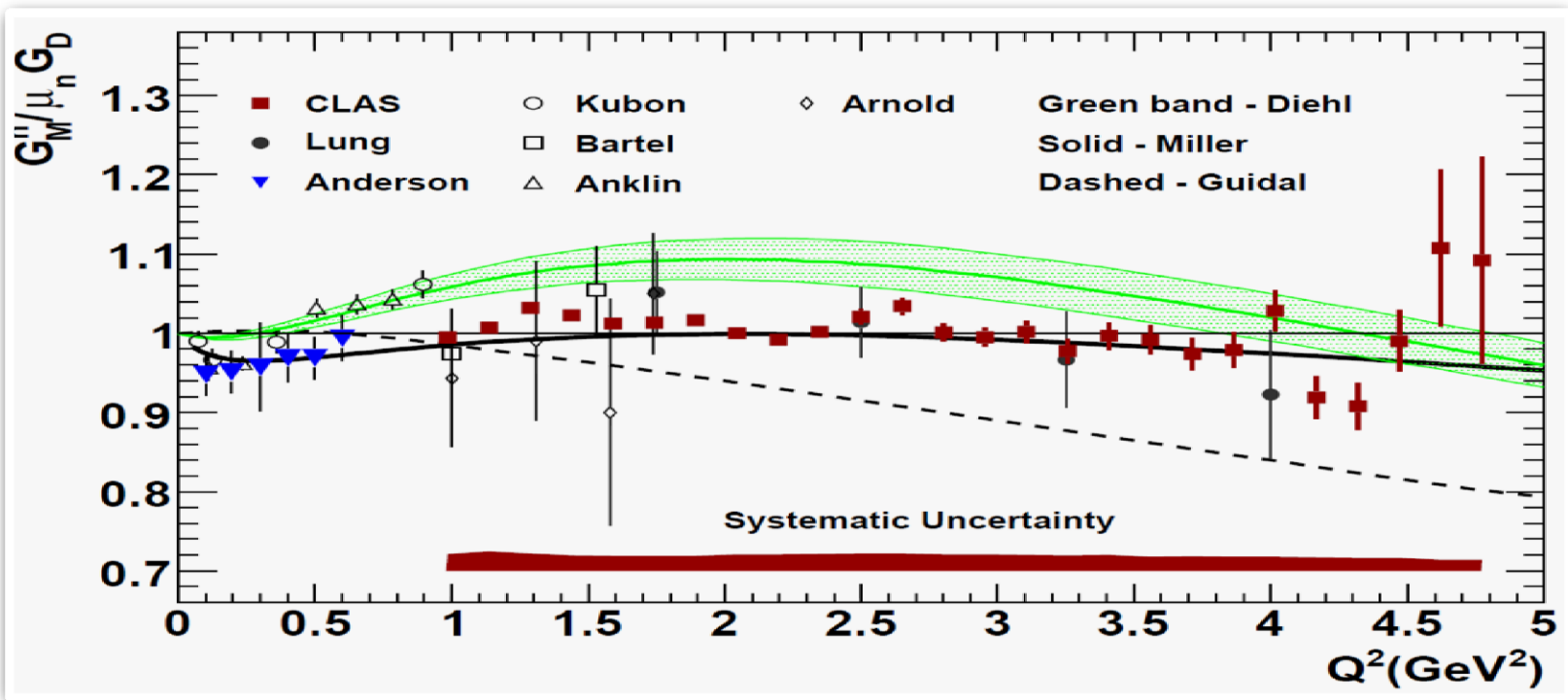
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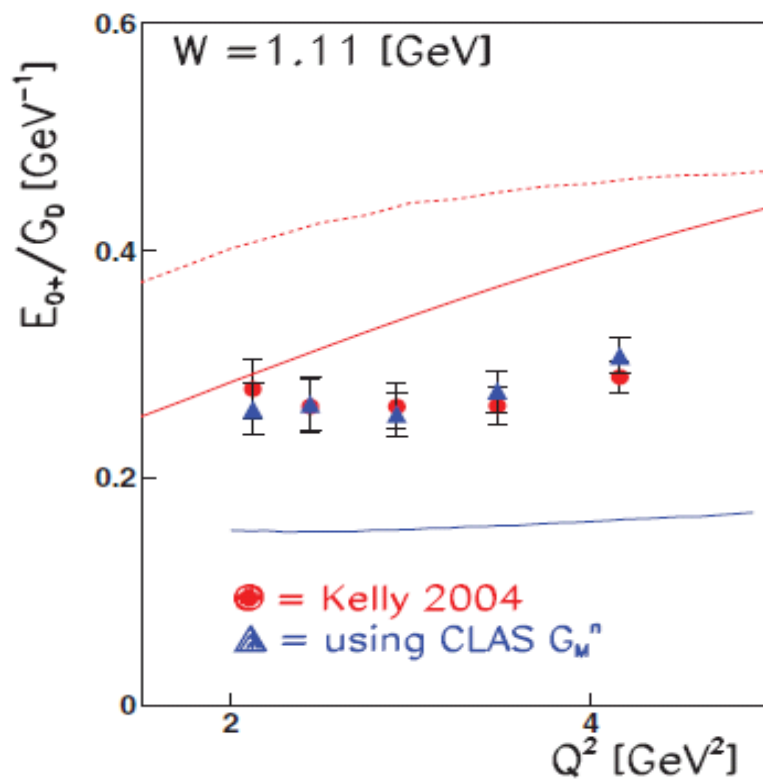
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Form factors and Multipole for $n\pi^+$ channel

