HADRON WAVE FUNCTIONS FROM LATTICE QCD

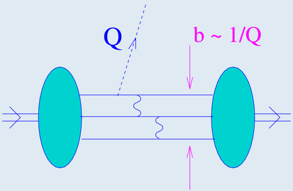
Vladimir M. Braun

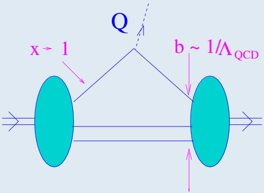
Institut für Theoretische Physik
Universität Regensburg

How to transfer a large momentum to a weakly bound system?

Heuristic picture:

- quarks can acquire large transverse momenta when they exchange gluons
- "hard" gluon exchanges can be separated from "soft" nonperturbative wave functions
- hard gluons can only be exchanged at small transverse separations





Hard rescattering:

Small b
Average $0 < x < 1$

Soft (Feynman):

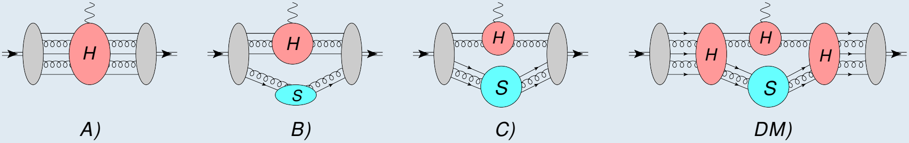
Average b
Large $x \to 1$

In practice three-quark states indeed seem to dominate, however

- "Squeesing" to small transverse separations occurs very slowly
- Helicity selections rules do not work. Orbital angular momentum?
- $\Rightarrow$ More complicated nonperturbative input needed

How to transfer a large momentum to a weakly bound system?

Can be formalized separating hard (H) and soft (S) momentum flow regions



Expect at $Q^2 \to \infty$:

$$F_1(Q^2) \sim \frac{\alpha_s^2(Q^2)}{\pi^2} \frac{1}{Q^4}\,, \qquad F_2(Q^2) \sim \frac{1}{Q^6}$$

- **A)**: $1/Q^4$, factorizable in terms of distribution amplitudes, only $F_1(Q^2)$
- **B)**: $1/Q^6(?)$, nonfactorizable, involves large transverse distances
- **C)**: $1/Q^8(?)$, nonfactorizable, involves large transverse distances
- **DM)** Duncan–Mueller: $1/Q^4$, nonfactorizable, contributes at NNLO

Main problem: leading term suppressed by $(\alpha_s/\pi)^2 \sim 0.01$

Light-cone wave functions vs. Distribution Amplitudes

- **Nucleon light-cone wave function** — Brodsky, Lepage

$$|P\uparrow\rangle^{\ell_z=0} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}}\, \psi^{L=0}(x_i,\vec{k}_i)\, \times$$
$$\times\Big\{ \big|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\uparrow(x_3,\vec{k}_3)\big\rangle - \big|u^\uparrow(x_1,\vec{k}_1)d^\downarrow(x_2,\vec{k}_2)u^\uparrow(x_3,\vec{k}_3)\big\rangle \Big\}$$

- **Leading-twist-three distribution amplitude**

Brodsky, Lepage, Peskin, Chernyak, Zhitnitsky

$$\Phi_3(x_1,x_2,x_3;\mu) \simeq 2\int^{\mu} [d^2\vec{k}]\; \psi^{L=0}(x_1,x_2,x_3;\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

can be defined using the OPE

$$\begin{aligned}\Phi_3(x_i;\mu) &= 120 f_N x_1x_2x_3\Big\{1 + \varphi_{10}(x_1-2x_2+x_3)L^{\frac{8}{3\beta_0}} \\ &\quad + \varphi_{11}(x_1-x_3)L^{\frac{20}{9\beta_0}} + \varphi_{20}\left[1+7(x_2-2x_1x_3-2x_2^2)\right]L^{\frac{14}{3\beta_0}} \\ &\quad + \varphi_{21}\,(1-4x_2)\,(x_1-x_3)\,L^{\frac{40}{9\beta_0}} + \varphi_{22}\left[3-9x_2+8x_2^2-12x_1x_3\right]L^{\frac{32}{9\beta_0}} + \ldots\Big\}\end{aligned}$$

