Exclusive electroproduction of pions

P. Kroll

Fachbereich Physik, Univ. Wuppertal and Univ. Regensburg

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Outline:

- **Introduction: The handbag approach**
- **Evidence for strong $\gamma_T^* \to \pi$ transitions**
- **Transversity in the handbag approach**
- **Pion electroproduction**
- **Vector mesons**
- **Summary**

Hard exclusive scattering within the handbag approach

rigorous proofs of collinear factorization in generalized Bjorken regime:

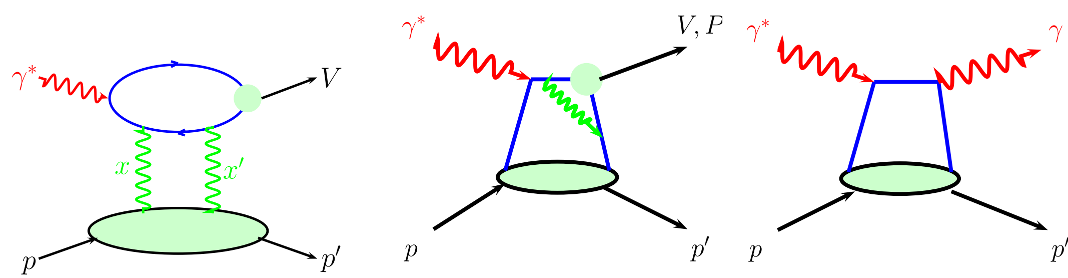
for $\gamma_L^* \to V_L(P)$ and $\gamma_T^* \to \gamma_T$ amplitudes $(Q^2, W \to \infty,\ x_{Bj}$ fixed$)$

Radyushkin, Collins et al, Ji-Osborne

hard subprocesses

$\gamma^* g \to V g\,,$

$\gamma^* q \to V(P,\gamma) q$



and GPDs and meson w.f.

(encode the soft physics)

$$\mathcal{M} \sim \int_{-1}^{1} dx\, \mathcal{H}(x,\xi,Q^2,t=0) K(x,\xi,t)$$

$$d\sigma/dt \sim \mid \mathcal{M} \mid^2 + \mathcal{O}(1/Q^2)$$

power corrections are theoretical not under control

Exp: strong power corrections from γ_T^* and $\gamma_L^* \to V_L(P)$

GPDs – a reminder

Müller et al (94), Ji(97), Radyushkin (97)

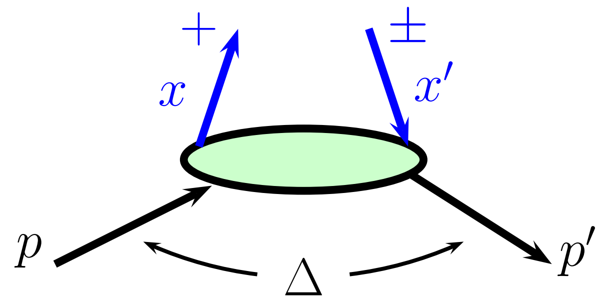
GPDs: $K = K(\bar{x}, \xi, t)$

$$K = H, E, \widetilde{H}, \widetilde{E}, H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$$

$$x = \frac{\bar{x}+\xi}{1+\xi} \quad x' = \frac{\bar{x}-\xi}{1-\xi}$$

for quarks ($\xi < \bar{x} < 1$) and gluons

(antiquarks for $-1 < \bar{x} < -\xi$, $q\bar{q}$ pairs $-\xi < \bar{x} < \xi$)



properties:

reduction formula $H^q(\bar{x}, \xi = t = 0) = q(\bar{x})$, $\widetilde{H}^q \to \Delta q(\bar{x})$, $H_T^q \to \delta^q(\bar{x})$

sum rules (proton form factors): $F_1^q(t) = \int d\bar{x} H^q(\bar{x}, \xi, t)$, $F_1 = \sum e_q F_1^q$

$$E \to F_2,\ \widetilde{H} \to F_A,\ \widetilde{E} \to F_P$$

polynomiality, universality, evolution, positivity constraints

Ji's sum rule $\quad J_q = \frac{1}{2} \int_{-1}^{1} d\bar{x}\, \bar{x}\, [H^q(\bar{x}, \xi, t = 0) + E^q(\bar{x}, \xi, t = 0)]$

FT $\mathbf{\Delta} \to \mathbf{b}$ ($\Delta^2 = -t$): information on parton localization in trans. position space

An almost model-independent argument

consider pion electroproduction

sum and difference of single-flip ampl.
($\sim \sqrt{-t'}$ for $t' \to 0$ by angular mom. conserv.)

$$\mathcal{M}^{N(U)}_{0+\mu+} = \frac{1}{2}\Big[\mathcal{M}_{0+\mu+} + (-)\mathcal{M}_{0+-\mu+}\Big] \qquad\qquad \mu = \pm 1$$

$$\Longrightarrow \qquad\qquad \mathcal{M}^{N(U)}_{0+-+} = +(-)\mathcal{M}^{N(U)}_{0+++}$$

