Using the Covariant Spectator Theory© (CST) to extract N* properties at high Q^2

South Carolina, August 13 - 15, 2012

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JLab and W&M

- Part I -- What is our goal?
  - Equivalence between quark-gluon and hadronic degrees of freedom
  - The goal is to determine the hadronic lagrangian
- Part II -- Using the CST to model hadronic vertices at high Q^2
  - Compute BARE $\gamma^* + N \to N^*$ vertices from the N and N* wave functions and quark form factors
- Part III -- Connection to DIS

Thanks to:
Yohanes Surya

Gilberto Ramalho
Teresa Pena

PART I: What is our goal?

What is our goal?

1. Fundamental theoretical assumption (proved?):
   - If [quarks and gluons] ⇔ [baryons and mesons (hadrons)] are COMPLETELY EQUIVALENT descriptions of the physics, then
   - What are the REQUIRED hadronic fields in the Lagrangian (in order that equivalence will work)?
     - N, Δ, •, •, •, • ? ["elementary" baryons]
     - π, ρ, ω, η, σ(?) , •, • ? ["elementary" mesons]
2. Using an accepted (dynamical and regularization) SCHEME compute

   $\gamma^* + N \rightarrow \pi + N;$

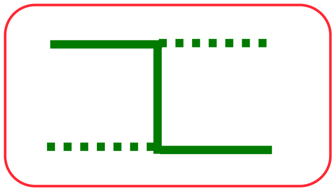
   $\gamma^* + N \rightarrow \rho + N,$

   $\gamma^* + N \rightarrow \pi + \pi + N,$

   $\gamma^* + N \rightarrow$ •, •, • .

2. The goal is to use the accepted SCHEME to find the bare baryon poles corresponding to the "elementary" baryons that appear in the hadronic Lagrangian: -- and to DETERMINE THE HADRONIC LAGRANGIAN

Lessons from the history of the Δ



- ★ Chew-Low theory (1955)
  - NO elementary Δ pole
  - Δ resonance generated by N exchange diagram (u channel pole)
- ★ Bootstrap: resonance feedback (1960's)
- ★ Discovery of quarks and recognition that the Δ is "elementary"
  - (i.e. it is REQUIRED for equivalence between hadronic and quark degrees of freedom)
- ★ The Δ - N mass difference enters into many estimates. Is it the "bare" mass difference or the "dressed" mass difference?

- ★ Conclusion: The existence of a bare delta pole is crucial to our understanding

Bare poles from various models (incomplete survey!)

Reference	Δ (1232) bare mass	D_{13} (1520) bare mass	P_{11} (1440) bare mass
Surya & Gross (1993&1996)	1318.6 (1993) 1301.8 (1996)	1513.5 (1993) 1520.4 (1996)	1431.8 (1993) 1431.8 (1996)
Sato & Lee (1996) Suzuki, et.al. (2010)	1299.07 (model L) 1318.52 (model H) 1391 (2010)	 1899 (2010)	 1763 (2010)
Döring, et al. (2011) Gasparyan, et.al. (2003)	1535 (2011) 1459 (2003)	 2236 (2003)	 NO POLE

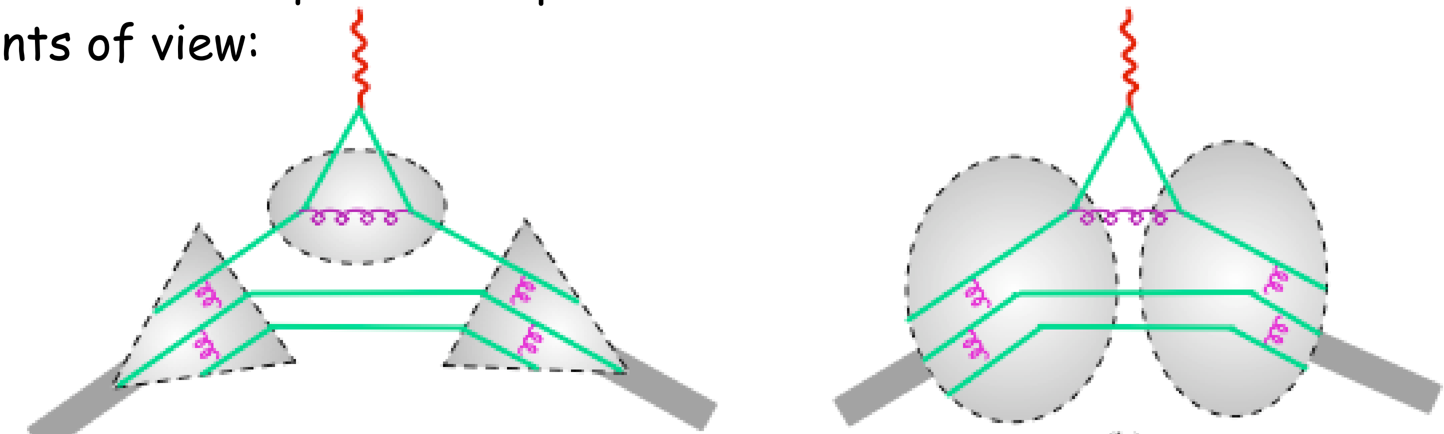
WARNING:

Our physical insight and understanding depends on our computational scheme

EXAMPLE: the "angular momentum theorem"

Does $F_2 > 0$ require $\ell > 0$? [Angular momentum theorem]

- Answer to this question depends on the formalism. There are two points of view:



(a) (b)

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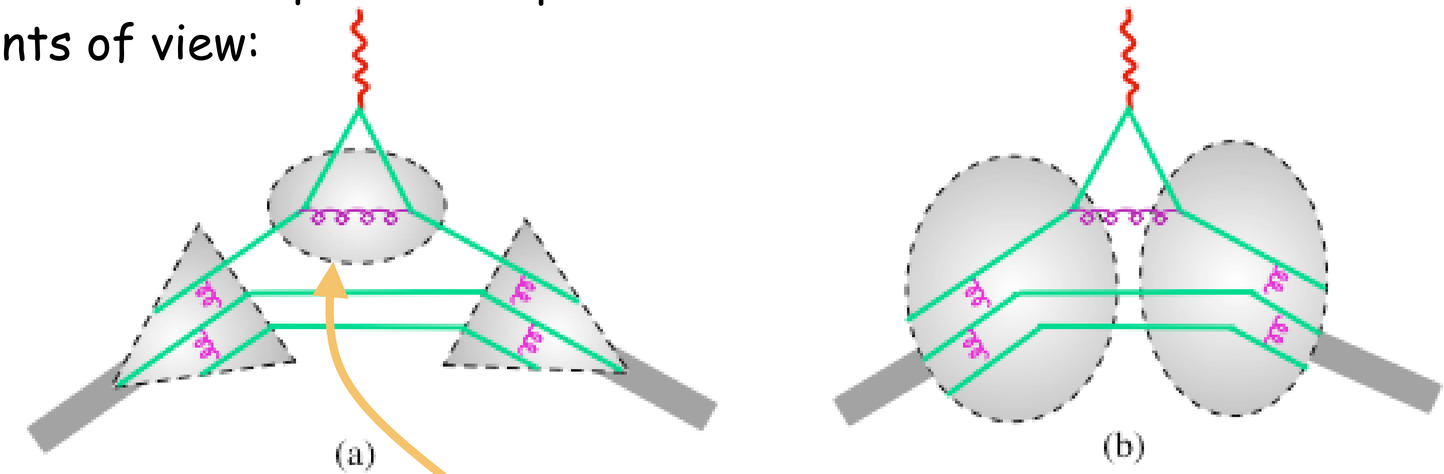

CST view: All interactions involving gluon exchange between the $q\bar{q}$ pair coupled to the photon are included in quark form factors; these produce the quark anomalous moments

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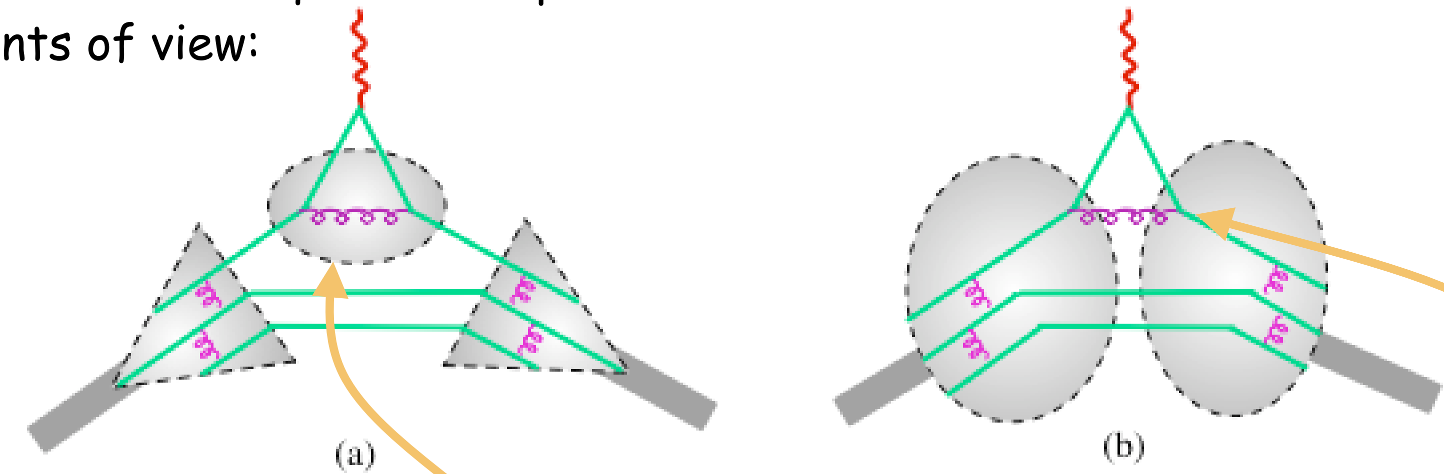

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Light-front view: nucleon wave function is a sum over Fock components; quark "structure" comes from higher Fock components of the hadronic wave function.

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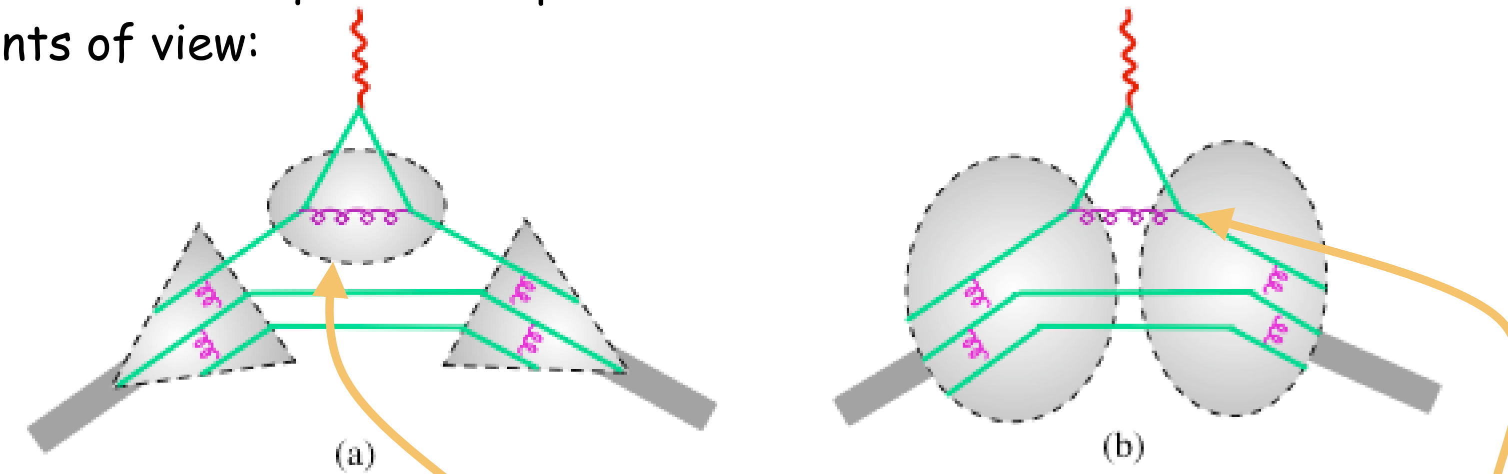

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Light-front view: nucleon wave function is a sum over Fock components; quark "structure" comes from higher Fock components of the hadronic wave function.