- $f_N(\mu_0)$: **wave function at the origin**
- $\varphi_{nk}(\mu_0)$: **shape parameters**

$L \equiv \alpha_s(\mu)/\alpha_s(\mu_0)$

Braun, Manashov, Rohwild

Wave functions vs. Distribution amplitudes (II)

- **Contributions of orbital angular momentum** Ji, Ma, Yuan, '03

$$|P\uparrow\rangle^{\ell_z=1} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}} \left[k_1^+\psi_1^{L=1}(x_i,\vec{k}_i) + k_2^+\psi_2^{L=1}(x_i,\vec{k}_i)\right] \times$$
$$\times\left\{\left|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\downarrow(x_3,\vec{k}_3)\right\rangle - \left|d^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)u^\downarrow(x_3,\vec{k}_3)\right\rangle\right\}$$

are related to higher-twist-four distribution amplitudes

Belitsky, Ji, Yuan, '03

$$\Phi_4(x_2,x_1,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_3}\, k_3^-\left[k_1^+\psi_1^{L=1} + k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$

$$\Psi_4(x_1,x_2,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_2}\, k_2^-\left[k_1^+\psi_1^{L=1} + k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$

$$k^\pm = k_x \pm ik_y$$

and, again, can be studied using OPE Braun, Fries, Mahnke, Stein '00

$$\Phi_4(x_i;\mu) = 12\lambda_1 x_1x_2 + 12f_N x_1x_2\left[1+\frac{2}{3}(1-5x_3)\right] + \ldots$$

$$\Psi_4(x_i;\mu) = 12\lambda_1 x_1x_3 + 12f_N x_1x_3\left[1+\frac{2}{3}(1-5x_2)\right] + \ldots$$

- **to this accuracy only one new nonperturbative constant $\lambda_1(\mu)$**

How to learn about DAs?

- **From experimental data on form factors, calculating contributions of large transverse distances in terms of DA using dispersion relations and duality LCSR**

  $\longrightarrow$ Nils Offen's talk on Wednesday

- **Moments of DAs can be calculated from first principles on the lattice**

  $\longrightarrow$ this talk

Regensburg Lattice Collaboration RQCD

Computational challenges

Cost of the simulation is proportional to

- number of points: $(L/a)^4$
- condition number of linear system: $1/m_\pi^2$
- $L^{1/2}/m_\pi$ from time integration in Hybrid Monte Carlo
- $1/a^{\geq 2}$ critical slowing down (autocorrelations)

Adjusting $L \propto 1/m_\pi$ this means

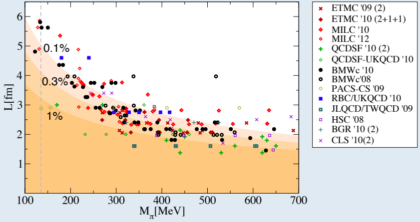
$$\boxed{\text{cost} \;\propto\; \frac{1}{a^{\geq 6} m_\pi^{7.5}}}$$

Huge progress in Hybrid Monte Carlo, solver, noise reduction, but

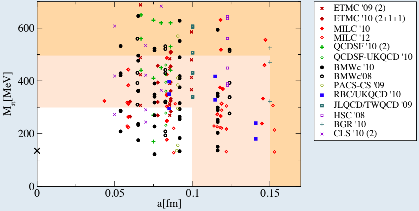
- Additional noise/signal problems and discretization errors from derivatives
- Nonperturbative renormalization and matching to $\overline{MS}$
- Separation of resonances and scattering states
- ...