like a one-particle-exchange of either Natural or Unnatural parity

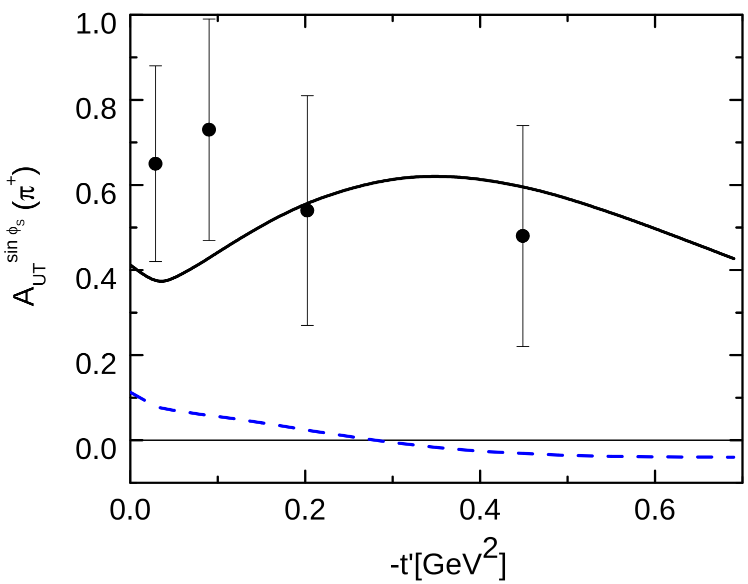
nucleon helicity flip: $\qquad \mathcal{M}_{0--+} \sim t' \qquad \mathcal{M}_{0-++} \sim const$
sum and difference inconvenient
(constant can be small - or zero - for dynamical reasons)

Experiment:

Pion photoproduction: cross section exhibits pronounced maximum at $t = 0$

$const.$ cannot be zero   Phillips (1967): Regge cuts necessary

Pion electroproduction



HERMES(09)

$Q^2 \simeq 2.5\,\mathrm{GeV}^2$, $W = 3.99\,\mathrm{GeV}$

$\sin\phi_s$ modulation very large

does not seem to vanish for $t' \to 0$

$A_{UT}^{\sin\phi_S} \propto \mathrm{Im}\Big[\mathcal{M}^*_{0-,++}\mathcal{M}_{0+,0+}\Big]$

n-f. ampl. $\mathcal{M}_{0-,++}$ required

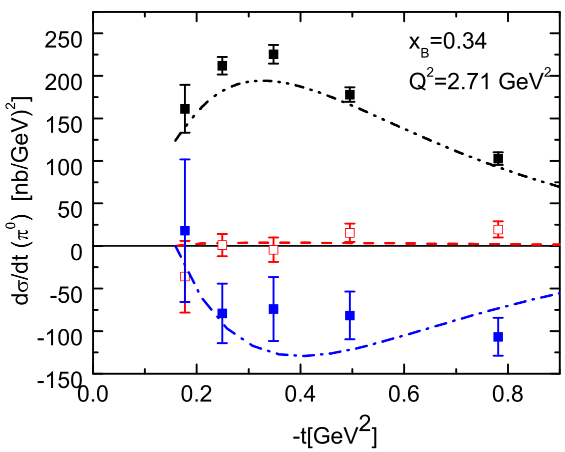
not vanishing in forward direction

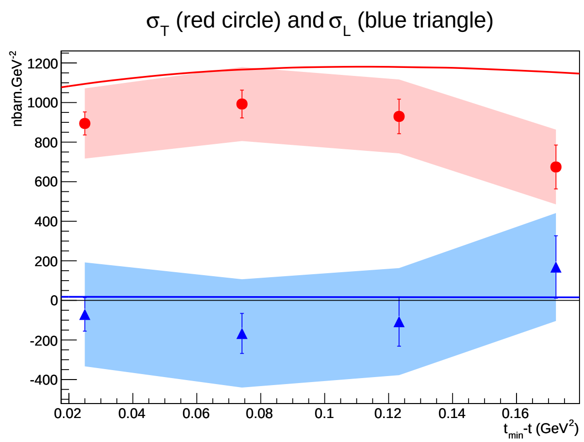
assumption: $|\mathcal{M}_{0--+}| \ll |\mathcal{M}_{0-++}|, |\mathcal{M}_{0+\pm++}|$

Transverse cross sections

$$\frac{d\sigma_T}{dt} \simeq \frac{1}{2\kappa}\left[|\mathcal{M}_{0-++}|^2 + 2|\mathcal{M}^N_{0+++}|^2 + 2|\mathcal{M}^U_{0+++}|^2\right] \quad \frac{d\sigma_{TT}}{dt} \simeq -\frac{1}{\kappa}\left[|\mathcal{M}^N_{0+++}|^2 - |\mathcal{M}^U_{0+++}|^2\right]$$

$$\Longrightarrow \left|\frac{d\sigma_{TT}}{dt}\right| \leq \frac{d\sigma_T}{dt} \leq \frac{d\sigma}{dt}$$





Hall A (preliminary)

$Q^2 = 1.76\,\mathrm{GeV}^2 \; x_B = 0.36$

π^0 data CLAS(12)

unsep. cross sec., $d\sigma_{LT}$, $d\sigma_{TT}$

$|\mathcal{M}^N_{0+++}|$ dominant (for $-t' > 0$)

(see forward dip)

$d\sigma_L/dt << d\sigma_T/dt$

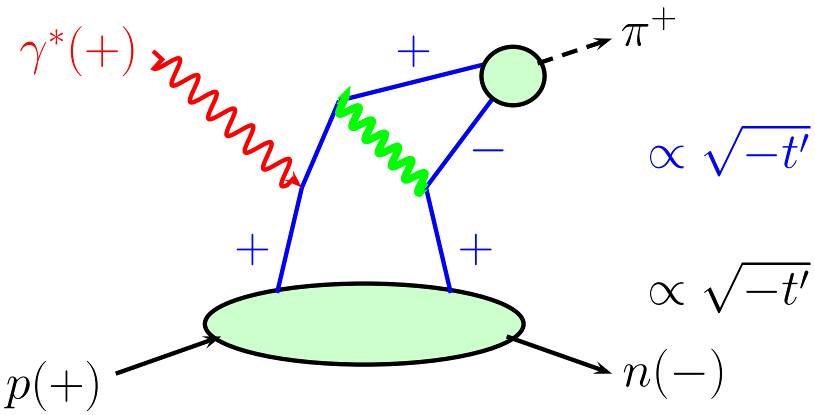
consistent with $d\sigma_{LT}/dt \simeq 0$