The light-front view requires $\ell > 0$ components just to give $\kappa_{\pm} \neq 0$

PART II: Using the

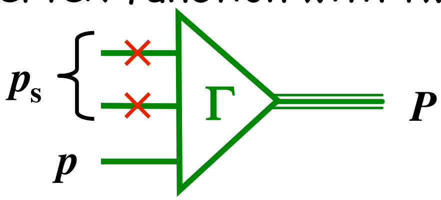
Covariant Spectator Theory© (CST)

to model the high Q^2 behavior of

γ^* + N $\rightarrow$ N* hadronic vertices

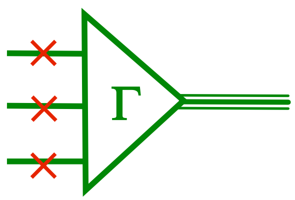
Baryon wave function in CST

- ★ The nucleon consists of 3 constituent quarks (CQ) with a size, mass, and form factor given by the dressing of the quark in the sea of gluons and $q\bar{q}$ pairs. The sea quarks can be neglected.
- ★ Using the Covariant Spectator© theory, the nucleon and Δ is described by a 3-CQ vertex function with two of the CQ on shell



$$\Psi_\alpha = \left(\frac{1}{m - \not{p} - i\varepsilon}\right)_{\alpha\beta} \Gamma_\beta(P, p_s)$$

- ★ Confinement insures that this vertex function is zero when all three quarks are on shell (i.e. there is no 3q scattering)



= 0

Hence, model Ψ directly (with no singularities at $p^2 = m^2$)

- ★ How should the wave function be modeled? Ockham's razor: *Start with the simplest case -- pure S-state with same spin-isospin structure as the nonrelativistic wave function.* See if it works!

Structure of the model baryon wave functions

- ★ For non-strange systems (u and d quarks only) fermion antisymmetry comes from the color factor $\varepsilon_{\alpha\beta\gamma}$. The rest of the wf must be symmetric.

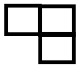
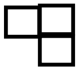
- ★ Construct nonrelativistic wave functions first; then generalize by replacing (for example)

$$\mathbf{k} \to k^{\mu} - \frac{(k \cdot P)P^{\mu}}{P^{2}}$$

$$\delta^{ij} \to -g^{\mu\nu} - \frac{P^{\mu}P^{\nu}}{P^{2}}$$

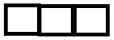
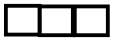
- ★ Starting from a simple S state spatial wave function, spin-isospin structure must be symmetric. Spin-isospin 1/2 requires superposition of mixed symmetry states

$$\left| p \uparrow \right\rangle = \frac{1}{\sqrt{2}}\left\{ \Phi_F^0 \Phi_S^0 + \Phi_F^1 \Phi_S^1 \right\}$$

Spin-isospin 3/2 requires pure symmetric states

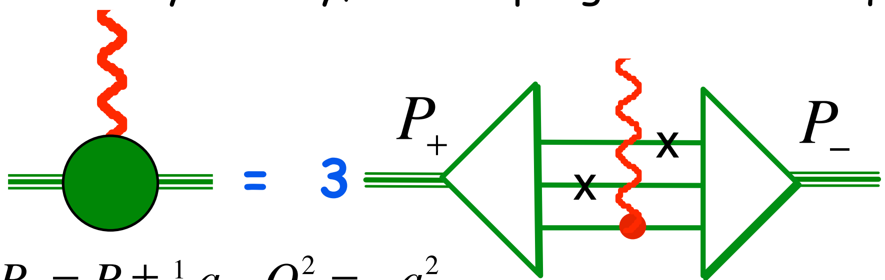
$$\left| \Delta \uparrow \right\rangle = \bar{\Phi}_F^1 \bar{\Phi}_S^1$$

- ★ When angular momentum components are added, F no longer necessarily equal to S.

Relativistic impulse approximation for the form factors

In the spectator theory, the photon couples to the off-shell quark, and because of the symmetry, the coupling to all three quarks is 3 times the coupling to one



$P_\pm = P \pm \frac{1}{2} q \quad Q^2 = -q^2$

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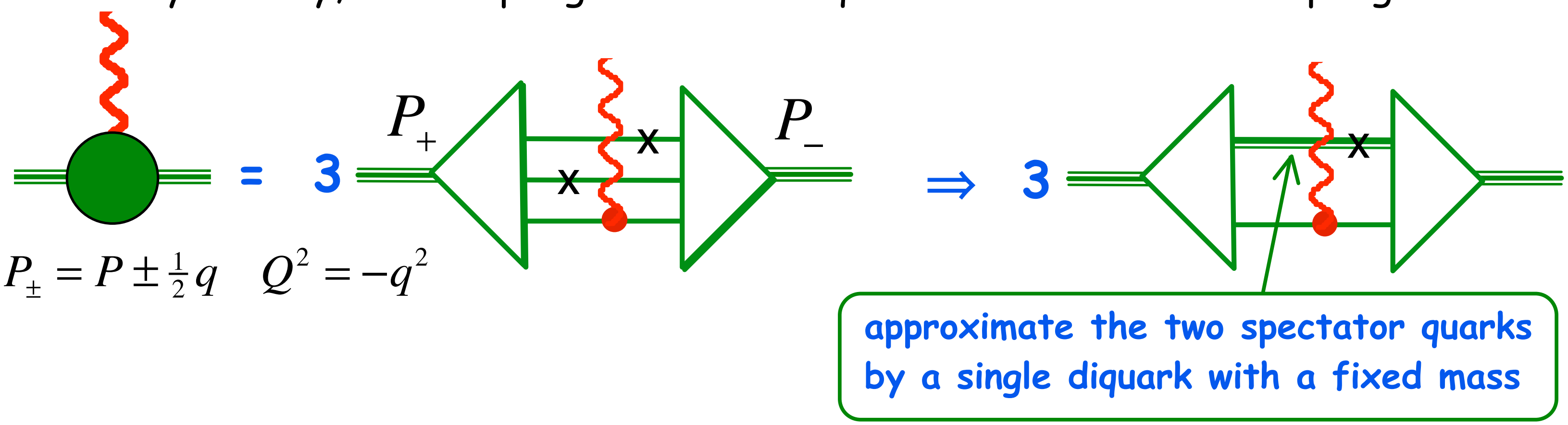

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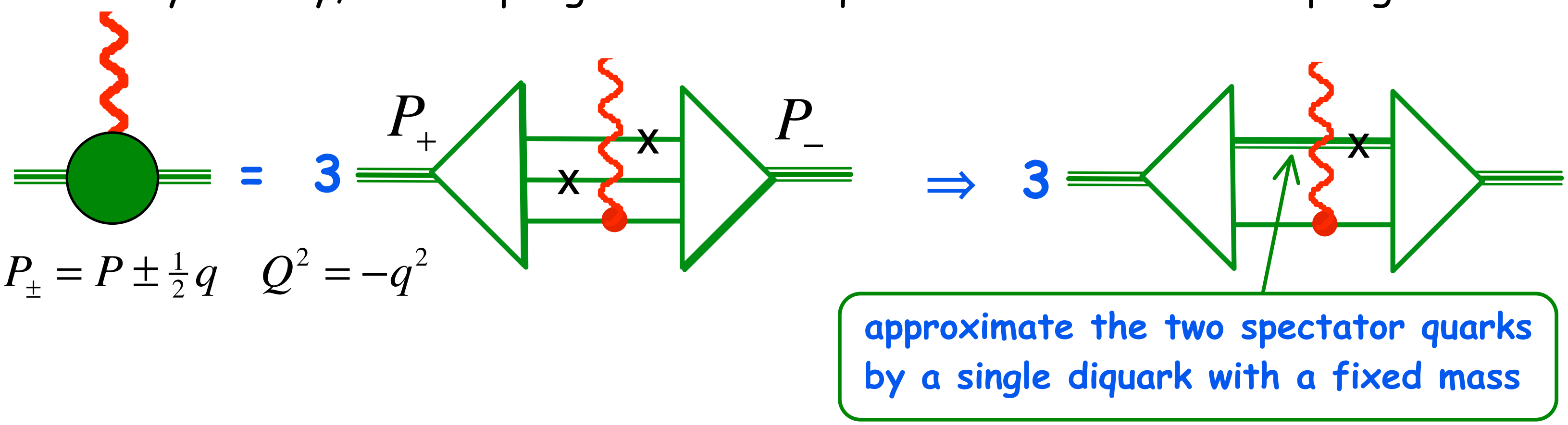


$P_\pm = P \pm \tfrac{1}{2} q \quad Q^2 = -q^2$

$$J^\mu = 3 \sum_\lambda \int \frac{d^3k}{(2\pi)^3 2E_s(k)} \bar{\Psi}_f(P_+, k)\, j_I^\mu(q)\, \Psi_i(P_-, k)$$

integrate over the (on-shell) spectator three momentum

quark current with form factors

Historical overview of our work

- Pure S-wave nucleon wave function shown to fit the nucleon form factors
  - Assumes constituent quarks with form factors
  - Does not violate of the "angular momentum theorem" because quarks have structure (recall previous slide)
- This wave function used to explain (or predict) transition form factors for $\gamma^* + N \to N^*$ where $N^* = \Delta(1236)$, $P_{11}(1440)$, $S_{11}(1535)$, and $\Delta(1600)$
- Calculations have been extended to the strange sector.
- Recently, we extracted a new N wave function (without any pion cloud contributions) directly from DIS. Best model gives 35% D-state!!
- Next generation of calculations:
  - Use N wave function extracted from DIS
  - Calculate pion cloud using constraints obtained from octet magnetic moments
  - Revise the $\gamma^* + N \to N^*$ calculations

Nucleon form factors with S-wave nucleons (2008)*

- ★ N wave function is S-wave only: 2 parameters
- ★ Quark form factors:

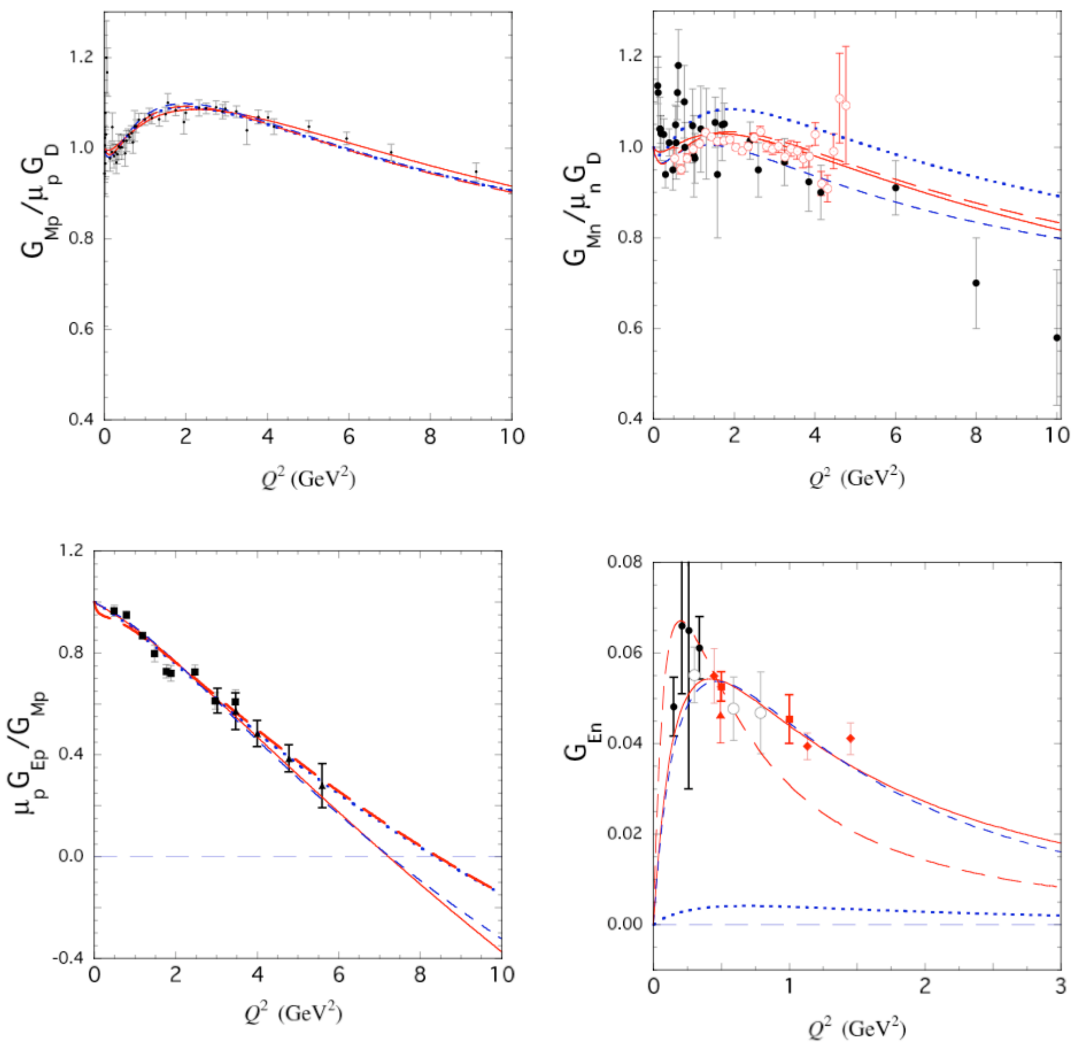
$$f_{1\pm} = \lambda + \frac{1-\lambda}{1+Q_0^2/m_v^2} + \frac{c_\pm Q_0^2/M_h^2}{\left(1+Q_0^2/M_h^2\right)^2}$$