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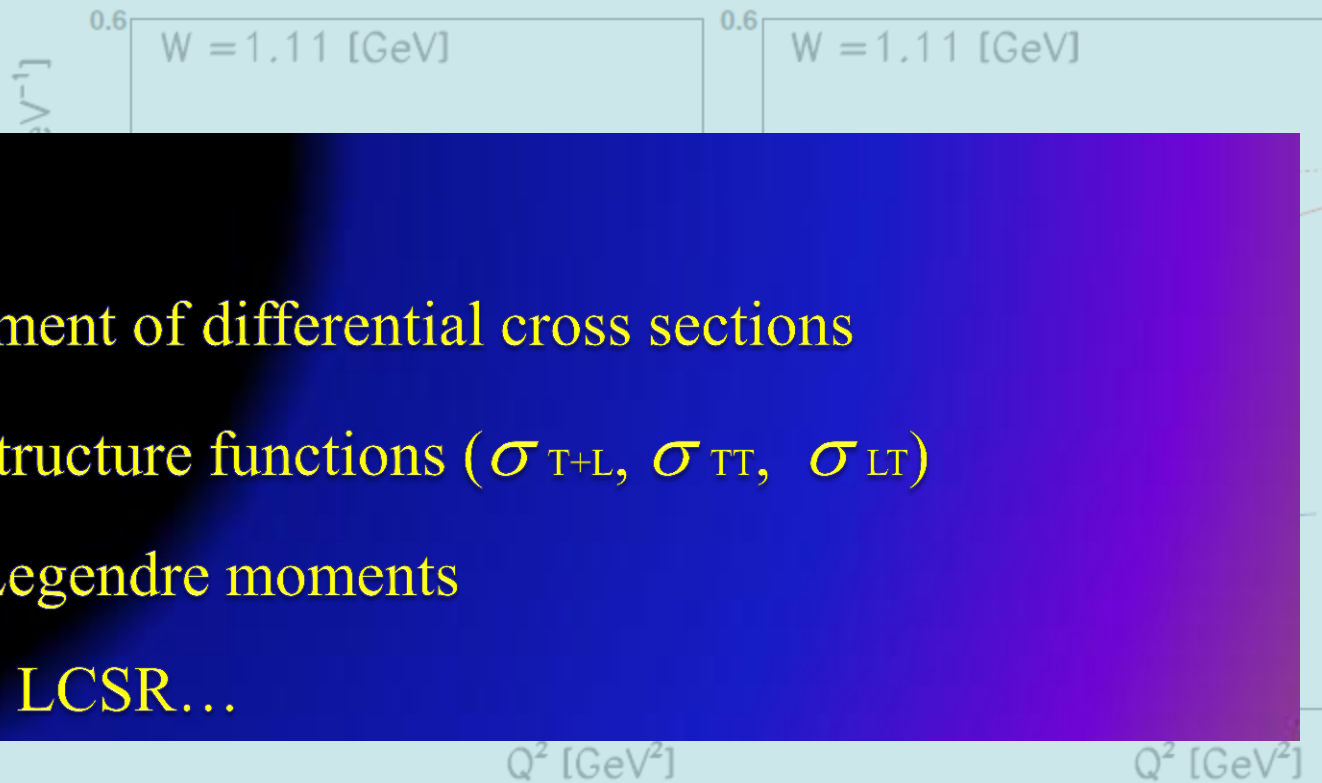
● Blue: E_{0+} using CLAS
 G_M^n measurement

○ Red: E_{0+} using G_M^n
 parameterization



Multipole extraction

Q^2 dependence of
the Normalized
 E_{0+} Multipole by
dipole F. F.



Using....

- (1) Measurement of differential cross sections
- (2) Extract structure functions (σ_{T+L} , σ_{TT} , σ_{LT})
- (3) Extract Legendre moments
- (4) Plug into LCSR...



Multipole analysis

Using six amplitudes (f_i)

** if $l_\pi = 1$

$$f_1 = E_{0+} + 3 \cos \theta_\pi^* (E_{1+} + M_{1+})$$

$$f_2 = 2M_{1+} + M_{1-}$$

$$f_3 = 3(E_{1+} - M_{1+})$$

$$f_4 = 0$$

$$f_5 = S_{0+} + 6 \cos \theta_\pi^* S_{1+}$$

$$f_6 = S_{1-} - 2S_{1+}$$



Multipoles Analysis

Helicity amplitudes
(H_i)

$$H_1 = \frac{-1}{\sqrt{2}} \cos \frac{\theta_\pi^*}{2} \sin \theta_\pi^* (f_3 + f_4)$$

$$H_2 = -\sqrt{2} \cos \frac{\theta_\pi^*}{2} \left[f_1 - f_2 - \sin^2 \frac{\theta_\pi^*}{2} (f_3 - f_4) \right]$$

$$H_3 = \frac{1}{\sqrt{2}} \sin \frac{\theta_\pi^*}{2} \sin \theta_\pi^* (f_3 - f_4)$$

$$H_4 = \sqrt{2} \sin \frac{\theta_\pi^*}{2} \left[f_1 + f_2 + \cos^2 \frac{\theta_\pi^*}{2} (f_3 + f_4) \right]$$

$$H_5 = \frac{-\sqrt{Q^2}}{|k_{\text{c.m.}}|} \cos \frac{\theta_\pi^*}{2} (f_5 + f_6)$$

$$H_6 = \frac{\sqrt{Q^2}}{|k_{\text{c.m.}}|} \sin \frac{\theta_\pi^*}{2} (f_5 - f_6)$$

I. G. Aznauryan, PRD 57, 2727 (1998)



Multipoles Analysis

Structure functions vs. Helicity amplitudes (H_i):

$$\sigma_T + \epsilon\sigma_L = \frac{1}{2} \sum_{i=1}^4 |H_i|^2 + \epsilon(|H_5|^2 + |H_6|^2),$$

$$\sigma_{TT} = \text{Re}(H_2^* H_3 - H_1^* H_4),$$

$$\sigma_{LT} = \frac{-1}{\sqrt{2}} \text{Re}[H_5^*(H_1 - H_4) + H_6^*(H_2 + H_3)].$$



Multipoles Analysis

Constraints :

* E_{0+} , S_{0+} are dominated in this regime.