Landscape of recent lattice simulations

from: C Hoelbling, arXiv:1410.3403



m_π vs. L



a vs. m_π

Study case: Pion Distribution Amplitude

Efremov-Radyushkin-Brodsky-Lepage-Chernyak-Zhitnitsky '79-'81

$$\langle 0|\bar{d}(z_2 n)\not{n}\gamma_5[z_2 n, z_1 n]u(z_1 n)|\pi(p)\rangle = if_\pi(pn)\int_0^1 dx\, e^{-i(pn)[z_1 x + z_2(1-x)]}\phi_\pi(x,\mu^2)$$

quark and antiquark momentum fractions

$$x_1 = x \qquad\qquad x_2 = 1 - x \qquad\qquad \boxed{\xi = x_1 - x_2}$$

moments

$$\langle \xi^2 \rangle \equiv \langle (x_1 - x_2)^n \rangle \;=\; \int_0^1 dx\,(2x-1)^2\,\phi_\pi(x,\mu^2)$$

$$a_2(\mu^2) \equiv \frac{7}{18}\langle C_2^{3/2}(2x-1)\rangle \;=\; \frac{7}{12}\int_0^1 dx\,\Big[5(2x-1)^2 - 1\Big]\,\phi_\pi(x,\mu^2)$$

Gegenbauer expansion

$$\phi_\pi(x,\mu^2) = 6x(1-x)\Big[1 + a_2(\mu^2)\,C_2^{3/2}(2x-1) + a_4(\mu^2)\,C_4^{3/2}(2x-1) + \ldots\Big]$$

Nonperturbative renormalization

bare operators

$$\mathcal{O}_\rho^{\overline{\rm MS}}(x) = \bar d(x)\gamma_\rho\gamma_5\, u(x)$$

$$\mathcal{O}^-_{\rho\mu\nu}(x) = \bar d(x)\left[\overleftarrow{D}_{\{\mu}\overleftarrow{D}_\nu - 2\overleftarrow{D}_{\{\mu}\overrightarrow{D}_\nu + \overrightarrow{D}_{(\mu}\overrightarrow{D}_\nu\right]\gamma_{\rho\}}\gamma_5\, u(x)$$

$$\mathcal{O}^+_{\rho\mu\nu}(x) = \bar d(x)\left[\overleftarrow{D}_{\{\mu}\overleftarrow{D}_\nu + 2\overleftarrow{D}_{\{\mu}\overrightarrow{D}_\nu + \overrightarrow{D}_{\{\mu}\overrightarrow{D}_\nu\right]\gamma_{\rho\}}\gamma_5\, u(x) \quad \neq \quad \partial_{\{\mu}\partial_\nu\mathcal{O}^{\overline{\rm MS}}_{\rho\}}(x)$$

The renormalized operators in $\overline{\rm MS}$ scheme

$$\mathcal{O}_\rho^{\overline{\rm MS}}(x) = Z_A\mathcal{O}_\rho(x)$$

$$\mathcal{O}^{\overline{\rm MS}-}_{4jk}(x) = Z_{11}\mathcal{O}^-_{4jk}(x) + Z_{12}\mathcal{O}^+_{4jk}(x) \qquad \boxed{\zeta_{ik} = \frac{Z_{ik}}{Z_A}}$$

$$\mathcal{O}^{\overline{\rm MS}+}_{4jk}(x) = Z_{22}\mathcal{O}^+_{4jk}(x)$$

$$\boxed{\textbf{lattice operators}} \quad \underset{\substack{\Uparrow\\ \text{nonperturbatively}}}{\Longrightarrow} \quad \boxed{\textbf{RI}' - \textbf{SMOM scheme}} \quad \underset{\substack{\Uparrow\\ \text{NLO (J. Gracey)}}}{\Longrightarrow} \quad \boxed{\overline{\textbf{MS}}\ \textbf{scheme}}$$

$$\langle\xi^2\rangle^{\overline{\rm MS}} = 0.2361(41)\boxed{(39)}, \qquad a_2^{\overline{\rm MS}} = 0.1364(154)\boxed{(145)}$$

error dominated by matching

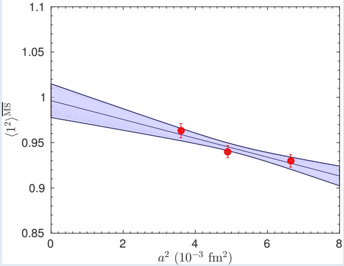
Discretization errors

Momentum sum rule for finite $a \sim 0.06 - 0.08$ fm is broken by lattice artifacts

$$a_2 \;=\; \frac{7}{12}\Big[5\left\langle (x_1 - x_2)^2\right\rangle - \left\langle (x_1 + x_2)^2\right\rangle\Big] \quad \xrightarrow{a\to 0} \quad \frac{7}{12}\Big[5\left\langle (x_1 - x_2)^2\right\rangle - 1\Big]$$