2 parameters $\begin{cases} \lambda & \text{fixed by DIS} \\ c_\pm & \text{fit} \end{cases}$

$$f_{2\pm} = \kappa_\pm \left(\frac{d_\pm}{1+Q_0^2/m_v^2} + \frac{(1-d_\pm)}{1+Q_0^2/M_h^2} \right)$$

1 parameter $\begin{cases} \kappa_\pm & \text{fixed by moments} \\ d_+ = d_- & \text{fit} \end{cases}$

- ★ Blue lines: NO pion cloud
- - - - Best phenomenology (5 parameters)
- ★ Red lines: with a pion cloud
- ★ Lesson: S-waves can explain data



*FG, Ramalho, Pena, PRC 77, 015202 (2008)

Results: γ^* + N -> Δ transition with PURE *S*-wave states*

- Three form factors, but ONLY $G^*_M \neq 0$ if BOTH the N and Δ wave functions are pure S-wave. Δ wave function has two new range parametrs.
- The value $G^*_M(0)$ cannot equal the correct value unless a separate pion cloud term is added, because of the Schwartz inequality

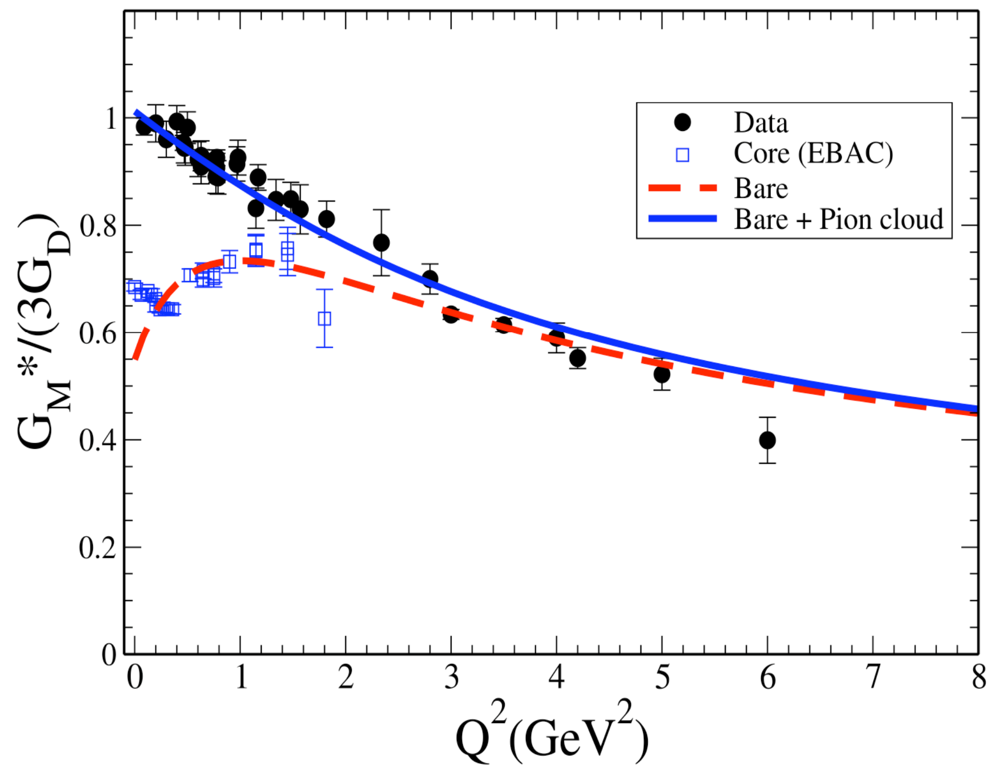
$$G^*_M(0) = \frac{8}{3\sqrt{3}}\left(\frac{m}{M+m}\right) j_- \int \psi_\Delta \psi_N$$
$$= 2.07 \int \psi_\Delta \psi_N, \quad \textbf{and}$$

$$\int \psi_\Delta \psi_N \leq \sqrt{\int |\psi_N|^2} \sqrt{\int |\psi_\Delta|^2} \leq 1$$

- Fit done with an empirical pion cloud term of the form

$$\frac{G^\pi_M}{3G_D} = \lambda_\pi \left(\frac{\Lambda^2_\pi}{\Lambda^2_\pi + Q^2}\right)^2$$

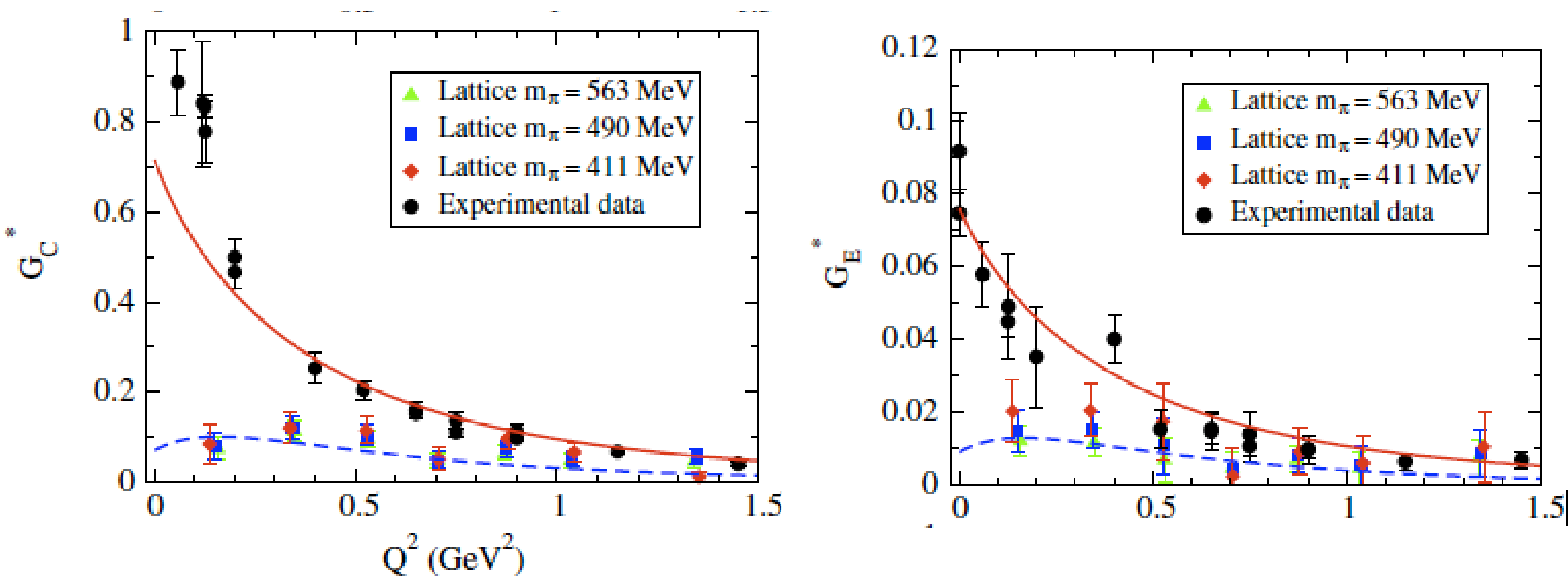
- Lessons:
  - quark core dominates large Q^2 region
  - Bare contributions agree with EBAC analysis



*Ramalho, Pena, FG, EPJA 36, 329 (2008)

Results: γ^* + N -> Δ transition with D-waves*

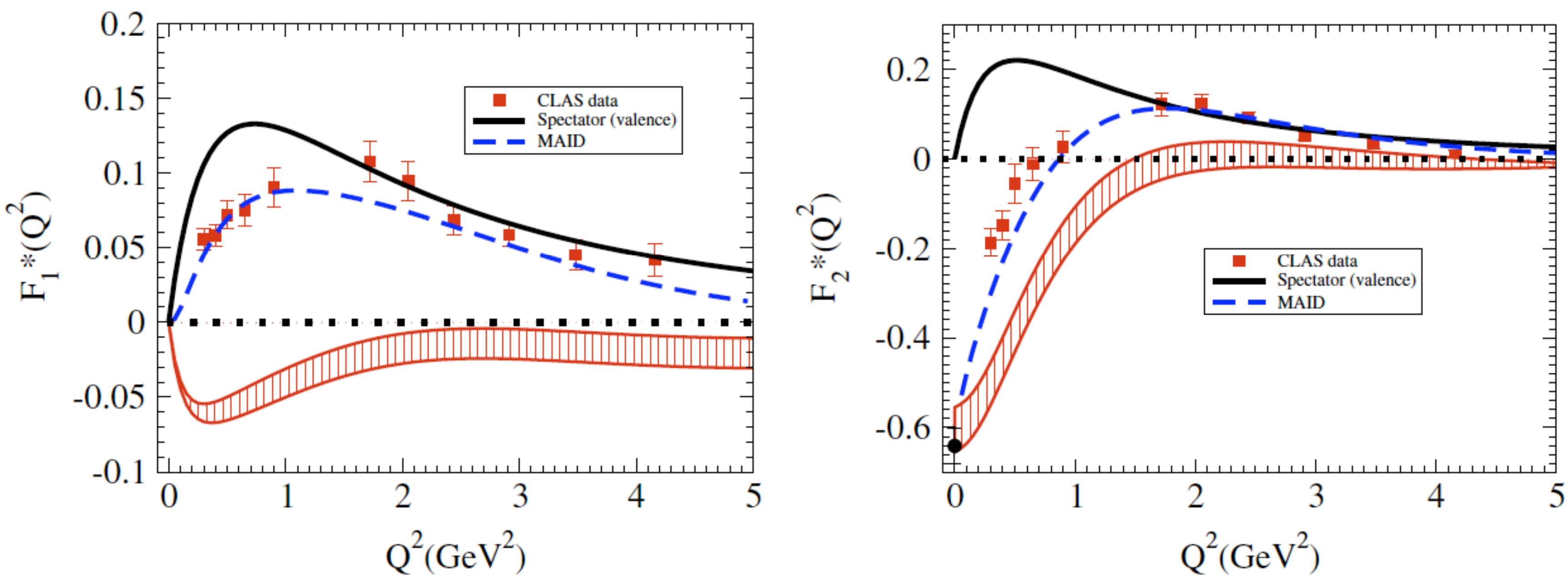
- Determine core D-wave admixtures by fitting G_C^* and G_E^* (0 for pure S-waves) to lattice data.
- Then add pion cloud to fit experimental data
- Lesson: lattice data can be used to fix quark core when both the core and the pion cloud are important



*Ramalho and Pena, PRD 80, 013008 (2009)

Results: γ^* + N -> P_{11} (1440)*

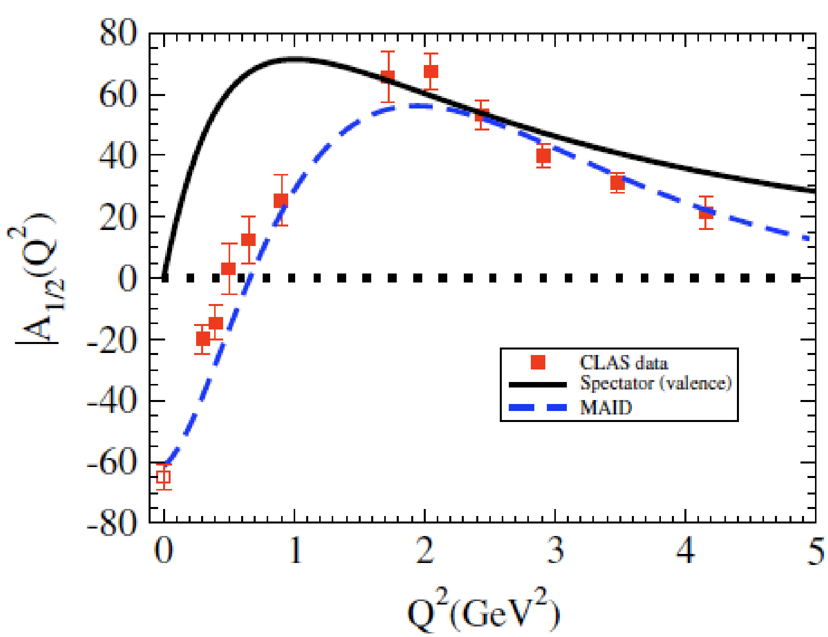
- One extra parameter in N* wave function fixed by orthogonality condition (N* is a radial excitation of N)
- Quark core transition amplitude fits high Q^2 data
- Pion cloud is estimated to be the difference between the MAID fit and the quark core (with error bands taken from the error bars in the data)