** M_{1-} , S_{1-} were used from MAID2007 model prediction.

*** for $I=3/2$ case, following correlation functions are acceptable.

$$\rightarrow G_{D'} = (1 + Q^2/\mu_{02})^2$$

$$\rightarrow G_M = 3 \cdot \exp(-0.21 \cdot Q^2) / (1 + 0.0273 \cdot Q^2 - 0.0086 \cdot Q^4) / G_{D'}$$

$$\rightarrow M_{1+} = (Y_0/52.437) \cdot G_M \cdot \sqrt{((2.3933 + Q^2)/2.46)^2 - 0.88} \cdot 6.786$$

$$\rightarrow E_{1+} = -0.02 \cdot M_{1+}$$

$$\rightarrow R_{SM} = -6.066 - 8.5639 \cdot Q^2 + 2.3706 \cdot Q^4 + 5.807 \cdot \sqrt{Q^2} - 0.75445 \cdot Q^4 \cdot \sqrt{Q^2}$$

$$\rightarrow S_{1+} = R_{SM} \cdot M_{1+} / 100.$$

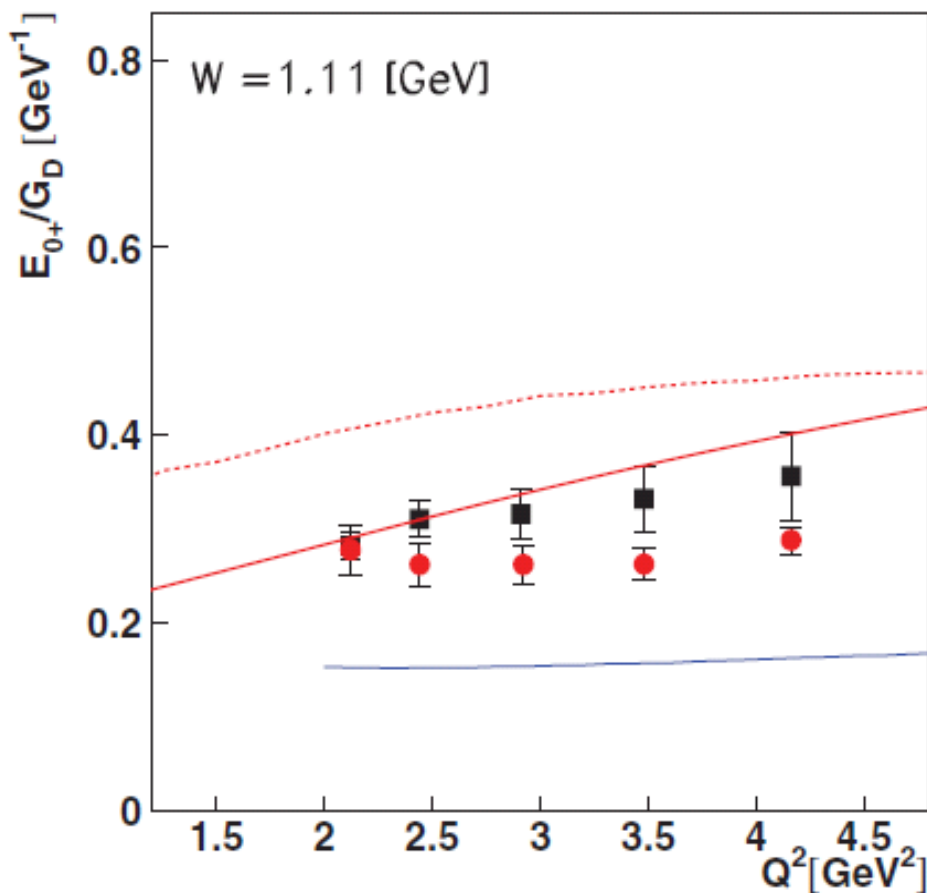
where, $\mu_{02} = 0.71$, Y_0 is the interpolation value from SAID model.

I. G. Aznauryan, PRD 57, 2727 (1998)

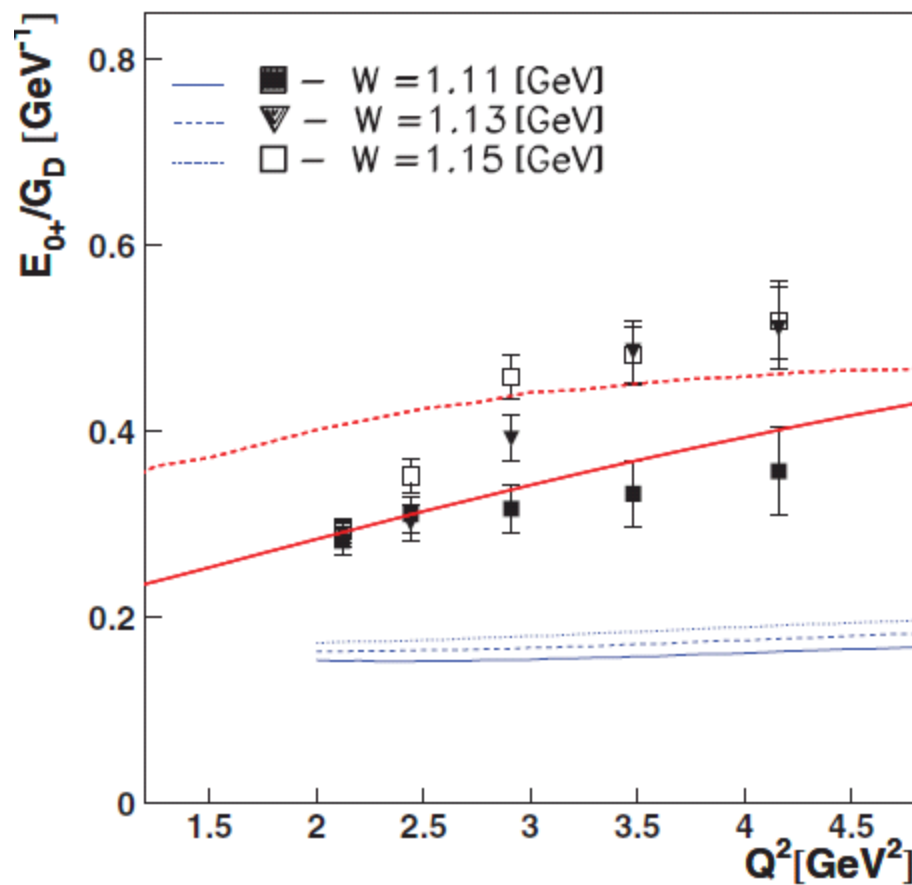


Multipoles extraction

Q^2 dependence of the Normalized E_{0+} Multipole by dipole F. F.