Two ways to the continuum limit:

$$\begin{array}{ccc} \left\langle \xi^2 \right\rangle^{\overline{\rm MS}}_{a\neq 0} & \Longrightarrow & a_2^{\overline{\rm MS}}|_{a\neq 0} \\ \Downarrow & & \Downarrow \\ \left\langle \xi^2 \right\rangle^{\overline{\rm MS}}_{a=0} & \Longrightarrow & a_2^{\overline{\rm MS}}|_{a=0} \end{array}$$



- A 6% deviation from unity results in a $25 - 30\%$ increase in the value of $a_2^{\overline{\rm MS}}$ at the same lattice spacing

Results

$\langle \xi^2 \rangle^{\overline{\mathrm{MS}}} = 0.2\,,$	$a_2 = 0$	asymptotic DA
$\langle \xi^2 \rangle^{\overline{\mathrm{MS}}} = 0.33(?)\,,$	$a_2^{\overline{\mathrm{MS}}} = 0.39(?)$	CZ model
$\langle \xi^2 \rangle^{\overline{\mathrm{MS}}} = 0.269(39)\,,$	$a_2^{\overline{\mathrm{MS}}} = 0.201(114)$	QCDSF 2006
$\langle \xi^2 \rangle^{\overline{\mathrm{MS}}} = 0.28(1)(2)\,,$	$a_2^{\overline{\mathrm{MS}}} = 0.233(29)(58)$	UKQCD 2010
$\langle \xi^2 \rangle^{\overline{\mathrm{MS}}} = 0.2361(41)(39)\,,$	$\boxed{a_2^{\overline{\mathrm{MS}}} = 0.1364(154)(145)}$	REG 2015

at the scale $\mu = 2\,\mathrm{GeV}$.

RQCD

Phys.Rev. D92 (2015) 1, 014504

V.M. Braun, S. Collins, M. Göckeler, P. Perez-Rubio, A. Schäfer, R. W. Schiel, A. Sternbeck

The nucleon

- **Nucleon couplings at the scale 2 GeV**

$$\begin{aligned} f_N &= 2.84(1)(33)\ 10^{-3}\ \mathrm{GeV}^2 \\ \lambda_1 &= -4.13(2)(20)\ 10^{-2}\ \mathrm{GeV}^2 \\ \lambda_2 &= 8.19(5)(39)\ 10^{-2}\ \mathrm{GeV}^2 \end{aligned}$$

◇ f_N 30% less than old estimates, bad for pQCD

◇ $\lambda_{1,2} \gg f_N$, large P-wave contributions in the WFs

Phys.Rev. D89 (2014) 9, 094511

V.M. Braun, S. Collins, B. Gläßle, M. Göckeler, A. Schäfer, R. W. Schiel, W. Söldner, A. Sternbeck, P. Wein

- **"Momentum fractions" carried by the valence quarks**

$$\langle x_1 \rangle = 0.372(7)(?)$$
$$\langle x_2 \rangle = 0.314(3)(?)$$
$$\langle x_3 \rangle = 0.314(7)(?)$$

◇ Discretization errors not seen in the data, probably smaller than statistics

◇ A "diquark" symmetry?

RQCD

Phys.Rev. D89 (2014) 9, 094511

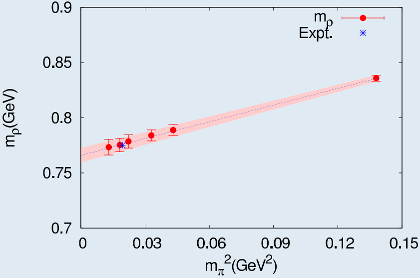
V.M. Braun, S. Collins, B. Gläßle, M. Göckeler, A. Schäfer, R. W. Schiel, W. Söldner, A. Sternbeck, P. Wein

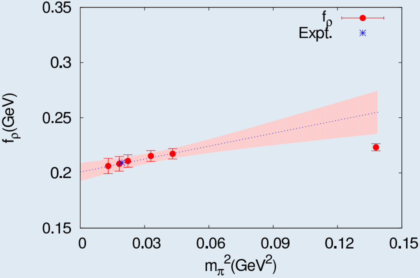
How to study resonances on the lattice?