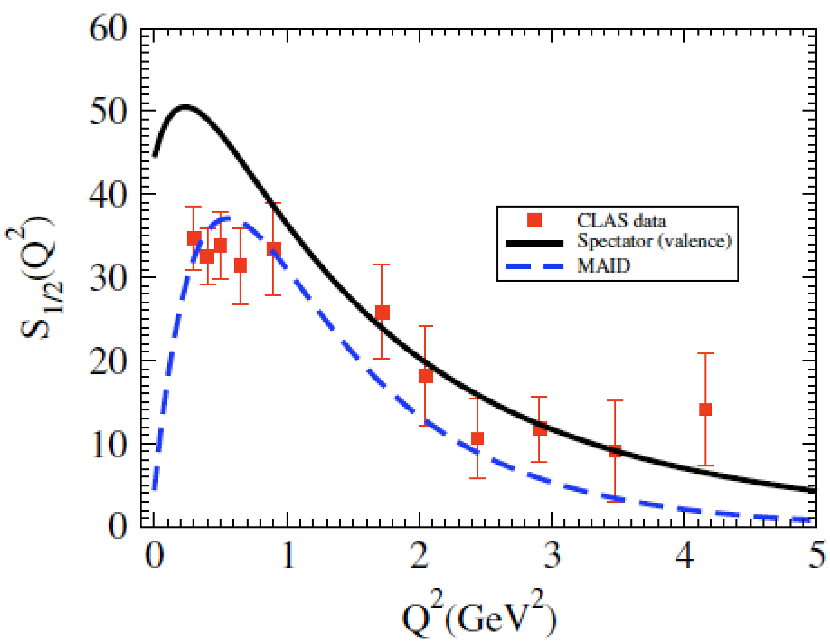


*Ramalho & Tsushima, PRD 81, 074020 (2010)

Results: γ^* + N -> P_{11} (1440)*

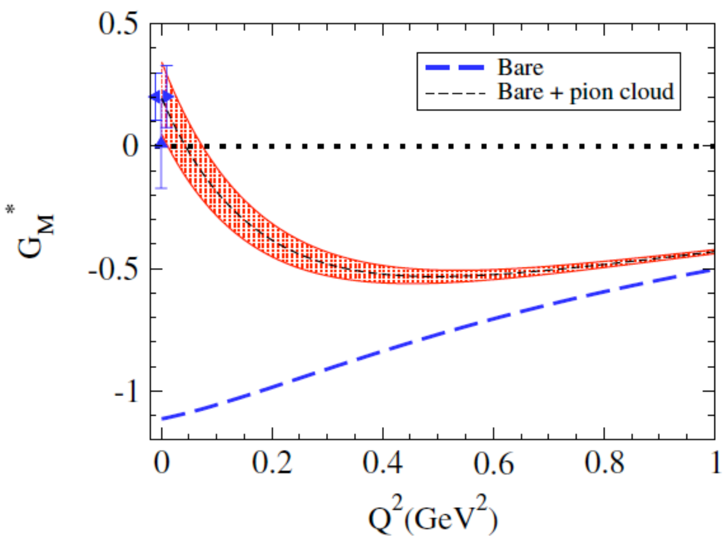
- Results for the helicity amplitudes
- Zero crossing for P_{11} must come from the pion cloud



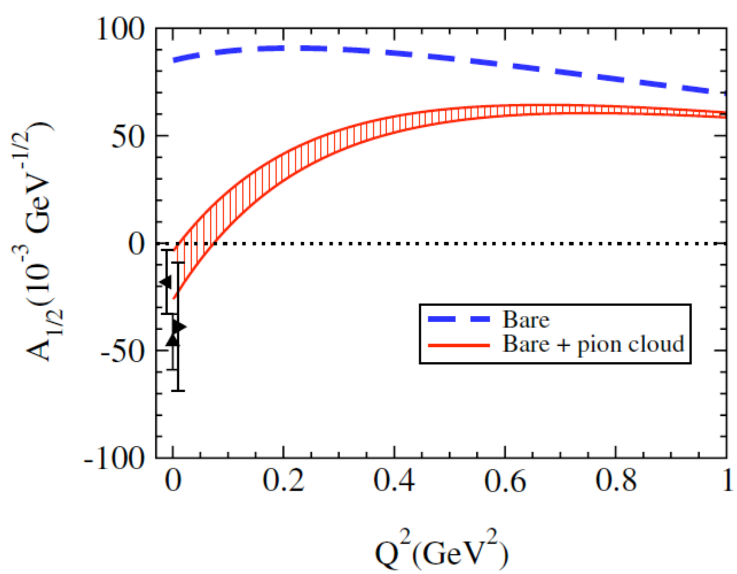


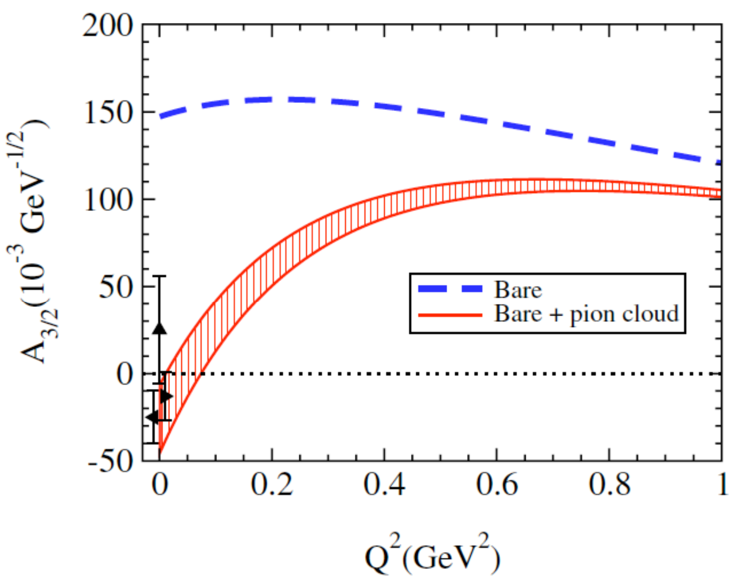
*Ramalho & Tsushima, PRD 81, 074020 (2010)

Results: $\gamma^* + N \rightarrow \Delta(1600)^*$



- ★ Treat Δ(1600) as redial excitation of the Δ(1236)
- ★ Calculate pion cloud contributions from the CBM using intermediate N, N*, Δ, and Δ*

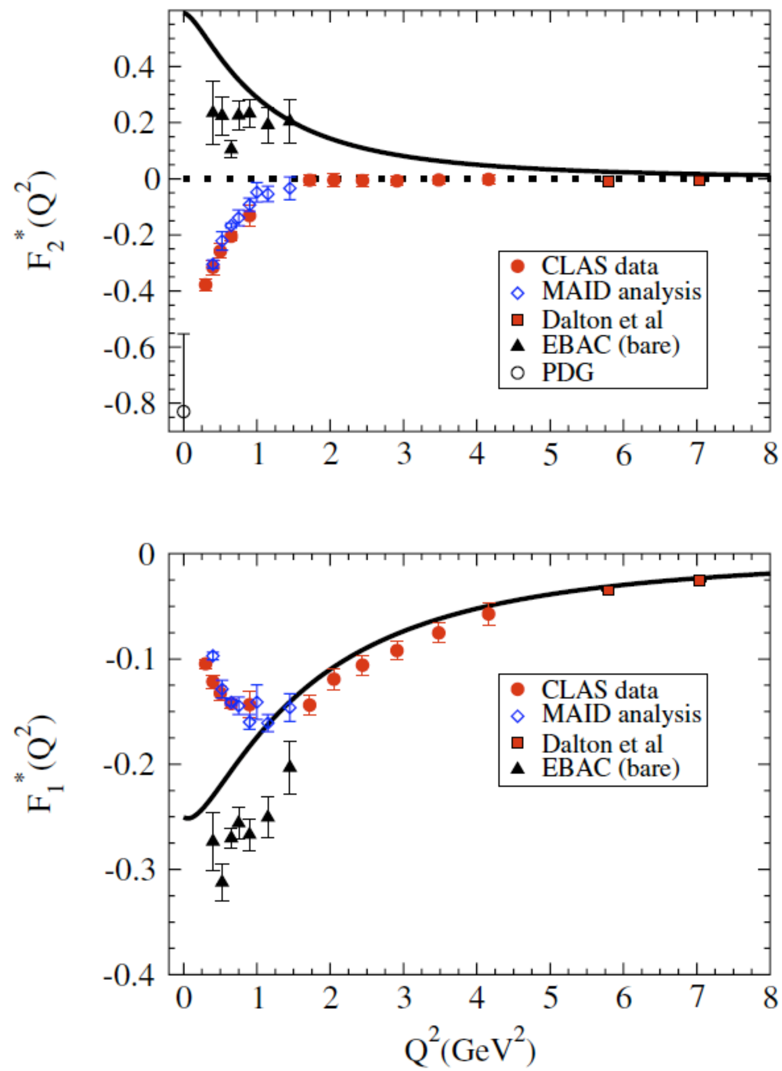




*Ramalho & Tsushima, PRD 82, 073007 (2010)

Results: $\gamma^* + p \rightarrow S_{11}(1535)$*

- Chose a radial wave function identical to the nucleon; only angular (P-wave) part is different. No free parameters
- Bare contribution is close to the bare contribution extracted from the EBAC analysis
- Meson cloud (pion+eta) is predicted to be large and of opposite sign



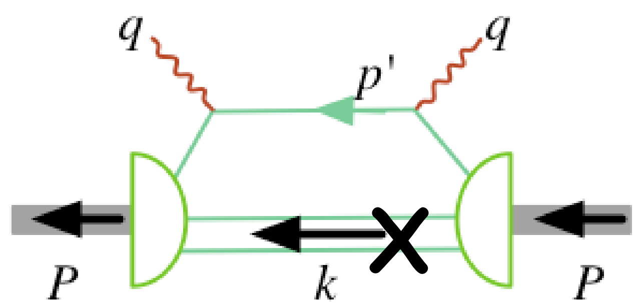
*Ramalho & Pena, PRD 84, 033007 (2011)

PART III:

Lessons from the study of DIS

The core wave functions can be determined from DIS (1)*

- DIS structure functions calculated from the handbag diagram



- Model nucleon wave functions depend on k = Mκ only through the covariant variable

$$\chi = \frac{2P \cdot k}{Mm_s} - 2 = 2\sqrt{1 + \frac{\kappa^2}{r^2}} - 2 \qquad r = \frac{m_s}{M}$$

- For S-wave nucleons, the structure function is

$$f_q(x) = \frac{Mm_s\lambda^2}{16\pi^2}\int_{\xi}^{\infty} d\chi \left[\psi_q(\chi)\right]^2 \quad \text{with} \quad \xi = \frac{(r+x-1)^2}{r(1-x)}$$

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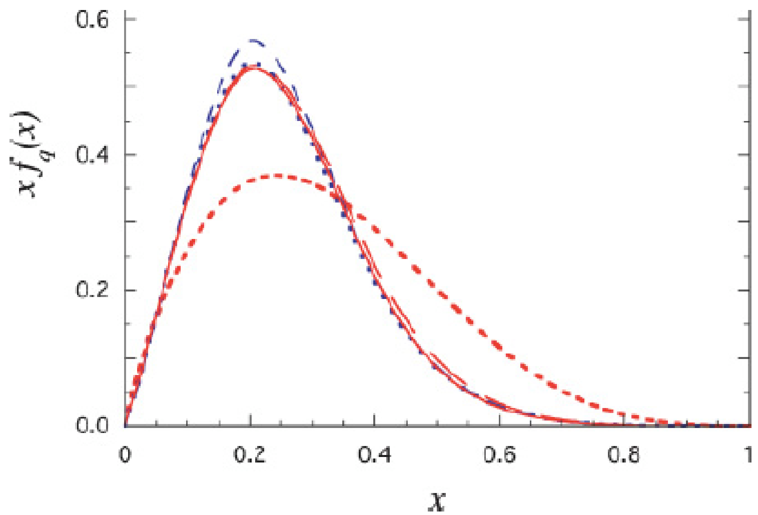
*FG, Ramalho, Pena, PRC 77, 015202 (2008)