Results of multipole
LCSR w/o pion-mass

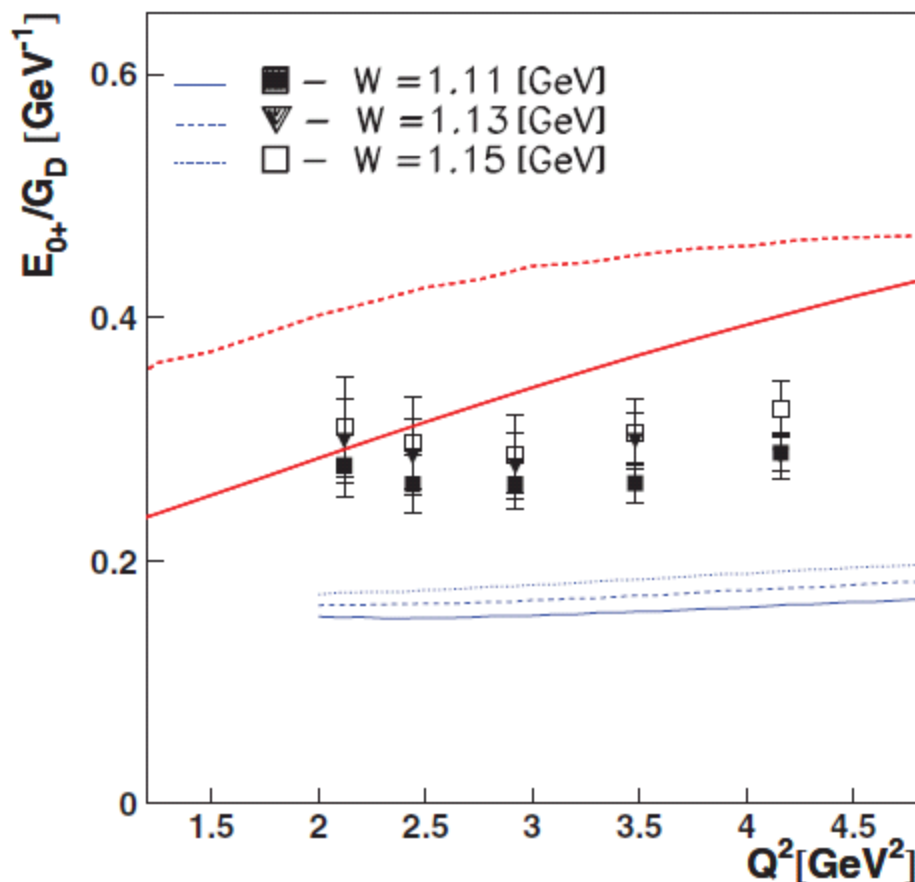


Results of multipole

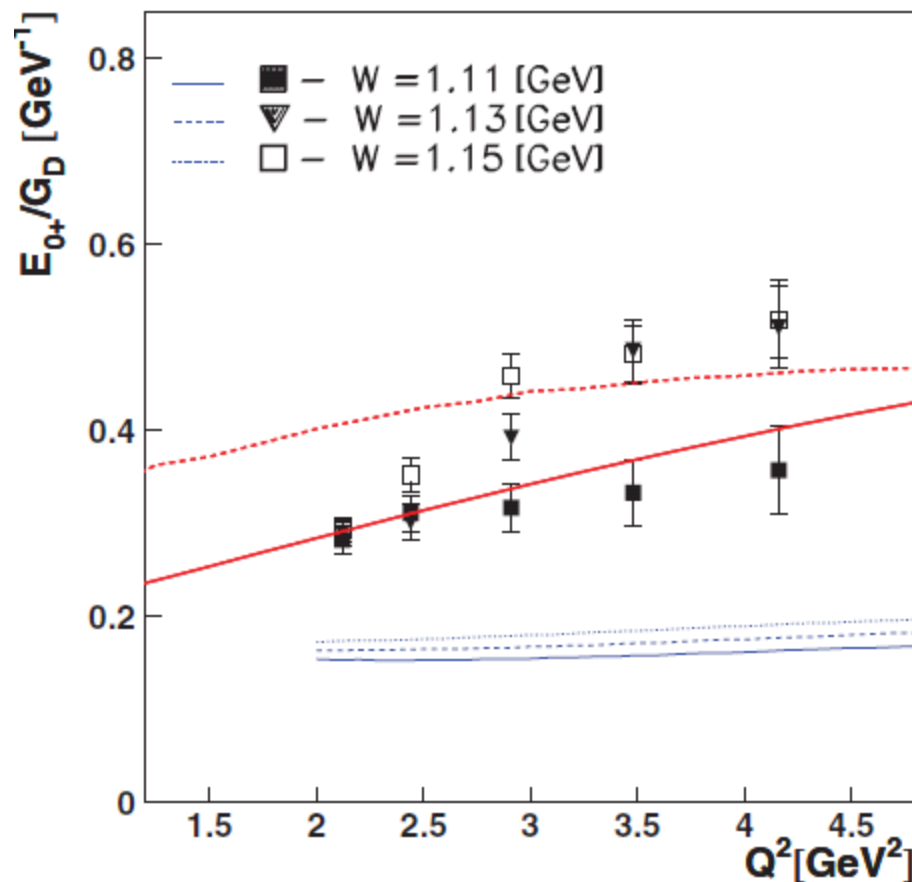


Multipoles extraction

Q^2 dependence of the Normalized E_{0+} Multipole by dipole F. F.