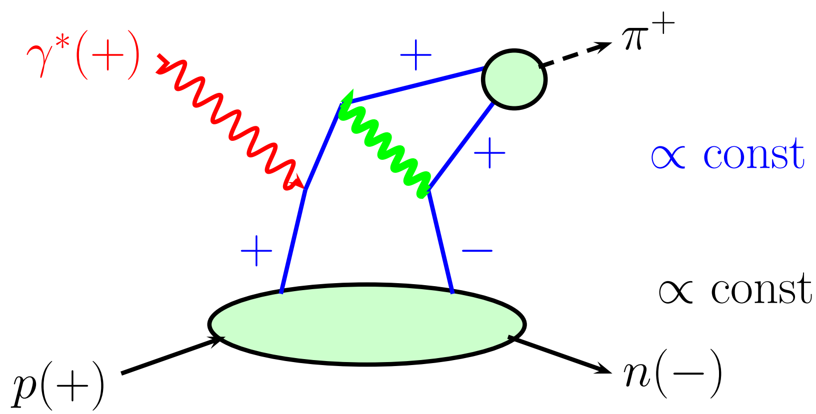
check: $d\sigma_T + d\sigma_{TT} \simeq \frac{A_{LL}^{\cos(0\phi)} d\sigma}{\sqrt{1-\epsilon^2}}$?

if not - $\mathcal{M}^U_{0+++}$, $\mathcal{M}_{0--+}$

experimental verification of transversity dominance

Handbag: can $\mathcal{M}_{0-,++}$ be fed by ordinary GPDs?





lead. twist pion wave fct. $\propto q' \cdot \gamma\gamma_5$
(perhaps including $\mathbf{k}_\perp$)

transversity GPDs required
go along with twist-3 w.f.

$\mathcal{M}_{0-,++} \propto t'$

$\mathcal{M}_{0-,++} \propto$ const

(forced by angular momentum conservation)

prominent role of transversity GPDs also claimed by Ahmad et al
analysis and results different

$\gamma_T^* \to \pi$ in the handbag approach

see Diehl01, GK10, GK11 $\qquad \bar{E}_T \equiv 2\widetilde{H}_T + E_T \qquad \mu = \pm 1$

$$\mathcal{M}_{0+\mu+} = e_0 \frac{\sqrt{-t'}}{4m} \int dx \Big\{ \big(H_{0+\mu-} - H_{0-\mu+}\big)\big(\bar{E}_T - \xi \widetilde{E}_T\big) + \big(H_{0+\mu-} + H_{0-\mu+}\big)\big(\widetilde{E}_T - \xi E_T\big) \Big\}$$

$$\mathcal{M}_{0-\mu+} = e_0 \sqrt{1-\xi^2} \int dx \Big\{ H_{0-\mu+} \Big[H_T + \frac{\xi}{1-\xi^2}\big(\widetilde{E}_T - \xi E_T\big)\Big] + \big(H_{0+\mu-} - H_{0-\mu+}\big) \frac{t'}{4m^2} \widetilde{H}_T \Big\}$$

with parity conservation: $\qquad \mathcal{M}_{0+\pm+} = \mathcal{M}^N_{0+++} \pm \mathcal{M}^U_{0+++}$

time-reversal invariance: $\widetilde{E}_T$ is odd function of ξ

N: $\bar{E}_T$ with corrections of order ξ^2 $\qquad$ U: order ξ

small $-t'$: $\mathcal{M}_{0-++}$ mainly H_T with corrections of order ξ^2

$\mathcal{M}_{0--+}$ suppressed by t/Q^2 due to H_{0--+} and by $t'/4m^2$ from $\tilde{H}_T$

handbag explains structure of ampl. at least at small ξ and small $-t'$

The twist-3 pion distr. amplitude

projector $q\bar{q} \to \pi$ (3-part. $q\bar{q}g$ contr. neglected) Beneke-Feldmann (01)

$\sim q' \cdot \gamma\gamma_5 \Phi + \mu_\pi \gamma_5 \Big[\Phi_P - \imath\sigma_{\mu\nu}(\dots \Phi'_\sigma + \dots \Phi_\sigma \partial/\partial \mathbf{k}_{\perp\nu})\Big]$

definition: $\langle \pi^+(q') \mid \bar{d}(x)\gamma_5 u(-x) \mid 0\rangle = f_\pi \mu_\pi \int d\tau \mathrm{e}^{iq'x\tau}\Phi_P(\tau)$

local limit $x \to 0$ related to divergency of axial vector current

$\Longrightarrow \mu_\pi = m_\pi^2/(m_u + m_d) \simeq 2\,\mathrm{GeV}$ at scale $2\,\mathrm{GeV}$ (conv. $\int d\tau \Phi_P(\tau) = 1$)

Eq. of motion: $\tau\Phi_P = \Phi_\sigma/N_c - \tau\Phi'_\sigma/(2N_c)$

solution: $\Phi_P = 1, \quad \Phi_\sigma = \Phi_{AS} = 6\tau(1-\tau)$ Braun-Filyanov (90)

$H^{\mathrm{twist}-3}_{0-,++} \neq 0$, Φ_P dominant, Φ_σ contr. $\propto t/Q^2$

in coll. appr.: $H^{\mathrm{twist}-3}_{0-,++}$ singular **$\mathbf{k}_\perp$** factorization (m.p.a.) regular