*FG, Ramalho, and Pena, PRD 85,093006 (2012)
FG, Ramalho, and Pena, PRD 85,093005 (2012)

The core wave functions can be determined from DIS (2)

★ Our original choice (2008) using wave function fit to form factors

$$\psi(\chi) = \frac{N}{(\beta_1 + \chi)(\beta_2 + \chi)}$$



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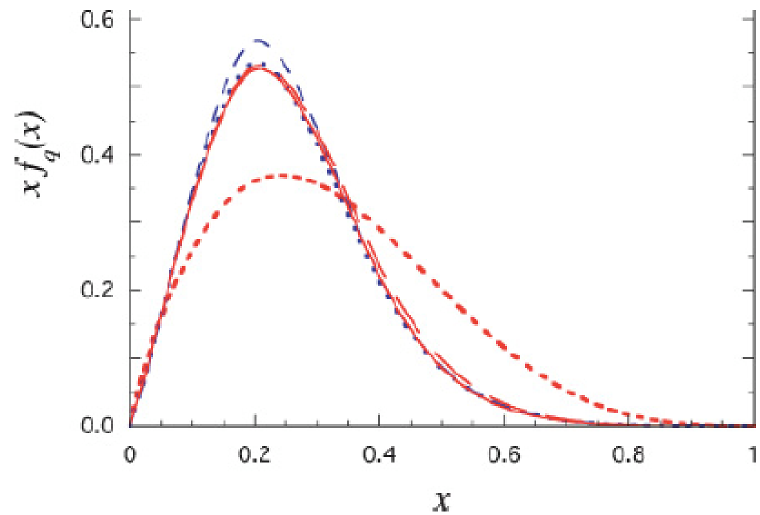
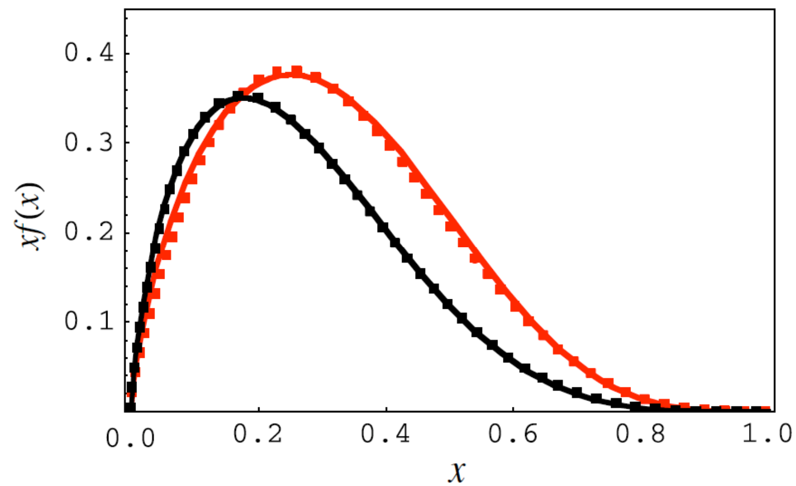


★ New choice (2012) using wave function fit directly to structure function

$$\psi(\chi) = N \frac{\beta \cos\theta + \chi \sin\theta}{\chi^{\alpha} (\beta + \chi)^{n-\alpha}}$$



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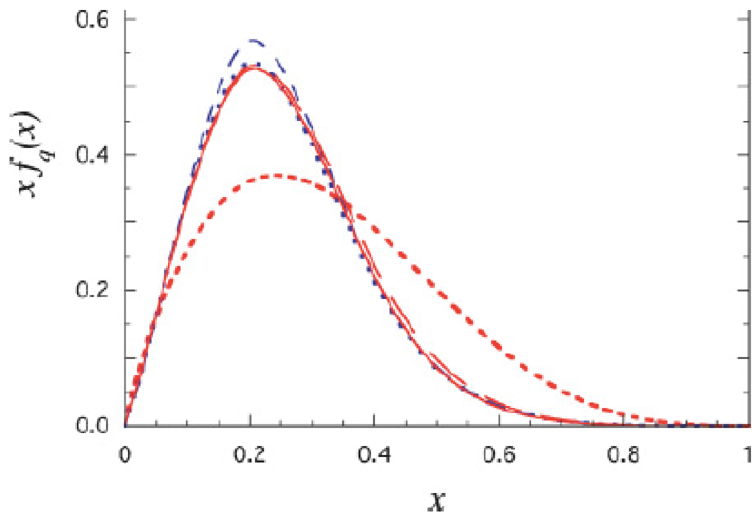

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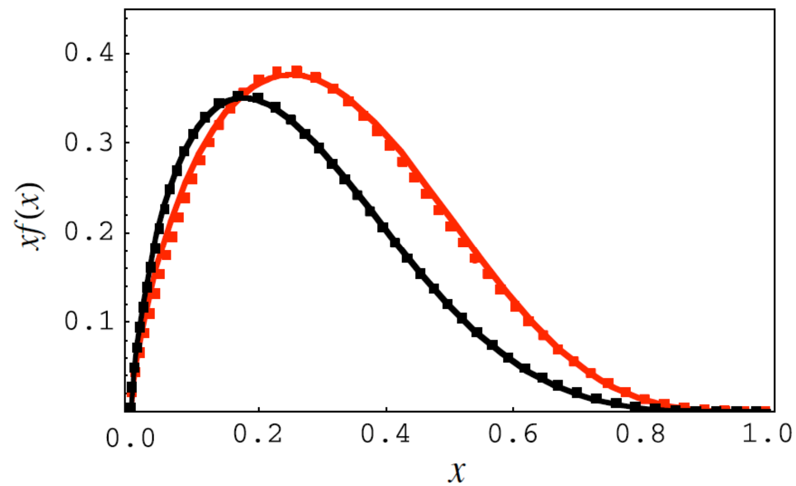
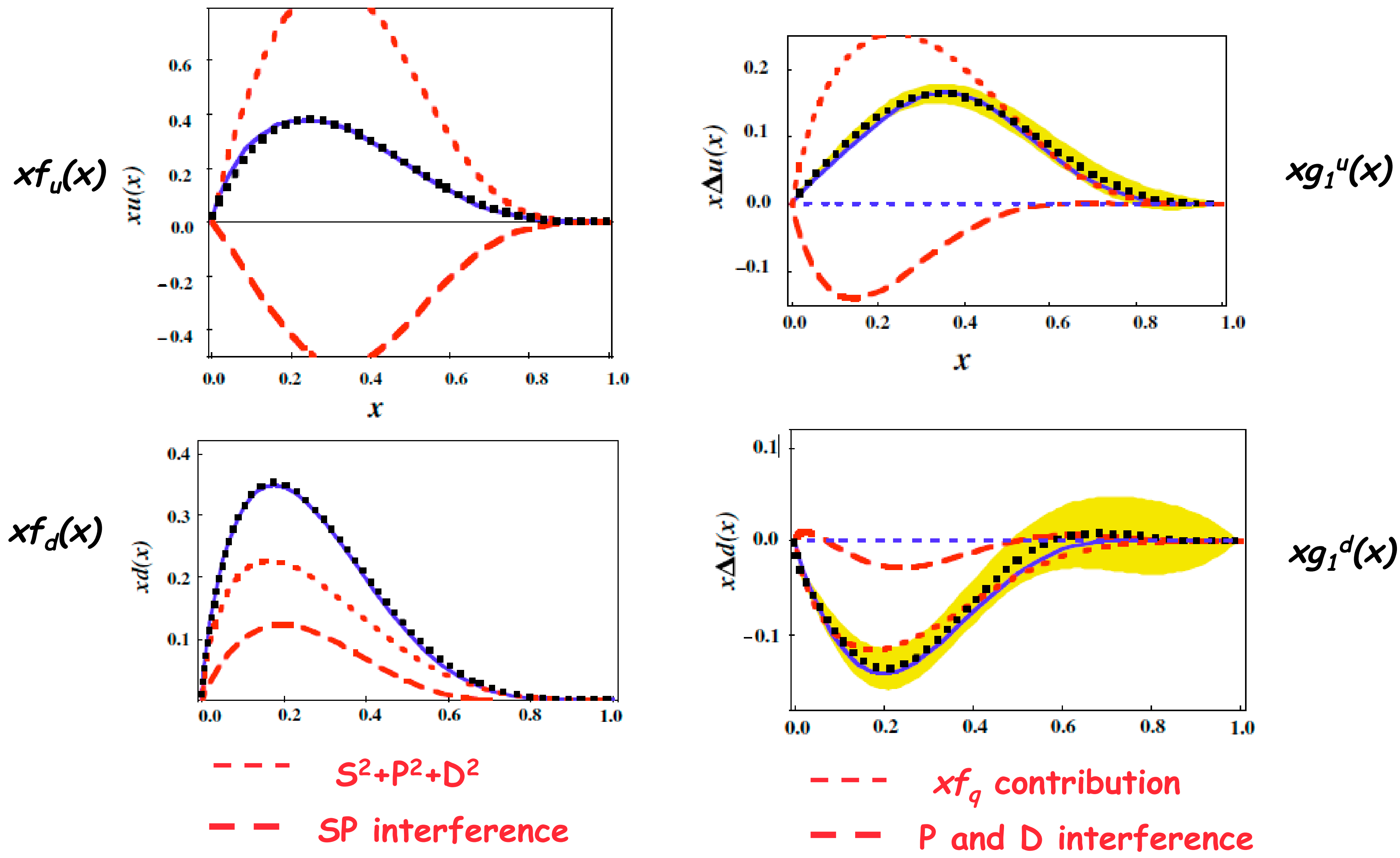
★ Then add P and D-states



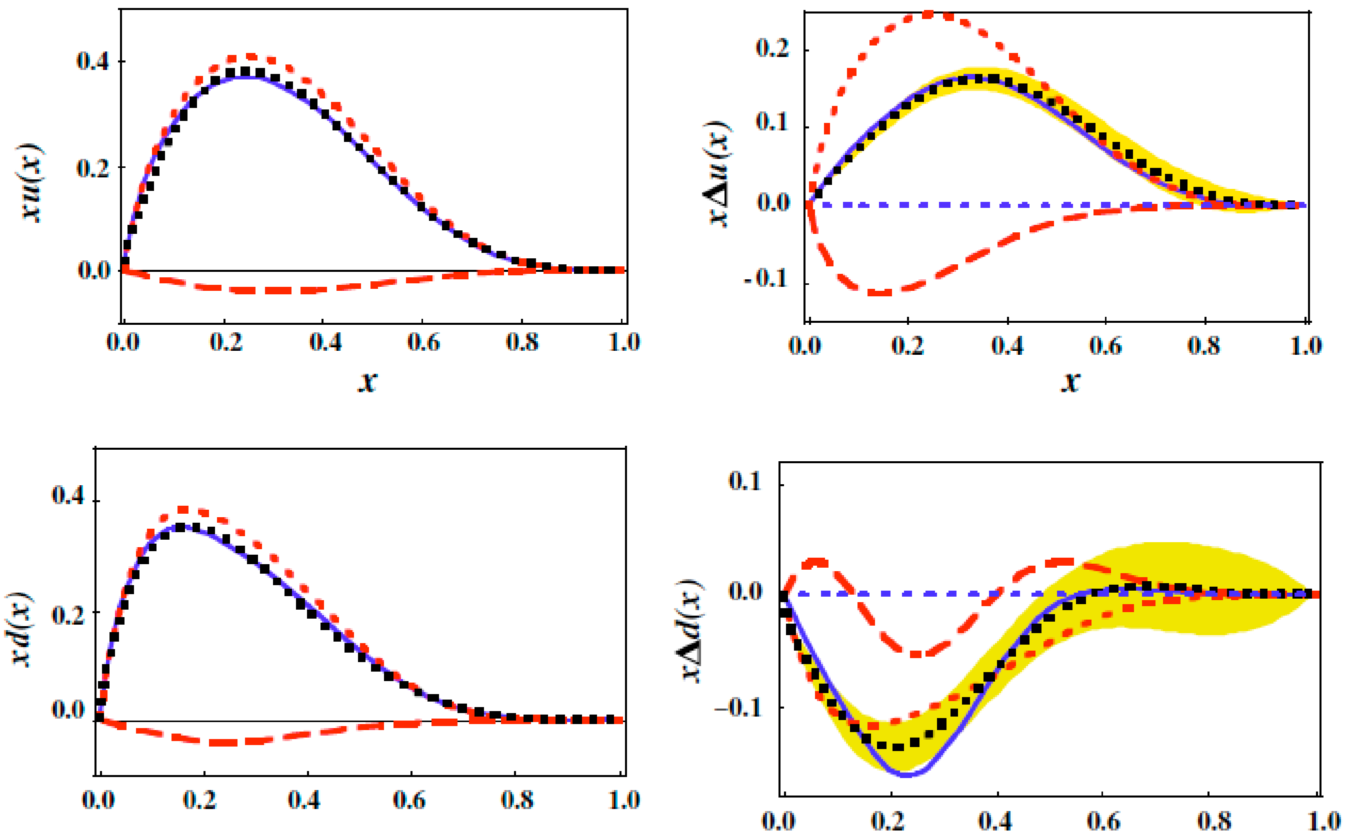
Solution 1 (18.5% P-state; 3.2% D-state): $f(x)$ and $g_1(x)$