LCSR w/o pion-mass

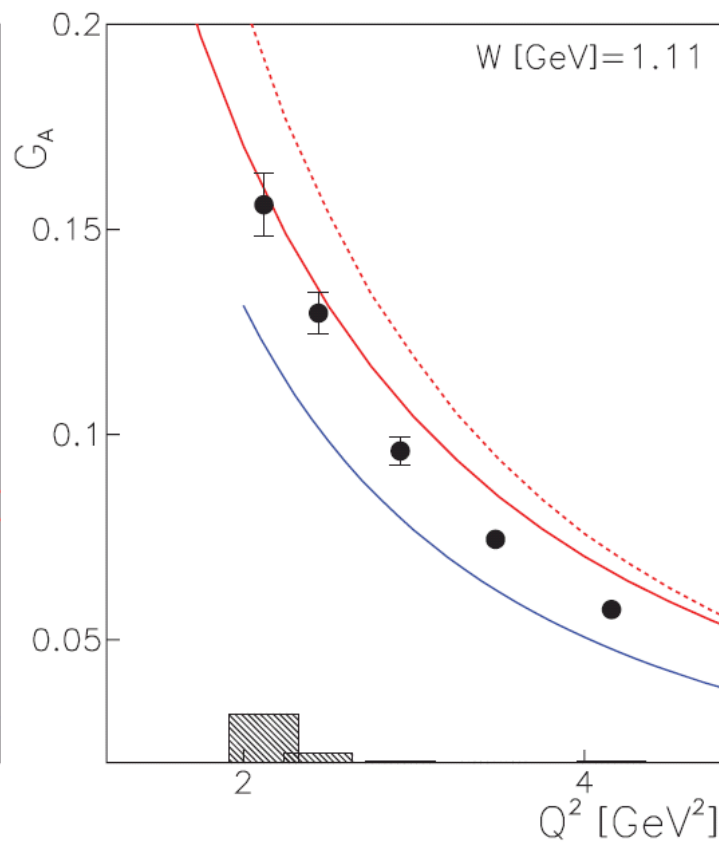
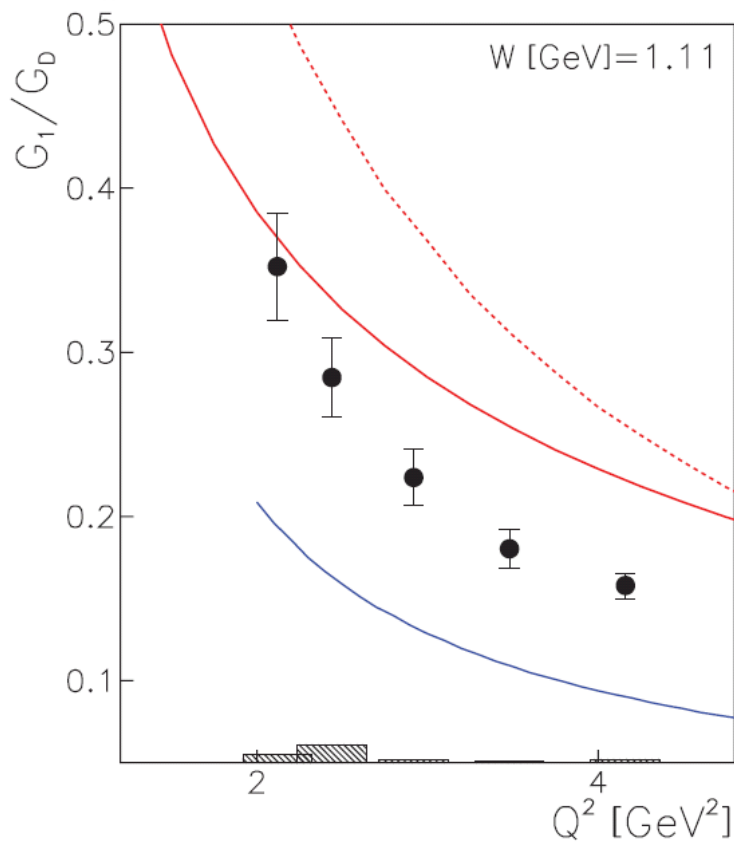


Results of multipole

W - dependence



G_1 and Axial form factor





Summary

- **As first time, E_{0+} multipole extraction and comparison near pion threshold $W = 1.11\text{--}1.15$ GeV at high $Q^2 = 2.12\text{--}4.16$ GeV² with two methods (LCSR, multipole fit) was performed.**
- **Multipole analysis gives us similar answer for extracting, E_{0+} multipole with LCSR method and showing 0.3 GeV⁻¹ and almost Q^2 independent at threshold.**
- **Independent of pion mass and G_n^E parameterization considerations, the $n\pi^+$ channel is dominated by the transverse s-wave multipole E_{0+} .**
- **These data give strong constraints on theoretical developments, especially on the extrapolation away from threshold and away from the chiral limit.**





Thank you for your attention ~!



Legendre –moment vs. F. F. for $n\pi^+$ channel

$$G_1^{\pi N} = G_1^{\pi^+ n}$$

$$G_M = G_M^n \approx \mu_n G_D(Q^2)$$

P.E. Bosted
Phys. Rev. C 51 (1995)

$$G_2^{\pi N} = G_2^{\pi^+ n}$$

$$G_E = G_E^n \approx 0$$

Assumption in LCSR
V.Braun PRD77(2008)

Due to low-energy theorem(LET) relates the S-wave multipoles or equivalently, the form factor G_1, G_2 @ threshold $m_{\pi} \equiv 0$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{Q^2 + 2m_N^2} G_M^n + \frac{1}{2} G_A$$

$$G_2^{\pi^+ n} = \frac{2\sqrt{2}g_A m_N^2}{Q^2 + 2m_N^2} G_E^n = 0$$

Scherer, Koch,
NPA534(1991)
Vainshtein, Zakharov
NPB36(1972)



Legendre moments vs. Form Factors

$$G_1^{\pi^+n} \quad G_2^{\pi^+n}$$

$$G_1^{\pi^+n} = x_1 + iy_1$$

$$G_2^{\pi^+n} = x_2 + iy_2$$

$$A_0 = D_0^{T+L} = \frac{1}{f_\pi^2} \left[\frac{4\vec{k}_i^2 Q^2}{m_p^2} |G_1^{\pi^+n}|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_p^2} Q^2 m_p^2 G_M^{n2} \right]$$

$$A_1 = D_1^{T+L} = \frac{1}{f_\pi^2} \frac{4c_\pi g_A |k_i| |k_f|}{W^2 - m_p^2} \left(Q^2 G_M^n \operatorname{Re}(G_1^{\pi^+n}) \right)$$

$$C_0 = C_0^{TT} = 0$$

$$D_0 = D_0^{LT} = 0$$

$$g_A \equiv 1.267$$

$$c_{\pi^+} \equiv \sqrt{2}$$

$$f_{\pi^+} \equiv 93 \text{ MeV}$$



l -moments vs. F. F. for $n\pi^+$ channel

$$G_1^{\pi N} = G_1^{\pi^+ n} \quad G_M = G_M^n \approx \mu_n G_D(Q^2)$$

$$G_2^{\pi N} = G_2^{\pi^+ n} \quad G_E = G_E^n \neq 0$$

P.E. Bosted
Phys. Rev. C 51
(1995)