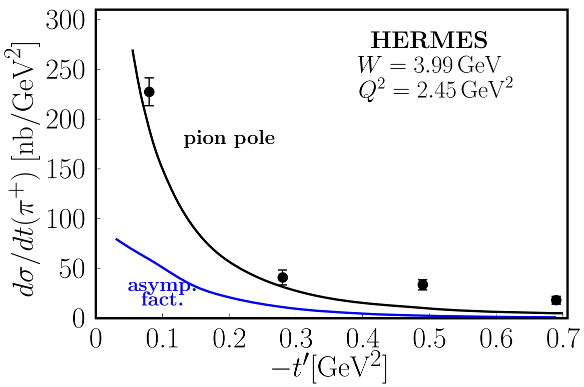
$$M_{0-++} = e_0\sqrt{1-\xi^2}\int dx H^{\mathrm{twist}-3}_{0-++} H_T\,, \qquad M_{0+\pm+} = -e_0\frac{\sqrt{-t'}}{4m}\int dx H^{\mathrm{twist}-3}_{0-++}\bar{E}_T$$

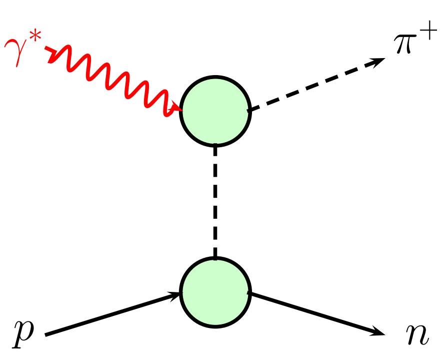
The pion pole

$$\mathcal{M}_{0+0+} = \frac{e_0}{2}\sqrt{1-\xi^2}\langle\widetilde{H} - \frac{\xi^2}{1-\xi^2}\widetilde{E}\rangle \qquad \mathcal{M}_{0-0+} = e_0\frac{\sqrt{-t'}}{4m}\xi\langle\widetilde{E}\rangle$$

leading amplitudes for $Q^2 \to \infty$

For π^+ production - pion pole:





$$\widetilde{E}^u_{\rm pole} = -\widetilde{E}^d_{\rm pole} = \Theta(|x| \le \xi)\frac{m f_\pi g_{\pi NN}}{\sqrt{2}\xi}\frac{F_{\pi NN}(t)}{m_\pi^2 - t}\Phi_\pi(\frac{x+\xi}{2\xi})$$

$$\Longrightarrow \frac{d\sigma_L^{\rm pole}}{dt} \sim \frac{-t}{Q^2}\left[\sqrt{2}e_0 g_{\pi NN}\frac{F_{\pi NN}(t)}{m_\pi^2 - t}Q^2 F_\pi^{\rm pert}(Q^2)\right]^2$$

handbag underestimates FF $F_\pi^{\rm pert.} \simeq 0.3 - 0.5 F_\pi^{\rm exp.}$

(F_π measured in π^+ electroproduction at Jlab)

Goloskokov-K(09): $F_\pi^{\rm pert} \to F_\pi^{\rm exp}$

knowledge of the sixties suffices to explain

π^+ data at small $-t$ and large Q^2

Parametrizing the GPDs

double distribution ansatz (Mueller *et al* (94), Radyushkin (99))

$$K_i(x,\xi,t) = \int_{-1}^{1} d\rho \int_{-1+|\rho|}^{1-|\rho|} d\eta\, \delta(\rho+\xi\eta-x)\, K_i(\rho,\xi=0,t) w_i(\rho,\eta) + D_i(x/\xi,t)\,\Theta(\xi^2-\bar{x}^2)$$

weight fct $w_i(\rho,\eta) \sim [(1-|\rho|)^2-\eta^2]^{n_i}$ ($n_g = n_{\rm sea} = 2, n_{\rm val} = 1$, generates ξ dep.)

zero-skewness GPD $K_i(\rho,\xi=0,t) = k_i(\rho)\exp\left[(b_{ki}+\alpha'_{ki}\ln(1/\rho))t\right]$

$k = q, \Delta q, \delta^q$ for $H, \widetilde{H}, H_T$ or $N_{ki}\rho^{-\alpha_{ki}(0)}(1-\rho)^{\beta_{ki}}$ for $E, \widetilde{E}, \bar{E}_T$

Regge-like t dep. (for small $-t$ reasonable appr.)

advantages: polynomiality and reduction formulas automatically satisfied

$H_{\rm val}$, $E_{\rm val}$ and $\widetilde{H}_{\rm val}$ at $\xi = 0$ from analysis of form factors (sum rules)

positivity bounds respected Diehl et al(04), Diehl-K (13)

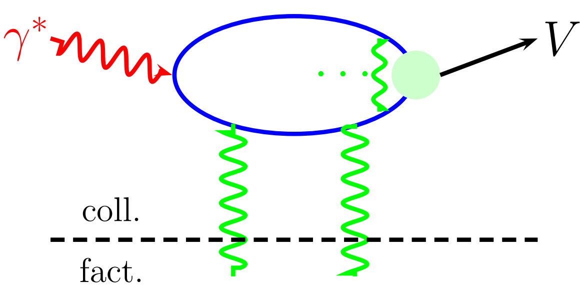
for H and E D-term; neglected

The subprocess amplitude for DVMP

mod. pert. approach - quark trans. momenta in subprocess

(emission and absorption of partons from proton collinear to proton momenta)

transverse separation of color sources $\Longrightarrow$ gluon radiation



LO pQCD

$+$ quark trans. mom.

$+$ Sudakov supp.