Solution 2 (0.6% P-state; 34.8% D-state): $f(x)$ and $g_1(x)$

$xf_u(x)$ $xg_1^u(x)$

$xf_d(x)$ $xg_1^d(x)$



S²+P²+D²

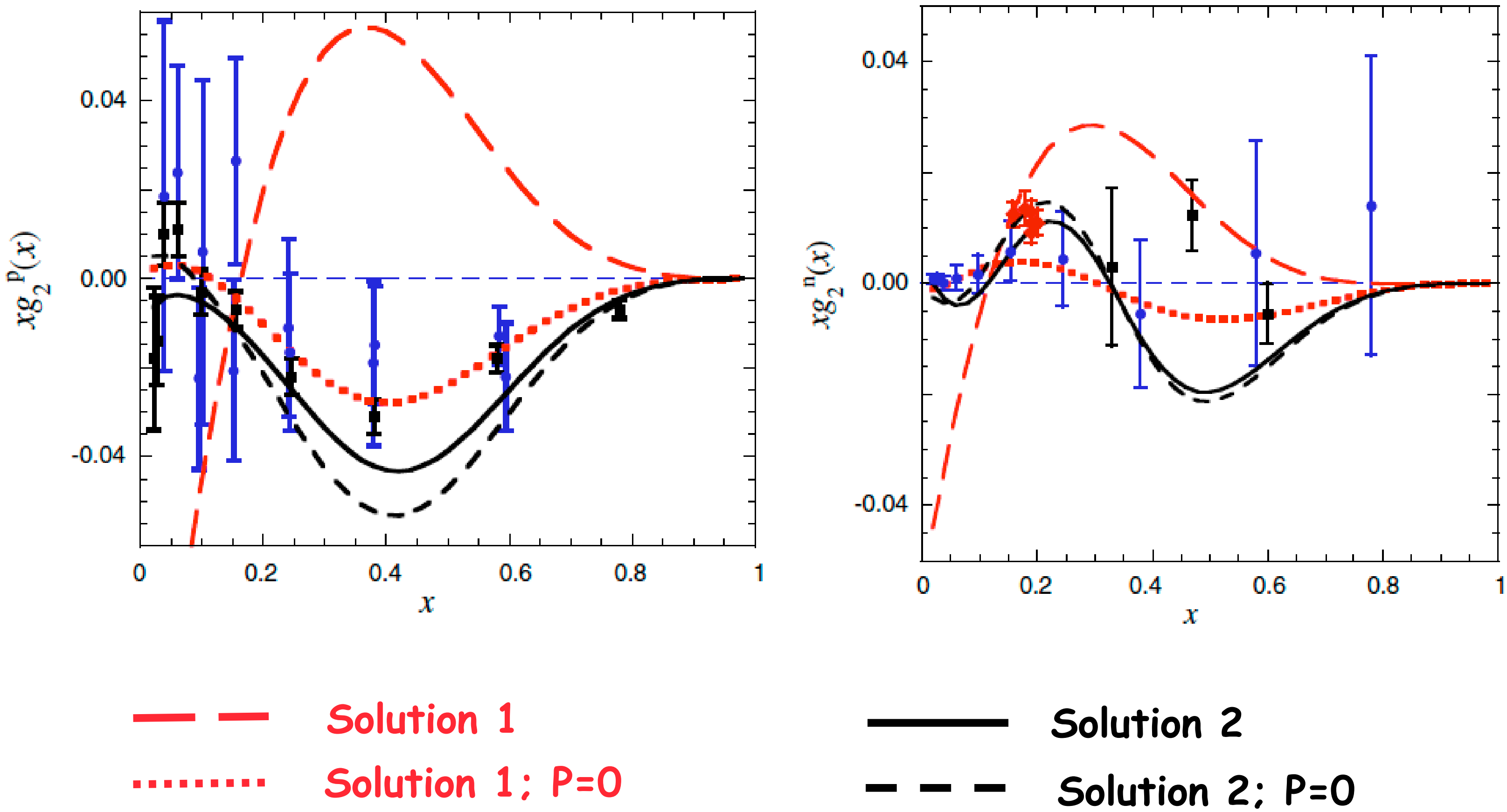
SP interference

xf_q contribution

P and D interference

Prediction for $g_2(x)$

★ Only Solution 2 (35% D state) gives a good result



Discussion and Implications

- ★ Existence and positions of "bare" N* poles are needed to construct the hadronic lagrangian -- the goal of this program (??)
- ★ High Q^2 vertex functions determined from CST modeled N and N* wave functions unify our understanding of
  - experimental data at high Q^2
  - LQCD (at larger pion masses)
  - coupled channel calculations
- ★ CST allows the nucleon wave function to be fixed directly from the experimental DIS data. The nucleon is expected to have a large D-state.
- ★ To do:
  - Calculate pion cloud contributions within the CST framework
  - Fix the quark form factors by refitting the nucleon form factors using new DIS determined wave function and new (theoretical) pion cloud contributions
  - Develop a covariant, gauge invariant, coupled-channel scheme for fitting photo-production data that uses CST vertex functions, parameterized quark form factors, and meson interactions consistent with pion cloud calculations.

END

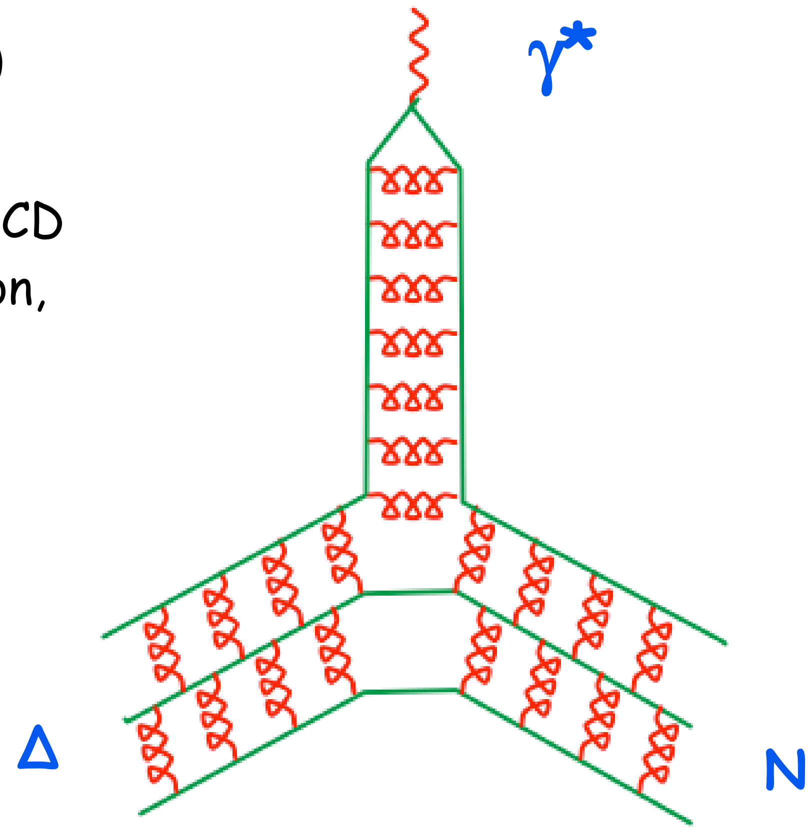
Theoretical issues to address when building a coupled channel SCHEME

- What form of relativistic dynamics, and what degrees of freedom?
  - Hadronic d.o.f.: hamiltonian instant form (Sato and Lee)
  - Hadronic d.o.f.: manifestly covariant (Bethe-Salpeter or Covariant spectator theory)
  - Point-like (current) quark d.o.f.: front form (Brodsky)
- Electromagnetic Gauge invariance (use Gross&Riska?)
- Orthogonality of the P_{11} states (including the nucleon)
- Consistent treatment of
  - s and u channels,
  - two and three body final states
  - pion cloud physics
- Beware! Interpretation of angular momentum depends on the form of dynamics and the degrees of freedom

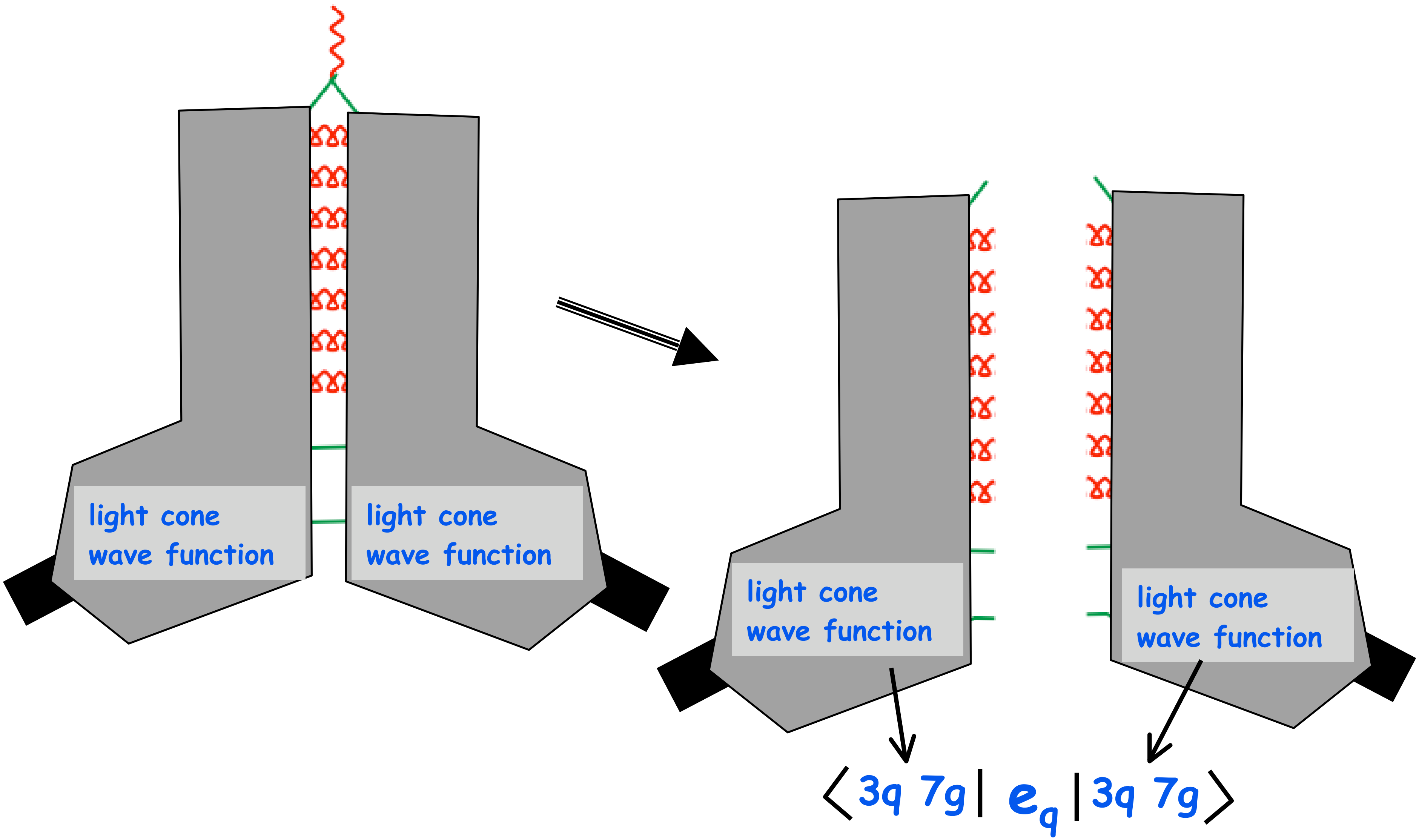
Two differing pictures -- with equivalent physics

- ★ Light front field theory (or quantum mechanics)
- ★ Covariant Spectator Theory (CST)