Due to low-energy theorem(LET) relates the S-wave multipoles or equivalently, the form factor G_1, G_2 @ threshold $m_{\pi} \equiv 0$

$$\frac{Q^2}{m_N^2} G_1^{\pi^+ n} = \frac{g_A}{\sqrt{2}} \frac{Q^2}{Q^2 + 2m_N^2} G_M^n + \frac{1}{2} G_A$$

$$G_2^{\pi^+ n} = \frac{2\sqrt{2}g_A m_N^2}{Q^2 + 2m_N^2} G_E^n$$



Legendre-moments vs. F. F.

$$G_1^{\pi^+n} \quad G_2^{\pi^+n}$$

$$G_1^{\pi^+n} = x_1 + iy_1$$

$$G_2^{\pi^+n} = x_2 + iy_2$$

$$A_0 = D_0^{T+L} = \frac{1}{f_\pi^2} \left[\frac{4\vec{k}_i^2 Q^2}{m_N^2} |G_1^{\pi N}|^2 + \frac{c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_N^2} Q^2 m_N^2 G_M^2 + \varepsilon_L \left(\vec{k}_i^2 |G_2^{\pi N}|^2 + \frac{4c_\pi^2 g_A^2 \vec{k}_f^2}{W^2 - m_N^2} m_N^4 G_E^2 \right) \right]$$

$$A_1 = D_1^{T+L} = \frac{1}{f_\pi^2} \frac{4c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} \left(Q^2 G_M \operatorname{Re}(G_1^{\pi N}) - \varepsilon_L m_N^2 G_E \operatorname{Re}(G_2^{\pi N}) \right)$$

$$C_0 = C_0^{TT} = 0$$

$$D_0 = D_0^{LT} = -\frac{1}{f_\pi^2} \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} Q m_N \left(G_M \operatorname{Re}(G_2^{\pi N}) + 4G_E \operatorname{Re}(G_1^{\pi N}) \right)$$

$$g_A \equiv 1.267$$

$$c_{\pi^+} \equiv \sqrt{2}$$

$$f_{\pi^+} \equiv 93 \text{ MeV}$$



Legendre moments vs. Form Factors

$$G_1^{\pi^+n} = x_1 + iy_1$$

$$G_2^{\pi^+n} = x_2 + iy_2$$

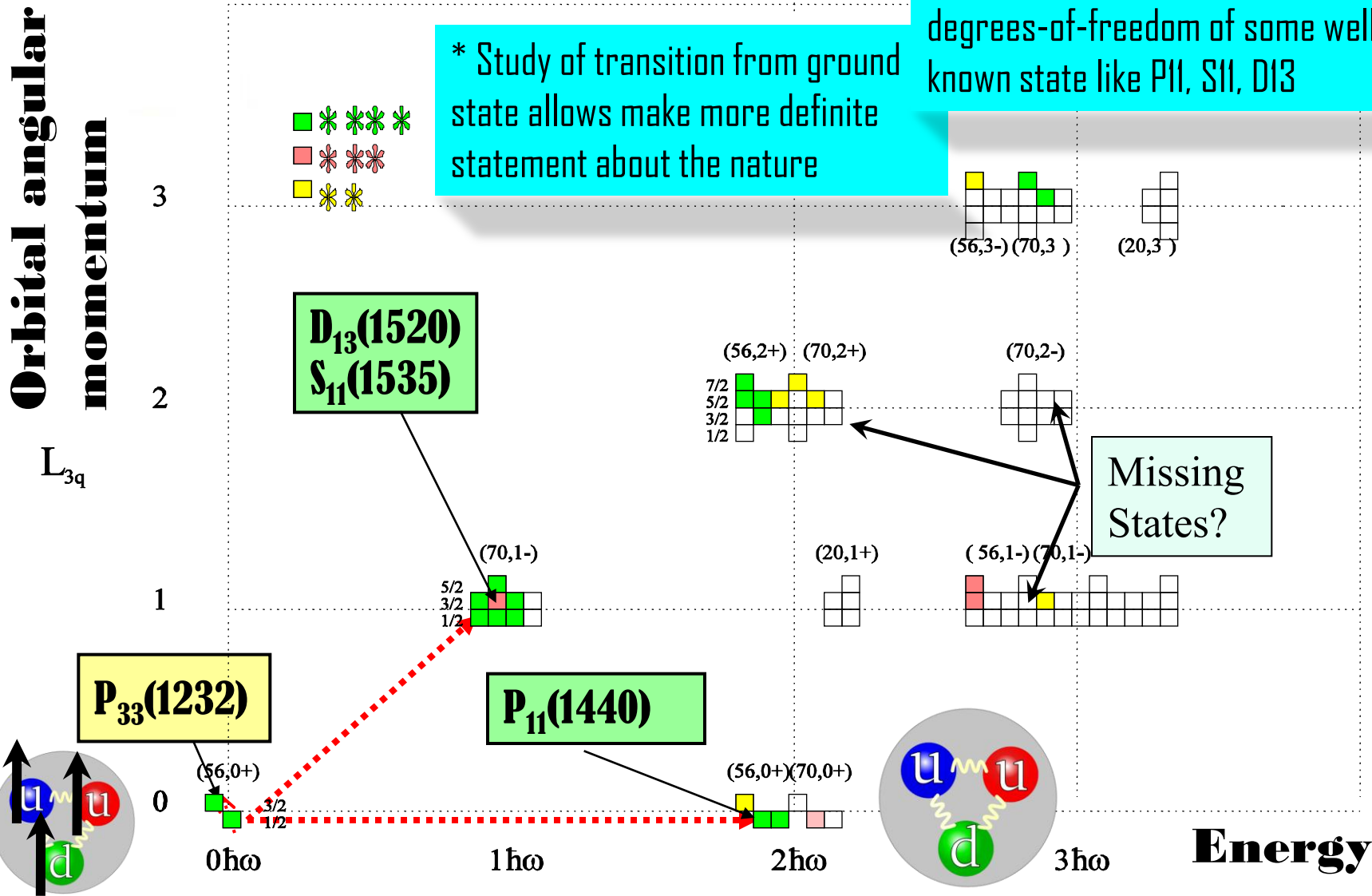
- * 3 Eqs. 4 parameter should be determined
- * Real parts x1, x2 can be determined by A1, D0 legendre coeff.
- * Imaginary parts y1, y2 can be determined in 2cases
- * Asymmetry helps to determine complete form factor

$$D_0' = D_0^{LT'} = -\frac{1}{f_\pi^2} \frac{c_\pi g_A |k_i| |k_f|}{W^2 - m_N^2} Qm_N \left(G_M \text{Im}(G_2^{\pi N}) - 4G_E \text{Im}(G_1^{\pi N}) \right)$$



SU(6)xO(3) Classification of Baryons

* There are questions about underlying degrees-of-freedom of some well known state like P11, S11, D13



* Study of transition from ground state allows make more definite statement about the nature



Multipoles Analysis

Alternative check !