- **Is finite volume always necessary?**

Chen et al., arXiv:1507.02541

Study uses RBC & UKQCD domain wall fermions, $m_\pi = (114 - 371)$ MeV





$f_\rho^{\rm lat} = 208.5 \pm 5.5 \pm 0.9$ MeV $\qquad f_\rho^{\rm exp} = 209.4 \pm 1.5$ MeV from $\tau \to \rho\nu_\tau$

Negative parity states from variational analysis using two interpolators

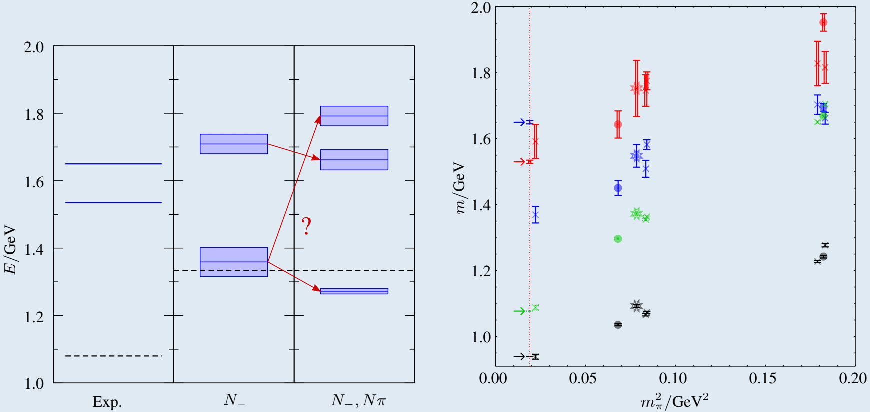
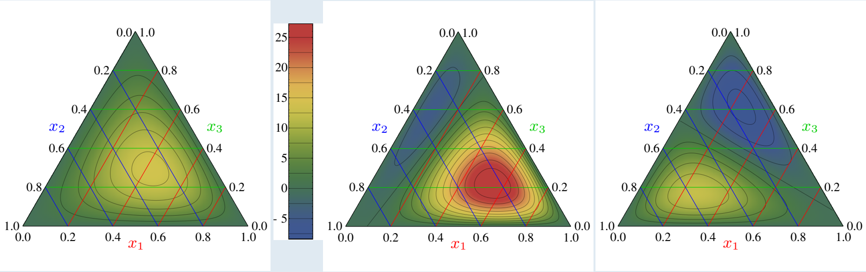


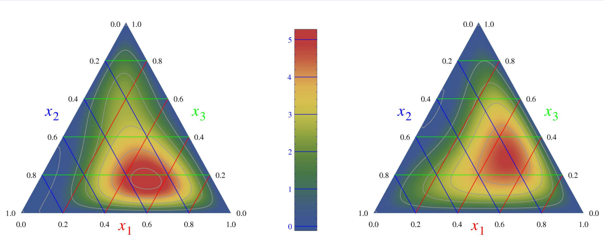
Figure: Left: Negative parity states from experiment, the two-state lattice variational analysis, and the three-state analysis including a five-quark operator. The dashed lines show the sum of the nucleon and pion masses. Figure taken from [Lang:2012db].
Right: Masses of the nucleon (black), $N^*(1650?)$ (blue), $N^*(1535?)$ (red) and the sum of the nucleon and pion masses (green)