$\Rightarrow$ asymp. fact. formula

(lead. twist) for $Q^2 \to \infty$

Sudakov factor Sterman et al(93)

$$S(\tau, \mathbf{b}_\perp, Q^2) \propto \ln \frac{\ln(\tau Q/\sqrt{2}\Lambda_{\rm QCD})}{-\ln(b_\perp \Lambda_{\rm QCD})} + \text{NLL}$$

resummed gluon radiation to NLL $\Rightarrow \exp[-S]$

provides sharp cut-off at $b_\perp = 1/\Lambda_{\rm QCD}$

$$\mathcal{H}^M_{0\lambda,0\lambda} = \int d\tau d^2b_\perp \, \hat{\Psi}_M(\tau, -\mathbf{b}_\perp)\, e^{-S} \hat{\mathcal{F}}_{0\lambda,0\lambda}(\bar{x}, \xi, \tau, Q^2, \mathbf{b}_\perp)$$

$\hat{\Psi}_M \sim \exp[\tau\bar{\tau} b_\perp^2/4a_M^2]$ LC wave fct of meson

$\hat{\mathcal{F}}$ FT of hard scattering kernel

e.g. $\propto 1/[k_\perp^2 + \tau(\bar{x}+\xi)Q^2/(2\xi)] \Rightarrow$ Bessel fct

Sudakov factor generates series of power corr. $\sim (\Lambda^2_{\rm QCD}/Q^2)^n$

from intrinsic $k_\perp$ in wave fct: series $\sim (a_M Q)^{-n}$

The role of H_T and $\bar{E}_T$ in pion leptoproduction

simplified picture: $H_T^d \simeq -1/2 H_T^u \qquad \bar{E}_T^d \simeq \bar{E}_T^u$

$\widetilde{H}, \tilde{E} \simeq 0 \qquad$ pion pole

supported by lattice QCD QCDSF-UKQCD(05,06)

transversity PDFs Anselmino et al(09)

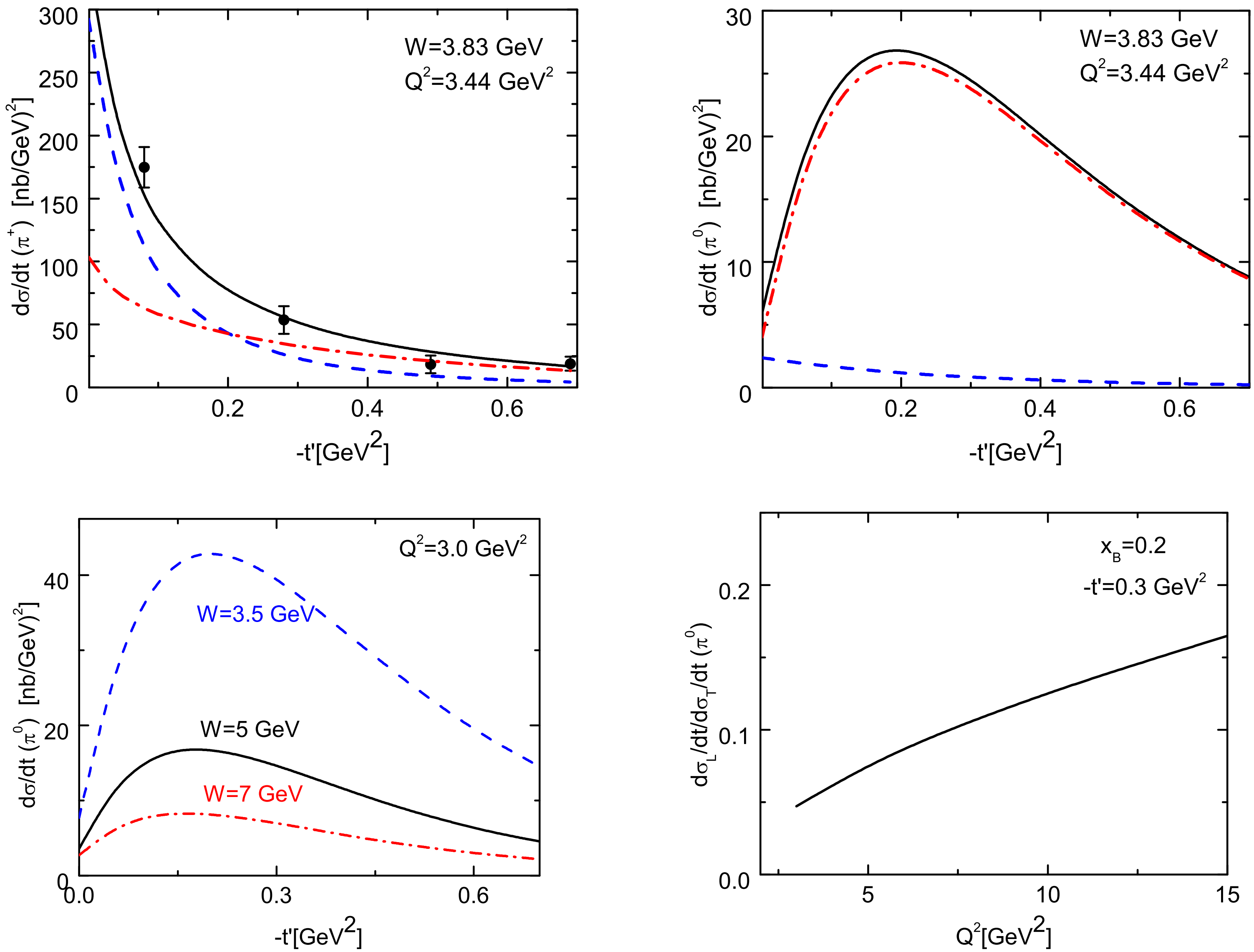
$\bar{E}_T$ related to Boer-Mulders fct $\quad \langle \cos(2\phi) \rangle$ in SIDIS − same pattern Burkhardt

π^+: pion pole $\to \sigma_L \qquad K_{\pi^+} = K^u - K^d; \qquad H_T^{\pi^+} = 3/2 H_T^u; \quad \bar{E}_T^{\pi^+} = 0$