Using six amplitudes (f_i)

** if $l_\pi = 1$

$$\begin{aligned}
 f_1 &= E_{0+} + 3 \cos(\theta) (E_{1+} + M_{1+}) \\
 f_2 &= 2 M_{1+} + M_{1-} \\
 f_3 &= 3 (E_{1+} - M_{1+}) \\
 f_4 &= 0 \\
 f_5 &= S_{0+} + 6 \cos(\theta) S_{1+} \\
 f_6 &= S_{1-} - 2 S_{1+}
 \end{aligned}$$

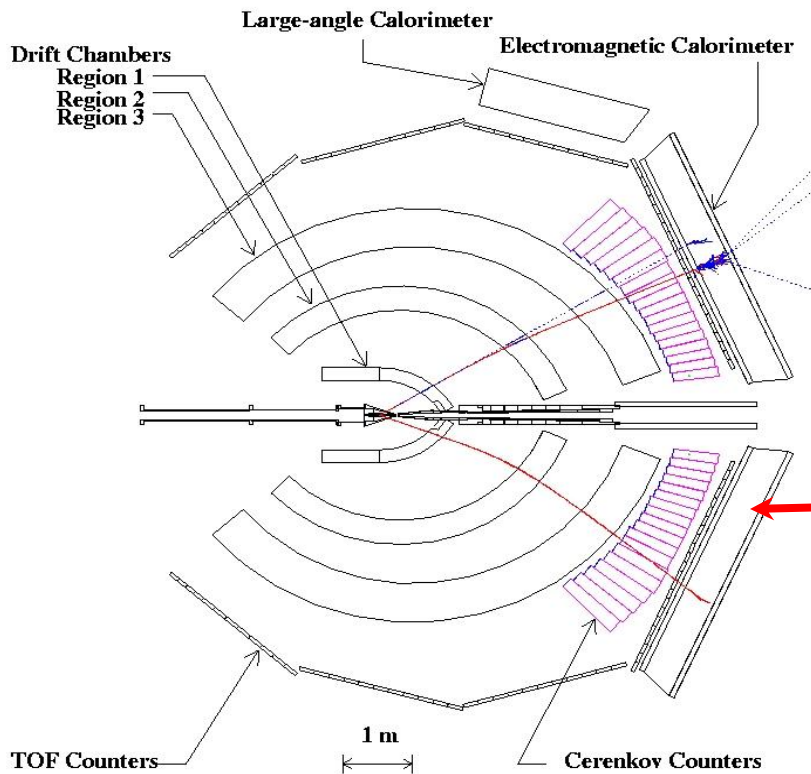
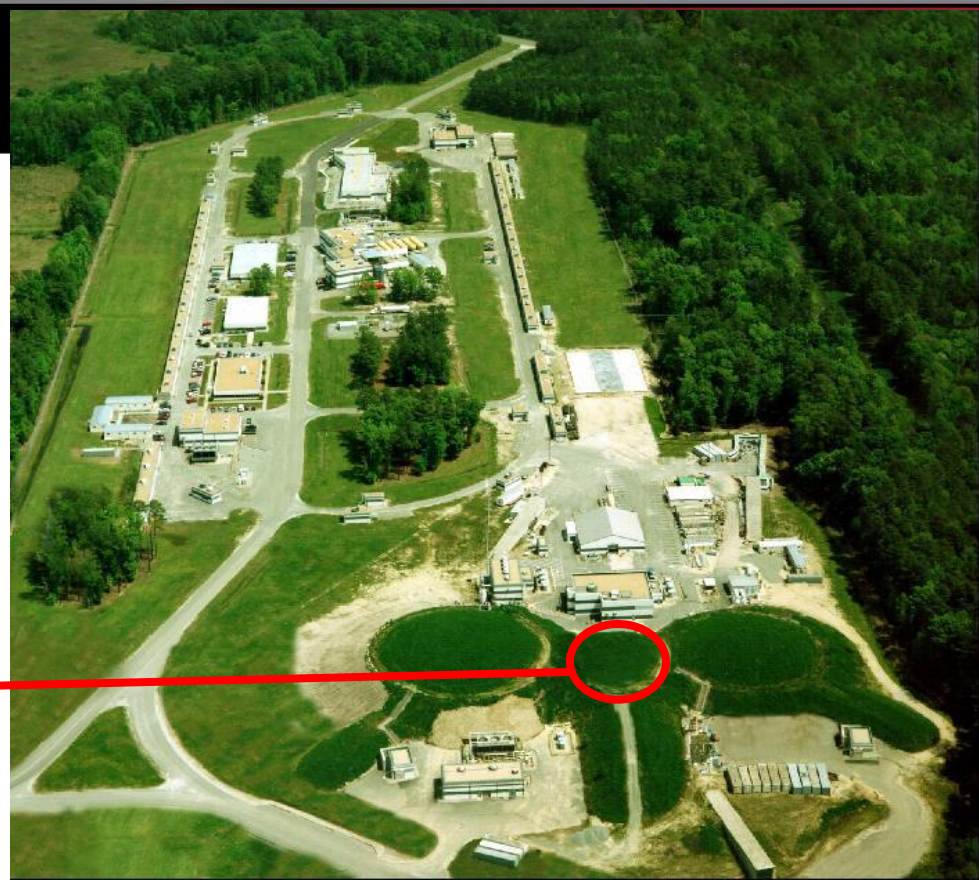
Helicity amplitudes (H_i)