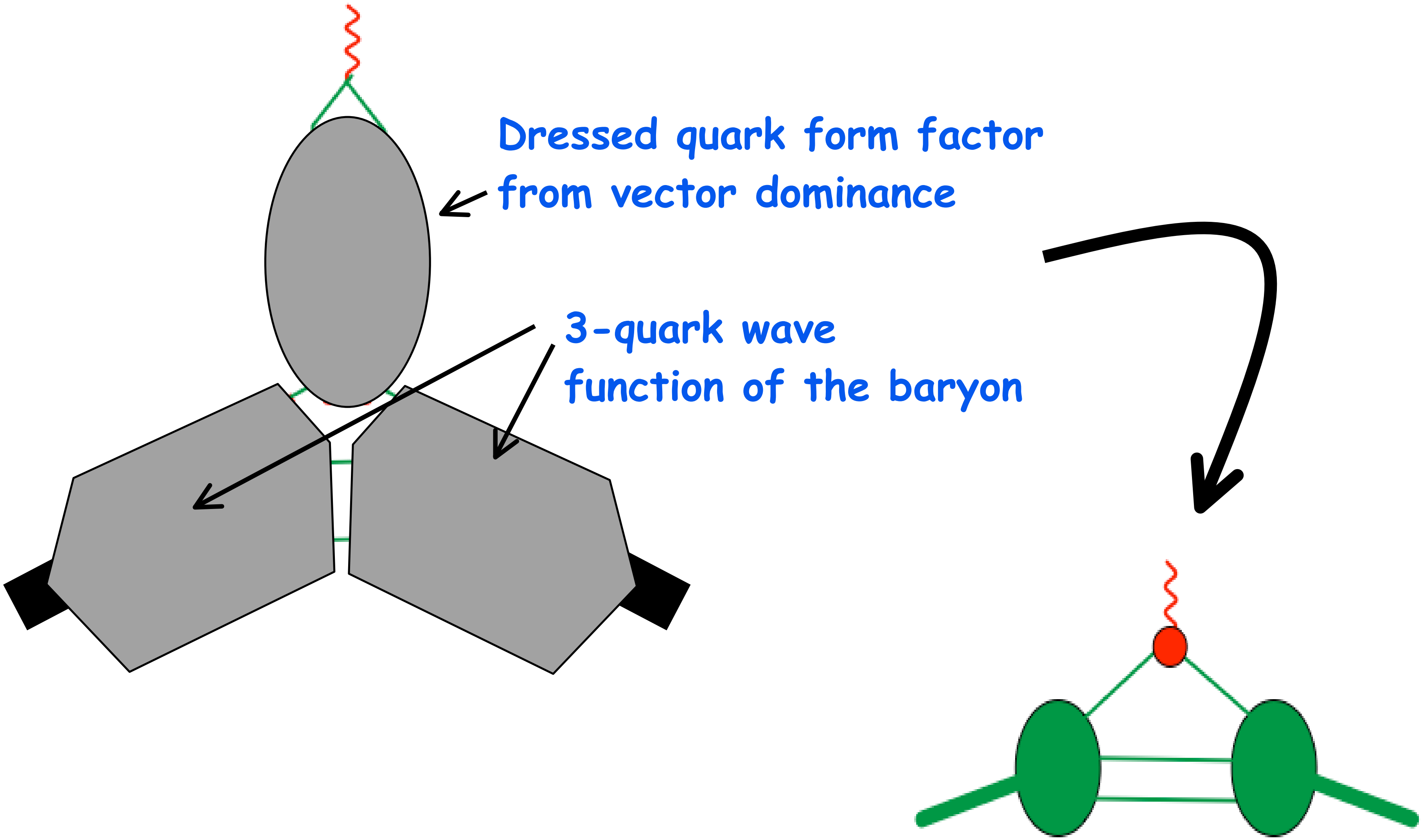
How do they describe a "typical" QCD diagram? (For the N $\rightarrow$ Δ transition, for example)



"Typical QCD diagram" : Light-Front interpretation

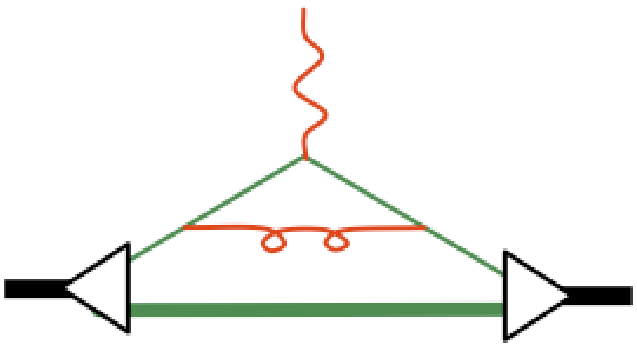


“Typical QCD diagram”: CST interpretation

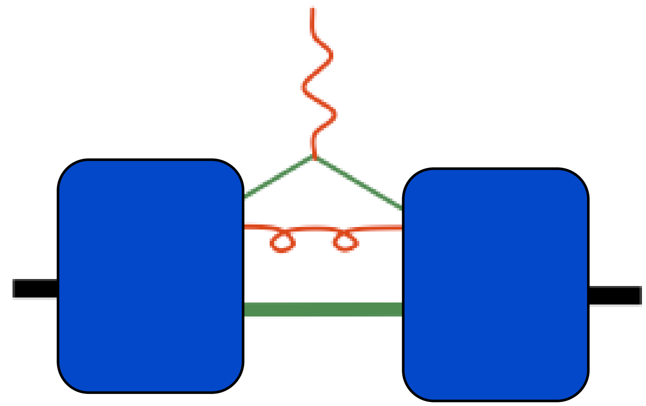


Angular momentum content depends on your point of view

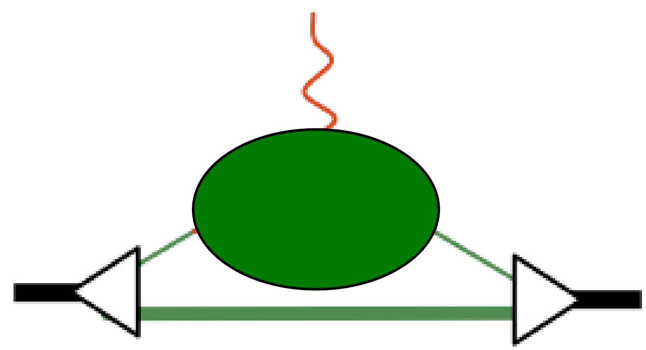
- ★ Two ways to interpret this process



Light cone; point-like quarks



$\langle N | qqqg \rangle \quad e_q \gamma^\mu \quad \langle qqqg | N \rangle$



CST; constituent quarks

$\langle N | qq \rangle \quad e_q F(q) \gamma^\mu \quad \langle qq | N \rangle$

Spin-isospin structure (2): nucleon

★ Spin-isospin structure of the NR nucleon wave function (cont'd)

introduce a mathematically compact form; suppress name of scalar diquark

$$|sf\rangle = \frac{1}{\sqrt{2}}\left\{\chi^s\chi^f \quad - \quad \frac{1}{3}\left[\sigma\cdot\xi_m^*\,\chi^s\right]\left[\tau\cdot\xi_F^*\,\chi^f\right]\right\}$$

(0,0) diquark
scalar spin
scalar flavor

(1,1) diquark
axial vector spin
vector flavor

$$\chi^{\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix} \qquad \chi^{-\frac{1}{2}} = \begin{pmatrix}0\\1\end{pmatrix}$$

★ Relativistic wave function, including spin-flavor structure

$$\Psi_N(sf) = \frac{1}{\sqrt{2}}\left\{u(P,s)\chi^f + \frac{1}{3}\left(\gamma^5\,\not{\varepsilon}_m^*\right)u(P,s)\left[\xi_F^*\cdot\tau\,\chi^f\right]\right\}\phi(P,p_s)$$

$$u(P,s) = N\begin{bmatrix}1\\ \dfrac{\sigma\cdot\mathbf{P}}{E_P+M}\end{bmatrix}\chi^s$$

★ When P = 0, the lower component is 0 and this reduces *exactly* to the nonrelativistic form

This is a fixed-axis state

Spin-isospin structure (3): Delta

★ Spin-isospin structure of the Delta wave function is pure (1,1); diquark with axial-vector spin and vector flavor

$$\Psi_\Delta(sf) = -\left[\xi_F^* \cdot T\, \tilde{\chi}^f\right] \varepsilon_m^{\mu *} w_\mu(P,s)\, \phi(P,p_s)$$

- where $\varepsilon_m^{\mu *}$ is a fixed-axis axial-vector polarization
- $w_\mu(P,s)$ is a Rarita-Schwinger wave function satisfying

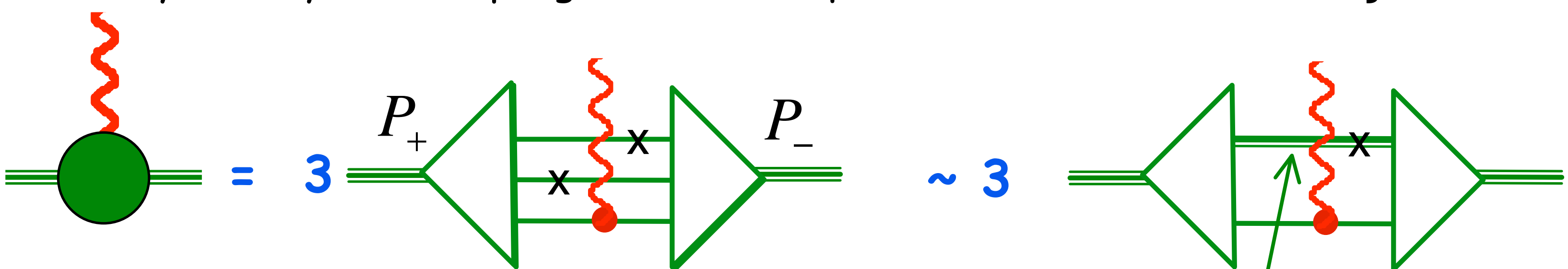
$$\gamma^\mu w_\mu = 0; \quad P^\mu w_\mu = 0$$

- T^i is an isospin 3/2 -> 1/2 transition operator
- $\tilde{\chi}^f$ is the isospin state of the Δ

★ when P = 0, the lower component is zero and this reduces *exactly* to the nonrelativistic form

Relativistic impulse approximation for the form factors

In the spectator theory, the photon couples to the off-shell quark, and because of the symmetry, the coupling to all three quarks is 3 times the coupling to one



$P_\pm = P \pm \frac{1}{2}q \quad Q^2 = -q^2$

approximate the two spectator quarks by a single diquark with a fixed mass

$$J_I^\mu = \bar{u}(P_+,\lambda')\,\frac{3}{2}\int \frac{d^3 p_s}{(2\pi)^3 2E_s(p_s)}\phi(P_+,p_s)\phi(P_-,p_s)\left\{ j_I^\mu - \frac{1}{9}\gamma^\nu\gamma^5\tau_j j_I^\mu \tau_j\gamma^5\gamma^{\nu'} D_{\nu\nu'}\right\} u(P_-,\lambda)$$

integrate over the (on-shell) spectator three momentum

quark currents with form factors

sum over the vector diquark polarization $D_{\nu\nu'} = \sum_\lambda \varepsilon_\nu \varepsilon^*_{\nu'}$

$$\phi(P\cdot p) = \frac{N_0}{\big(\alpha_1 + \chi(P\cdot p)\big)\big(\alpha_2 + \chi(P\cdot p)\big)}$$

Quark form factors (based on vector meson dominance)

★ The quark currents are

$$j_I^\mu = j_1\left(\gamma^\mu - \frac{\not{q}\, q^\mu}{q^2}\right) + j_2\,\frac{i\sigma^{\mu\nu}q_\nu}{2M},$$

+ is isoscalar

− is isovector

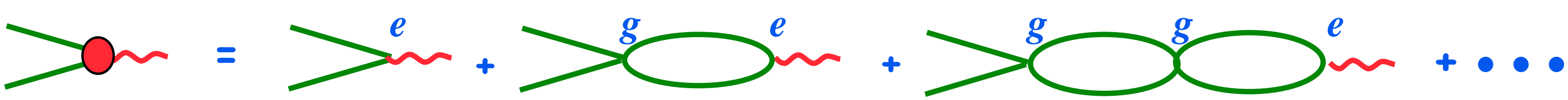
with 4 form factors:

$$j_i = \tfrac{1}{6} f_{i+}(Q^2) + \tfrac{1}{2}\tau_3 f_{i-}(Q^2)$$

$f_{1\pm}(0) = 1$

$f_{2\pm}(0) = \kappa_\pm$ quark anomalous moment

★ The quark form factors come from vector dominance



in a simple bubble model

$$f(Q^2) = e + gB(Q^2)e + gB(Q^2)gB(Q^2)e + \cdots = e + \frac{gB(Q^2)e}{1 - gB(Q^2)}$$

if $gB(Q^2) = \dfrac{\lambda^2}{\Lambda^2 + Q^2}$, then $f(Q^2) = e + \dfrac{\lambda^2 e}{\Lambda^2 - \lambda^2 + Q^2}$

