N^* Form Factors and Distribution Amplitudes in QCD

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based on arxiv:1310.1375
arxiv:1505.05759

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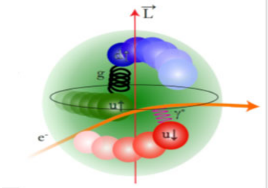
What is it all about?

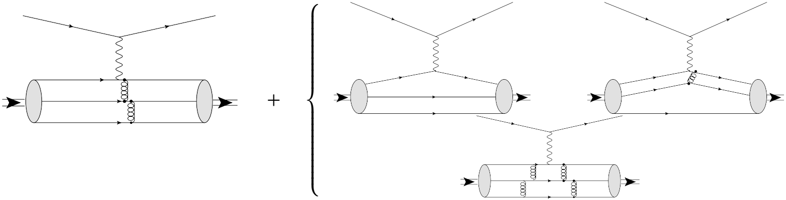


- described by form factors

$$\langle N(p+q)|j_\mu^{em}|N(p)\rangle$$
$$= \bar{N}(p+q)\left[F_1(Q^2)\gamma_\mu - \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(Q^2)\right]N(p)$$

- $F_1(Q^2)$ can formally be calculated via

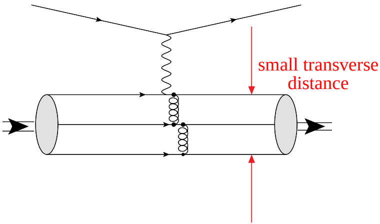
$F_1(Q^2) \sim$



factorizable

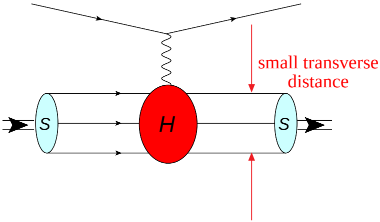
nonfactorizable

Factorizable and non-factorizable?



- quarks can be treated as collinear

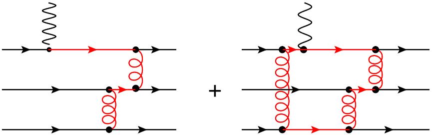
Factorizable and non-factorizable?



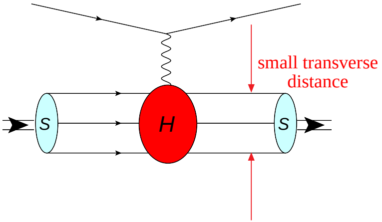
- quarks can be treated as collinear
- can be factorized in hard and soft part

$$F_1(Q^2) \sim \phi_N \otimes T_H \otimes \phi_N$$

- formally leading in $\frac{1}{Q^2}$
- T_H perturbatively calculable partonic amplitude

$T_H =$  $+ \cdots$

Factorizable and non-factorizable?



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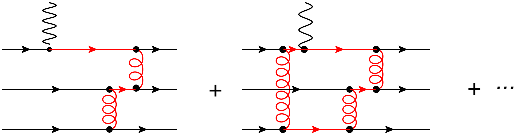
$T_H =$ 

- ϕ_N leading distribution amplitude
  - describes longitudinal momentum distribution for lowest Fock-state

What is the problem?

- formally leading part suppressed by $\left(\frac{\alpha_s}{\pi}\right)^2 \sim 0.01$
- fails miserably to describe experimental data
- non-factorizable part dominant at current energies
- non-factorizable part cannot be reduced to simpler quantities
  exception: Duncan, Mueller part
- cannot be calculated from first principles

What are the solutions?

- calculate overlap integral of baryon wave functions with model functions
  quark models, ADS/QCD

- TMD factorization with Sudakov suppression of large transverse distances
  models for TMDs, complicated nonperturbative input

- calculate everything in terms of distribution amplitudes using dispersion relations and duality
  LCSRs

- calculate parameters from first principles on the lattice
  Only few parameters possible

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TMDs vs. DAs

- s-wave light-cone wave function

$$|P\uparrow\rangle^{l_z=0} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1\, x_2\, x_3}} \psi^{L=0}(x_i, \vec{k}_i)$$
$$\times \quad \{|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\uparrow(x_3,\vec{k}_3)\rangle - |u^\uparrow(x_1,\vec{k}_1)d^\downarrow(x_2,\vec{k}_2)u^\uparrow(x_3,\vec{k}_3)\rangle\}$$