Distribution amplitudes from lattice RQCD vs. LCSRs

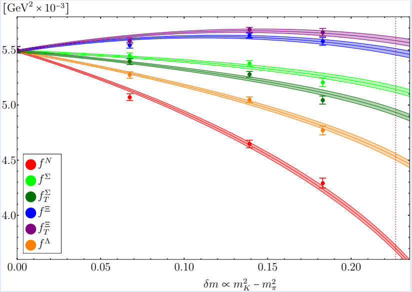


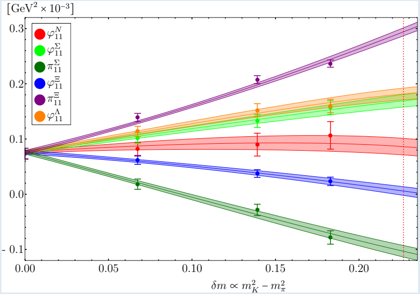
Nucleon $\qquad$ $N^*(1650?) + N\pi$ $\qquad$ $N^*(1535?)$

Nucleon DA
from LCSRs
for comparison
(N. Offen)



• $\frac{1}{2}^+$ baryon octet





◇ Zero on the horizontal axis corresponds to the $SU(3)_f$ symmetric point $m_u = m_d = m_s$

◇ The physical limit is taken by following the line $m_u + m_d + m_s =$ const

RQCD

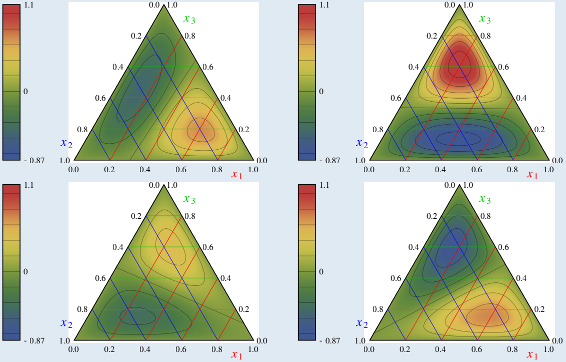
paper in preparation

V.M. Braun, S. Collins, M. Gruber, M. Göckeler, F. Hutzler, A. Schäfer, J. Simeth, W. Söldner, A. Sternbeck, P. Wein

- **Deviation from the asymptotic DA**

N Λ

Σ Ξ



paper in preparation

V.M. Braun, S. Collins, M. Gruber, M. Göckeler, F. Hutzler, A. Schäfer, J. Simeth, W. Söldner, A. Sternbeck, P. Wein

- **Feasibility study in progress**

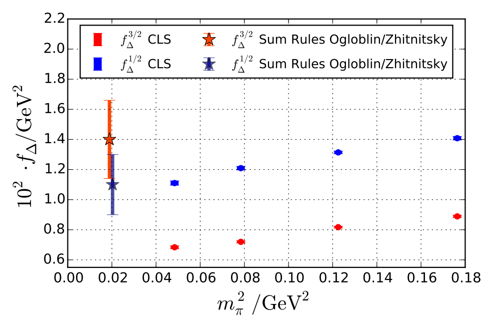


Figure: Raw data for the couplings (wave function at the origin) of the $\Delta(1232)$ resonance with helicity 1/2 and 3/2. The existing QCD sum rule estimates [Chernyak:1987nv] are shown for comparison

work in progress

V.M. Braun, S. Collins, M. Gruber, M. Göckeler, F. Hutzler, A. Schäfer, J. Simeth, W. Söldner, A. Sternbeck, P. Wein

Summary and outlook

Lattice calculations of moments of the DAs are becoming mature

- Lattice operators, nonperturbative renormalization, $\overline{\mathrm{MS}}$-like scheme, matching
- Parity projectors for "moving" baryons
- One-loop chiral perturbation theory
- Code writing, LibHadronAnalysis, ...

A consistent picture is emerging

- Valence quark distributions in the nucleon are not far from "asymptotic" form
- Large P-wave contributions (quark orbital angular momentum)
- Qualitatively different valence quark distributions in Nucleon and $N^*(1535)$
- Large $SU(3)_f$ breaking

Outlook

- $N_f = 2 + 1$ CLS configurations with $a \to 0.04$ fm, continuum extrapolation
- $\frac{1}{2}^+$ baryon octet, $\Delta(1232)$
- Bottlenecks: Two-loop matching, very high statistics needed for the second moments