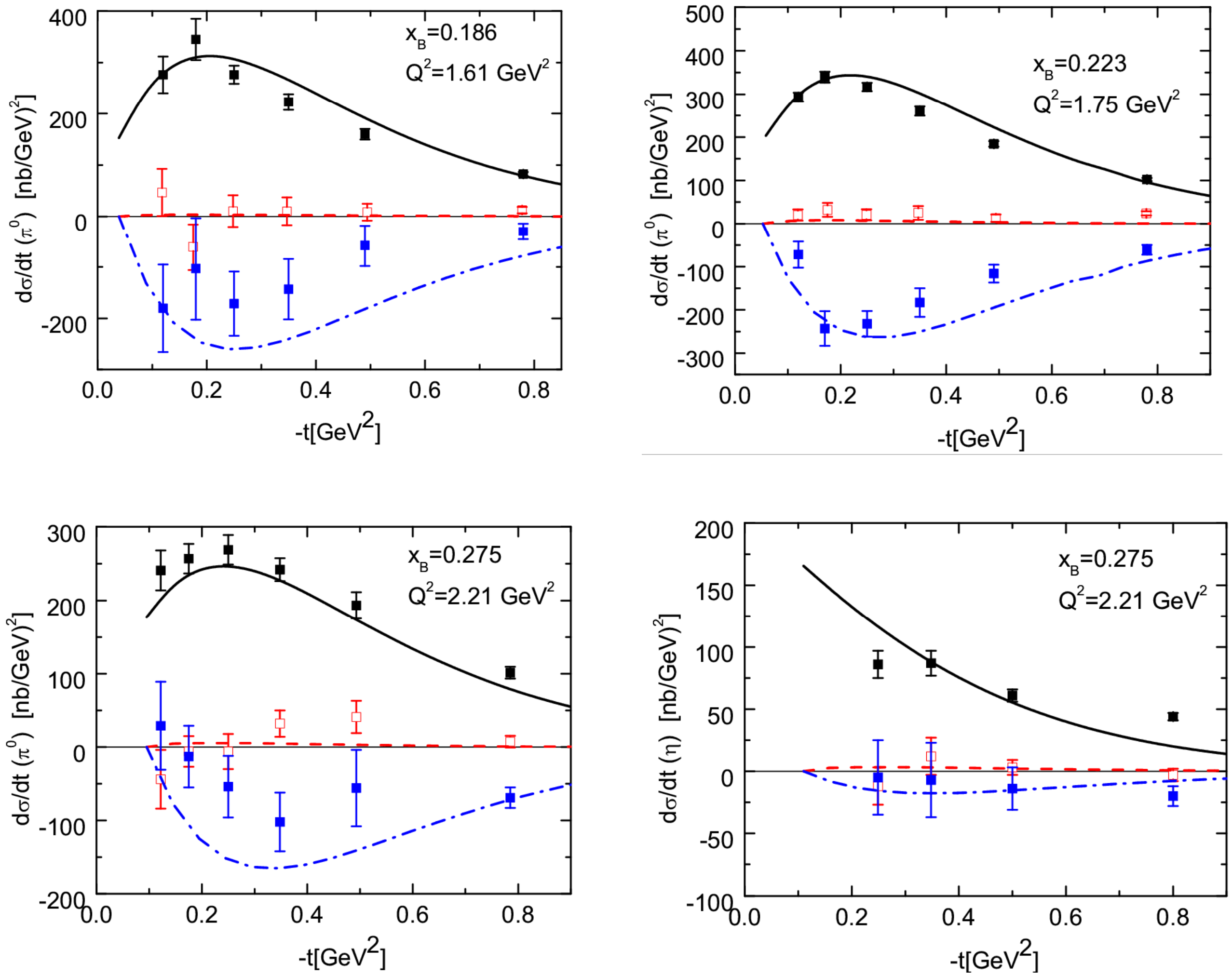
π^0: $\qquad \sigma_L = 0 \qquad K_{\pi^0} = e_u K^u - e_d K^d; \qquad H_T^{\pi^0} = 1/2 H_T^u; \quad \bar{E}_T^{\pi^0} = \bar{E}_T^u$

	$d\sigma_L$	$d\sigma_T$	$d\sigma_{LT}$	$d\sigma_{TT}$	$A_{UT}^{\sin\phi_s}$
π^+	large	H_T large	large	0	large
π^0	0	$\bar{E}_T$ large	0	large	0

Results for pion production

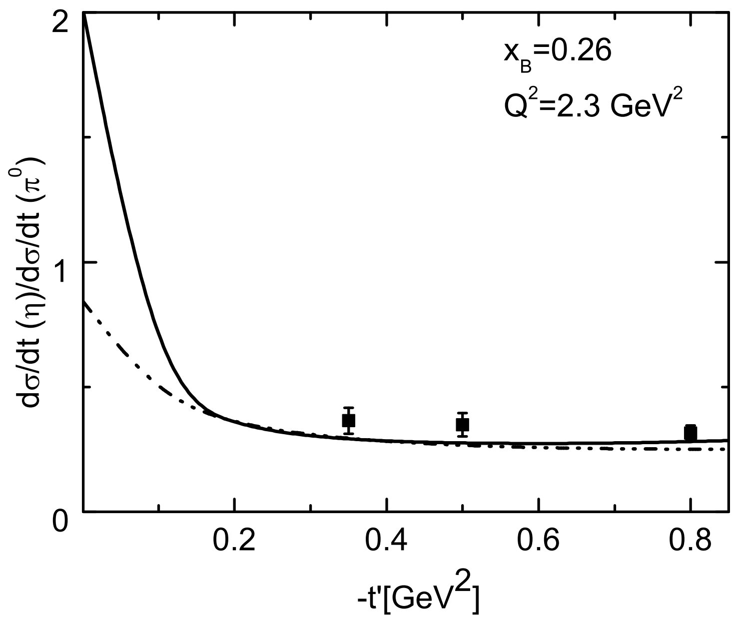


Goloskokov-K (10),(11) optimized for small ξ and large W



Bedlinsky et al (12)