★ We use

$$f_{1+} = \lambda + \frac{1-\lambda}{1 + Q_0^2/m_v^2} + \frac{c_\pm\, Q_0^2/M_h^2}{\left(1 + Q_0^2/M_h^2\right)^2}$$

3 parameters: λ fixed by DIS; $c_\pm$ fit

$$f_{2\pm} = \kappa_\pm\left(\frac{d_\pm}{1 + Q_0^2/m_v^2} + \frac{(1 - d_\pm)}{1 + Q_0^2/M_h^2}\right)$$

4 parameters: $\kappa_\pm$ fixed by moments; $d_\pm$ fit

The sum over the diquark polarization is tricky

- First, must be in a collinear frame so that the fixed-axes (tied to the nucleon and Δ momentum) are identical

- Then, the most general form is

$$D^{\mu\nu} = \sum_{\lambda} \varepsilon_{+}^{\mu} \varepsilon_{-}^{\nu *} = \left(-g^{\mu\nu} + \frac{P_{-}^{\mu} P_{+}^{\nu}}{} \right)$$

$$+ a_1 \left(P_{-} - \frac{bP_{+}}{M_{+}^{2}} \right)^{\mu} \left(P_{+} - \frac{bP_{-}}{M_{-}^{2}} \right)^{\nu}$$

with $b = \left(P_{+} \cdot P_{-} \right)$ **and** $a = \dfrac{M_{+} M_{-}}{b\left(M_{+} M_{-} + b \right)}$

- For equal masses this becomes

$$D^{\mu\nu} = -g^{\mu\nu} - \frac{P_{+}^{\mu} P_{-}^{\nu}}{M^{2}} + 2 \frac{P^{\mu} P^{\nu}}{P^{2}}$$ **where** $P = \tfrac{1}{2}\left(P_{+} + P_{-} \right)$

Results: Nucleon form factors

★ Nucleon Form factors

★ Four models χ^2/N

I (4 parameters)

$\alpha_1 \quad \alpha_2$ quarks with isospin symmetry

$c_+ = c_-$

$d_+ = d_-$ ············ 9.26

II (5 parameters)

$\alpha_1 \quad \alpha_2$ break charge symmetry

$c_+ \neq c_-$ — — — — 1.36

$d_+ = d_-$

Best phenomenology!

III (6 parameters)

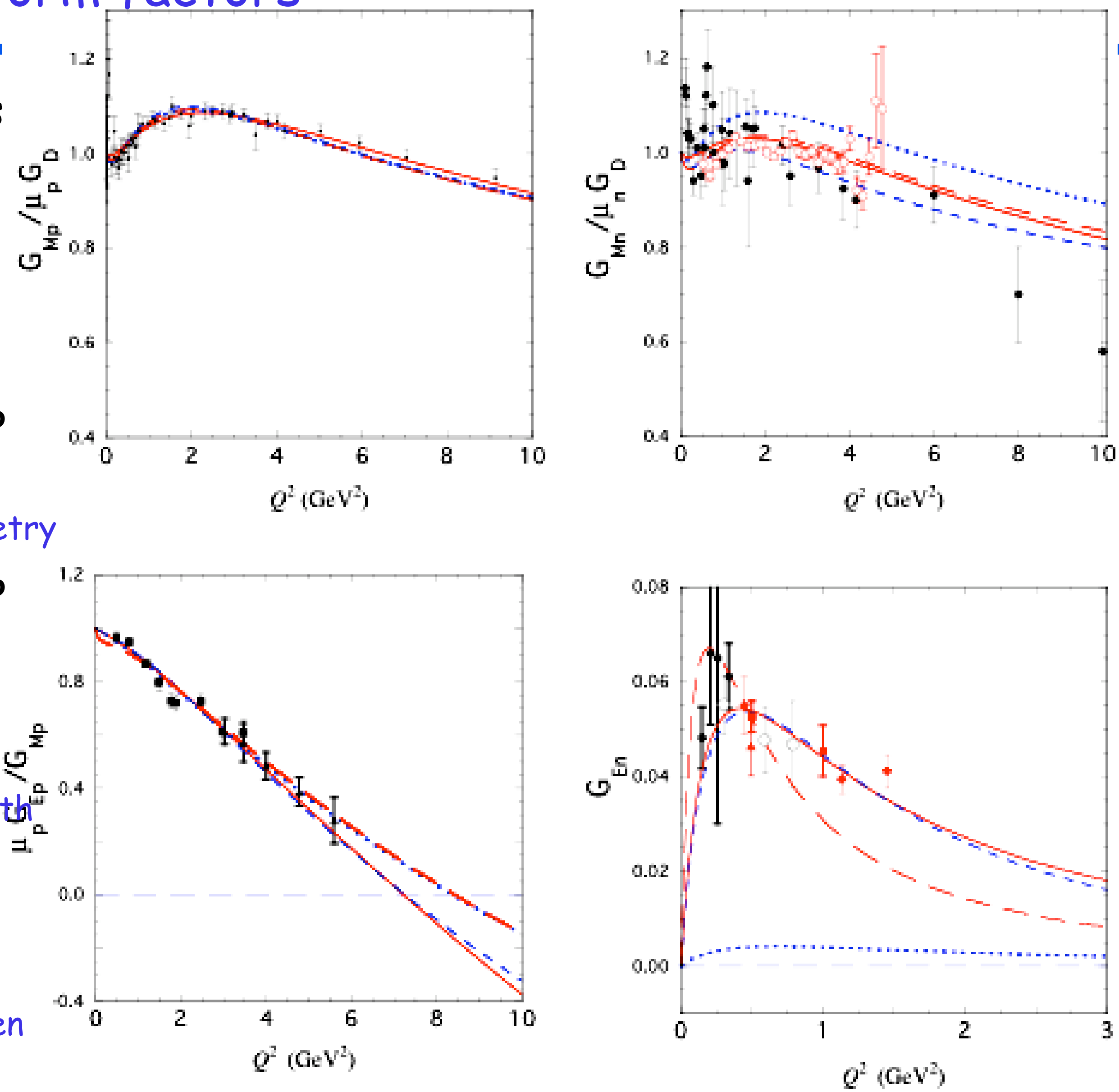
isospin symmetry with pion clouds

— — — - 1.85

IV (9 parameters)

pion cloud and broken isospin symmetry

——— 1.03



Quark distribution function from DIS*

- ★ Our model predicts the quark distribution amplitude measured in DIS
- ★ Our normalization gives

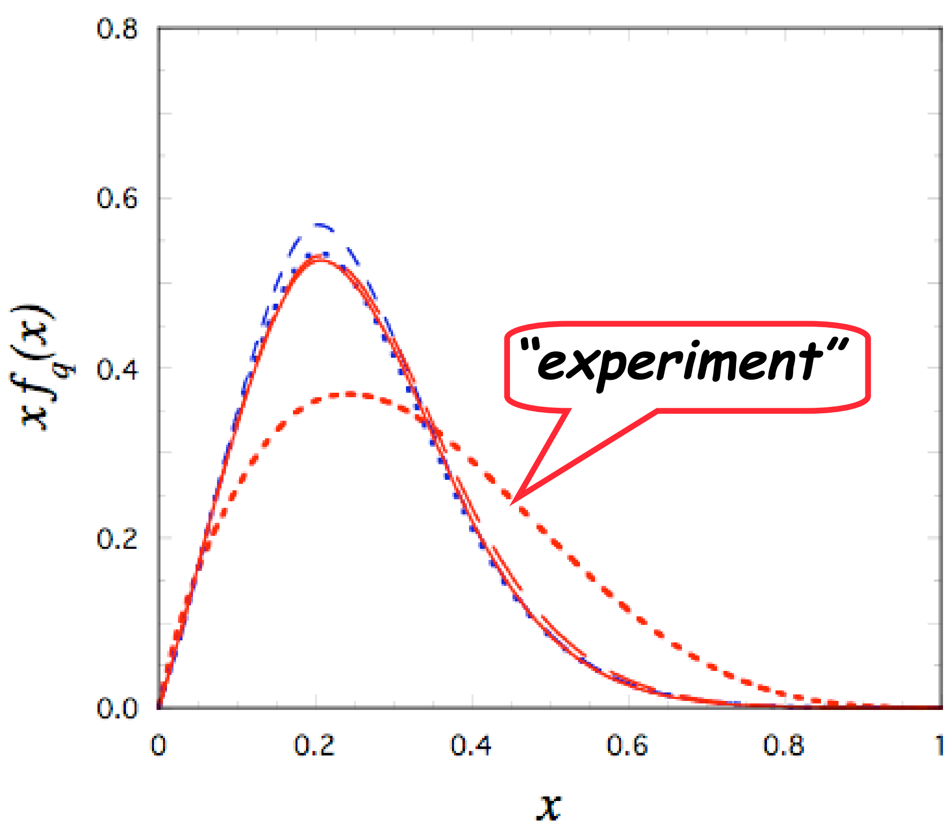
$$1 = \int_{-\infty}^{1} dx f(x) \quad \textbf{instead of} \quad 1 = \int_{0}^{1} dx f(x)$$

- ★ Choose quark charge at $Q^2=\infty$ to be $\lambda>1$, where

$$\int_0^1 dx f(x) = \frac{1}{\lambda^2} < 1$$

- ★ Choose diquark mass to give experimental momentum fraction

$$\frac{\langle xf \rangle}{\langle f \rangle} = \frac{\int_0^1 dx\, xf(x)}{\int_0^1 dx f(x)} = 0.171$$



The core wave functions can be determined from DIS*

- DIS structure functions calculated from the handbag diagram
- Model nucleon wave functions depend on k = Mκ only through the covariant variable

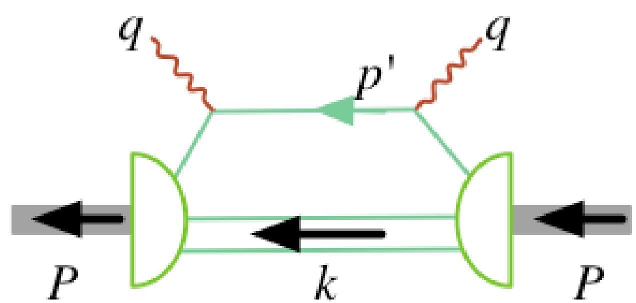
$$\chi = \frac{2P\cdot k}{Mm_s} - 2 = 2\sqrt{1+\frac{\kappa^2}{r^2}} - 2 \qquad r = \frac{m_s}{M}$$

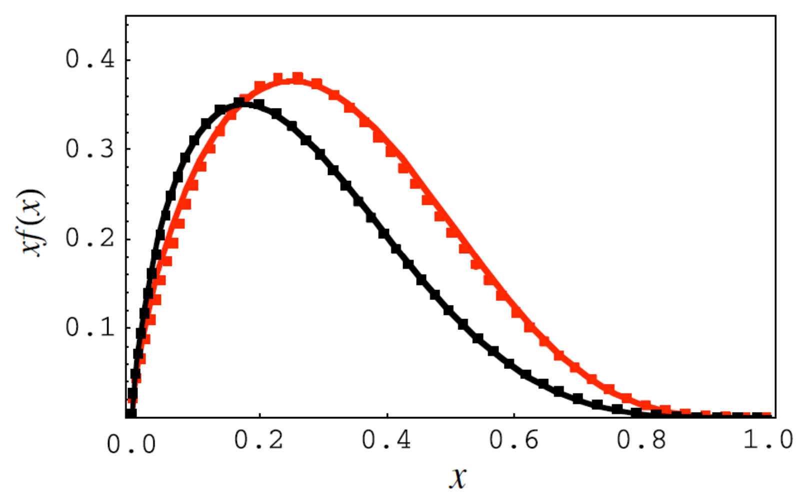
- In DIS limit, x dependence emerges naturally. We chose m_s = M to map k = 0 to x = 0

$$\kappa \geq |\kappa_{\min}| \qquad \kappa_{\min} \equiv \frac{r^2 - (1-x)^2}{2(1-x)}$$

- Choose functional form of wave function and adjust parameters to fit xf(x)
- DIS choice is

$$\psi(\chi) = N\frac{\beta\cos\theta + \chi\sin\theta}{\chi^\alpha(\beta+\chi)^{n-\alpha}}$$





*FG, Ramalho, and Pena, PRD 85,093006 (2012)
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