$$\begin{aligned}
 H_1 &= (-1/\sqrt{2}) \cos(\theta/2) \sin(\theta) (f_3 + f_4) \\
 H_2 &= -1 \sqrt{2} \cos(\theta/2) (f_1 - f_2 - \sin(\theta) (f_3 - f_4)) \\
 H_3 &= (1/\sqrt{2}) \sin(\theta/2) \sin(\theta) (f_3 - f_4) \\
 H_4 &= \sqrt{2} \sin(\theta/2) (f_1 + f_2 + (\cos(\theta/2))^2 (f_3 + f_4)) \\
 H_5 &= -1 (\sqrt{Q^2}/\text{abs}(k_{\text{cm}})) \cos(\theta/2) (f_5 + f_6) \\
 H_6 &= (\sqrt{Q^2}/\text{abs}(k_{\text{cm}})) \sin(\theta/2) (f_5 - f_6)
 \end{aligned}$$

I. G. Aznauryan, PRD 57, 2727 (1998)



JLab & CLAS



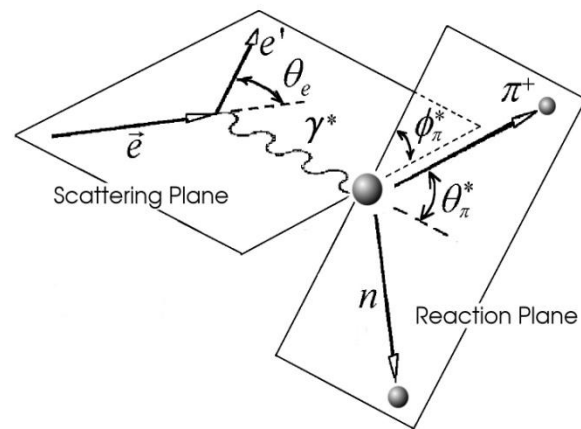
• Experiment (Oct.2001 - Jan.2002)

$E_0 = 5.754 \text{ GeV}$

$B_I = 3375 \text{ A}$

LH2 target

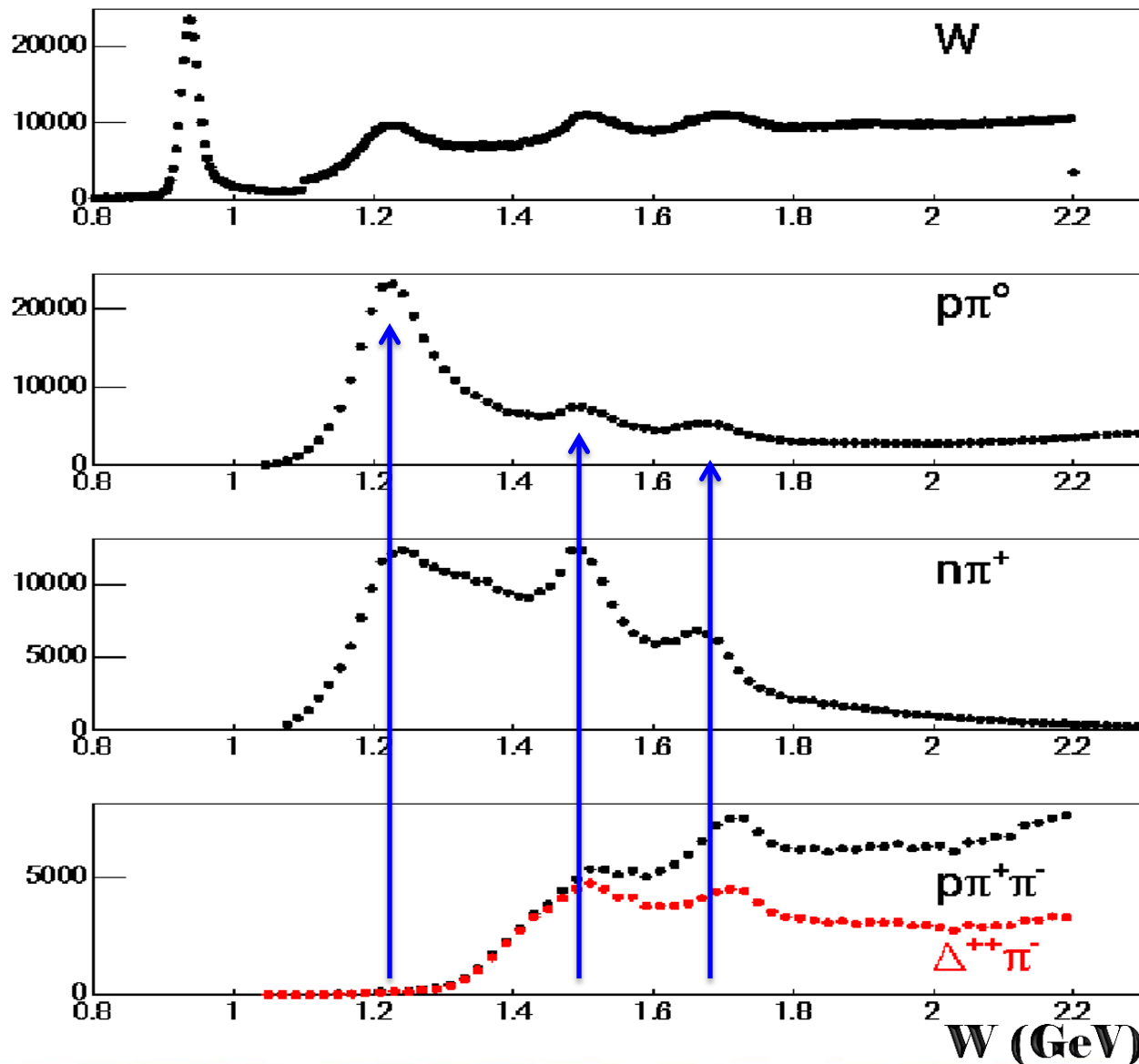
Almost 4π angular coverage





Single and double pion electroproduction

▪ Provides information that is complementary to the $N\pi$ channel





Single and double pion electroproduction

▪ Provides information that is complementary to the $N\pi$ channel