η/π^0 ratio



data CLAS (prel.) unseparated (longitinal, transverse) cross sections

$$\frac{d\sigma(\eta)}{d\sigma(\pi^0)} \simeq \left(\frac{f_\eta}{f_\pi}\right)^2 \frac{1}{3} \left|\frac{e_u\langle K^u\rangle + e_d\langle K^d\rangle}{e_u\langle K^u\rangle - e_d\langle K^d\rangle}\right|^2 \qquad (f_\eta = 1.26 f_\pi)$$

if K^u and K^d have opposite sign: $\eta/\pi^0 \simeq 1$ $\qquad (\eta = (\cos\theta_8 - \sqrt{2}\sin\theta_1)\eta_q)$

if K^u and K^d have same sign: $\eta/\pi^0 < 1$

$t' \simeq 0$ $\widetilde{H}, H_T$ dominant (see also Eides et al(98) from asym. factor. formula for all t')

$t' \neq 0$ $\bar{E}_T$ dominant

Reanalysis of pion electroproduction data

up to now: rather estimates than analysis

new (preliminary) exp. information:

- σ_L, σ_T for π^0 production (settles dominance of $\gamma_T^* \to \pi$)
- A_{LL}, A_{UL}, A_{LU} for π^+ and π^0 from CLAS
- expected π^0 cross section from COMPASS

$\widetilde{H}$ from Diehl-K (13) based on DSSV (11)

$H_T, \bar{E}_T$ parametrized as before (more transversity GPDs ? see A_{LL})

DIFFICULTY:

large $-t_0 = 4m^2\xi^2/(1-\xi^2)$ implied (e.g. $Q = W = 2$ GeV: $t_0 \simeq -1\,\mathrm{GeV}^2$)

implies corrections in ξ (see also Braun et al (14))

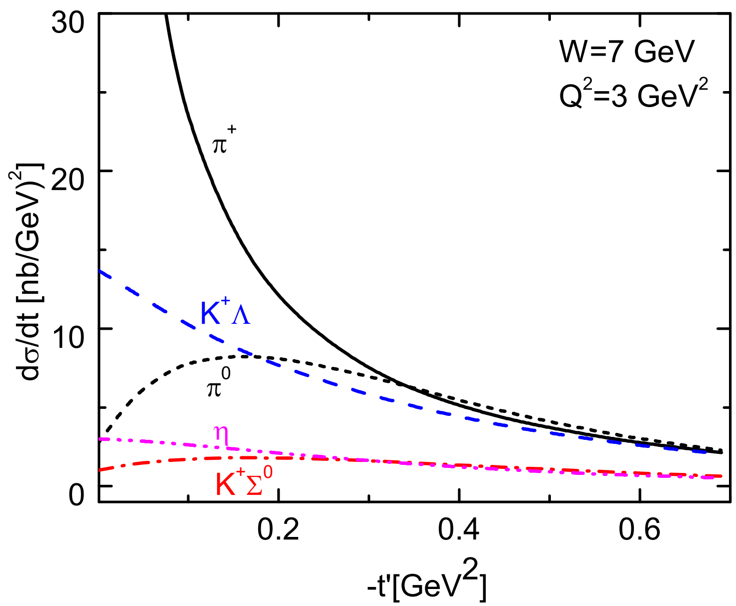
$$\xi = \frac{x_{\rm Bj}}{2-x_{\rm Bj}}\Big[1 + \frac{2}{2-x_{\rm Bj}}\frac{m_\pi^2}{Q^2} - 2x_{\rm Bj}^2\frac{1-x_{\rm Bj}}{2-x_{\rm Bj}}\frac{m^2}{Q^2} + 2x_{\rm Bj}\frac{1-x_{\rm Bj}}{2-x_{\rm Bj}}\frac{t}{Q^2}\Big]$$

handbag approach requires $-t \ll Q^2$, Q is the hard scale

for $-t \gtrsim Q^2$ factorization different (subprocess and generalized form factors)

Strangeness production

e.g. $\gamma^* p \to K^+ \Lambda(\Sigma^0)$



similar to π^+ production

Kaon pole (smaller than pion pole)

and

twist-3 effect with

$\mu_K = m_K^2/(m_u + m_s) \simeq 2.0\,\text{GeV}$

would probe $\widetilde{H}$, $\widetilde{E}$ and H_T for flavor symmetry breaking in sea

e.g.

$$K_{p\to\Sigma^0} = -K_v^d + (K^s - K^{\bar{d}})\,,$$

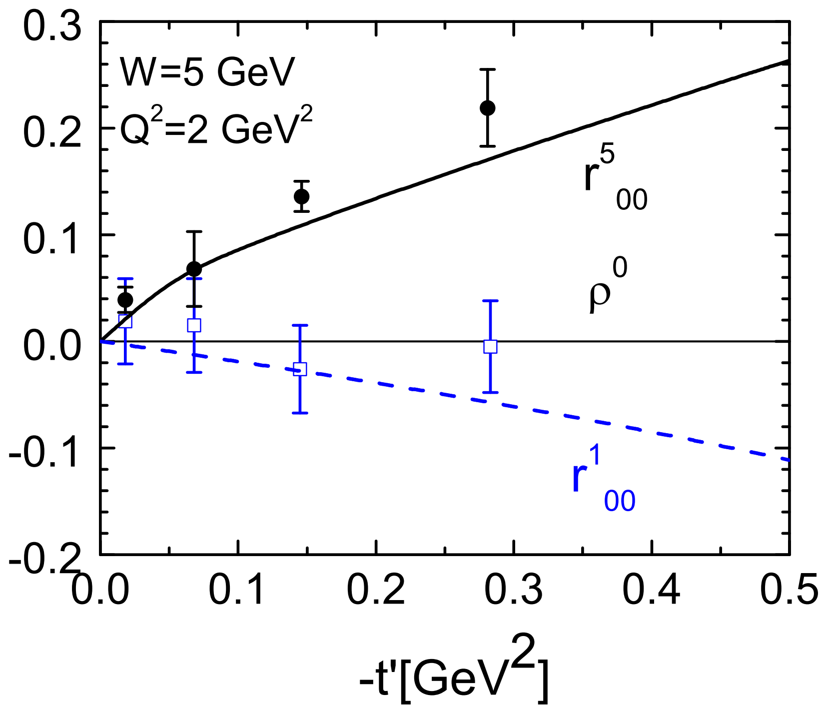
$$K_{p\to\Lambda} = -\frac{1}{\sqrt{6}}\Big[2K_v^u - K_v^d + (2K^{\bar{u}} - K^{\bar{d}} - K^s)\Big]$$

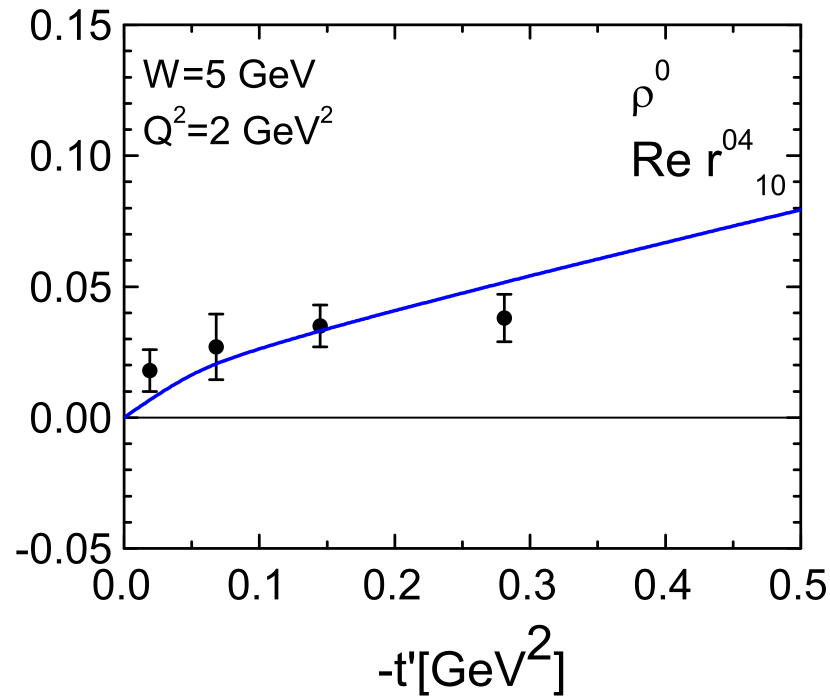
Transversity in vector meson electroproduction

as for pions: $\gamma_T^* \to V_L$ amplitudes, same subprocess amplitude
except $\Psi_\pi \to \Psi_V$, i.e. $f_\pi \to f_V$, $\mu_\pi/Q \to m_V/Q$

$\gamma_T^* \to V_L$ amplitudes of about the same strength as the $\gamma_T^* \to \pi$ ones but
competition with $\langle H \rangle$ (for gluons and quarks) instead with $\langle \widetilde{H} \rangle$ ($|\langle H \rangle| \gg |\langle \widetilde{H} \rangle|$)
$\Longrightarrow$ small transversity effects for vector mesons
to be seen in some of the SDMEs and in spin asymmetries
examples from Goloskokov-K(13,14) estimates, not fits

Spin density matrix elements





SDME from HERMES(09)

$r^1_{00} \sim -|\langle \bar{E}_T \rangle|^2$

$r^5_{00} \sim \mathrm{Re}\big[\langle \bar{E}_T \rangle^* \langle H \rangle_{LL}\big]$

$\mathrm{Re} r^{04}_{10} \sim \mathrm{Re}\big[\langle \bar{E}_T \rangle^* \langle H \rangle_{TT}\big]$

$\langle H \rangle_{LL(TT)}$ convolution of H with $\gamma^*_L \to V_L$ $(\gamma^*_T \to V_T)$ subprocess ampl.

Gluon transversity?

only non-flip subprocess ampl. $\gamma^* g \to V g$ with gluon helicity-flip $\mathcal{H}_{--,++}$ (helicities ± 1)

$\Longrightarrow$ contribution to $\gamma_T^* \to V_{-T}$ amplitudes $\mathcal{M}_{-\mu\nu'\mu\nu}$

SDME (HERMES(09), H1(09)): $\gamma_T^* \to V_{-T}$ ampl. are small, compatible with zero

consistent with small gluon transv. GPDs

not in contradiction with large quark transv. GPDs:

gluon and quark transv. GPDs evolve independently with scale

Hoodbhoy-Ji(98), Belitsky et al(00)

gluon transv. contribution to $\gamma_T^* \to \gamma_{-T}$ DVCS at NLO

Hoodbhoy-Ji(98), Belitsky-Müller (00)

From pion leptoproduction we learn about $\widetilde{H}$ and $\tilde{E}$

state of the art 10-15 years ago

obsolete now   it is to be revised:

From pion leptoproduction

we learn about H_T and $\bar{E}_T$