- leading twist distribution amplitude

$$\phi_N(x_1, x_2, x_3; \mu) = 2\int^{\mu} [d^2\vec{k}]\psi^{L=0}(x_1, x_2, x_3, \vec{k}_1, \vec{k}_2, \vec{k}_3)$$

- defined as

$$\epsilon^{ijk}\langle 0|u^i_\alpha(z_1)u^j_\beta(z_2)d^k_\gamma(z_3)|N(P)\rangle \sim (C\not{P})_{\alpha\beta}N_\gamma \int e^{iP\cdot(x_1 z_1 + x_2 z_2 + x_3 z_3)}\phi_N(x_i;\mu)$$

- can be expanded using OPE

$$\phi_N(x_i;\mu) = 120 f_N x_1 x_2 x_3 \left[1 + \varphi_{10}(\mu)(x_1 - 2x_2 + x_3) + \varphi_{11}(\mu)(x_1 - x_3)\right.$$
$$\left. + \quad \cdots \right]$$

$\varphi_{ij}(\mu)$ shape parameters, related to local operators

What are light-cone sum rules?

- ▷ use analyticity, operator product expansion and quark-hadron duality
- ▷ calculate non-factorizable and factorizable part in terms of same nucleon DAs
- ▷ no double counting
- ▷ quark-hadron duality is only model assumption
- ▷ systematic improvement possible

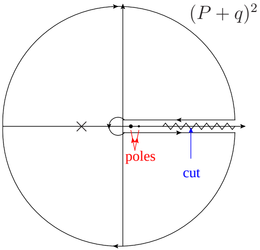
though limited accuracy ~ 10-20 %

Dispersion relations?

- correlation function is analytic

$$T_\mu(P,q) = \int d^4x e^{iqx} \langle 0 | T\{\eta(0) j_\mu^{em}(x)\} | N(P) \rangle$$

- except on positive real axis



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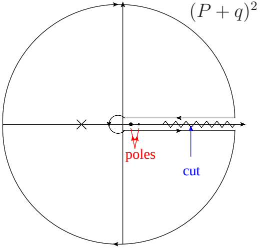

- away from positive real axis it can be expanded around the light cone

$$\int d^4x e^{iqx} \langle 0 | \epsilon^{ijk} T\{u^i(C\gamma_\nu)u^j(\gamma_5\gamma^\nu d^k)_\gamma (e_u \bar{u}\gamma_\mu u + e_d \bar{d}\gamma_\mu d)(x)\} | N(P) \rangle$$

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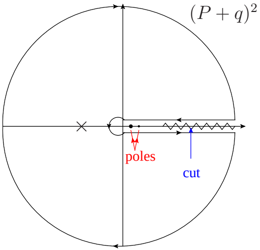

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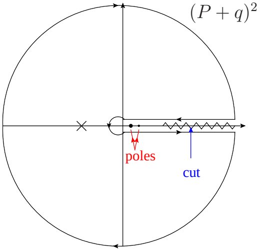

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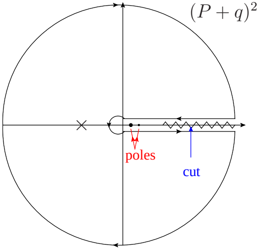
$$e_d \int d^4x e^{iqx} (C\gamma_\nu)_{\alpha\beta} (\gamma_5 \gamma^\nu \frac{-i\not{x}^T}{2\pi^2 x^4} \gamma_\mu)_{\gamma\delta} \langle 0 | \epsilon^{ijk} u^i_\alpha u^j_\beta d^k_\delta(x) | N(P) \rangle$$

Dispersion relations?



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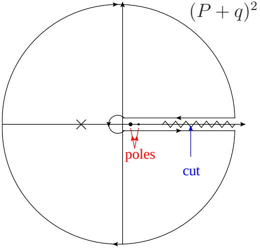

- away from positive real axis it can be expanded around the light cone

$$T_\mu(P,q) = \sum_t \int [dx] (C_\mu^t(x_i, P, P+q))_{\gamma\delta} \, \phi_N^t(x_i) \, N_\delta$$

Dispersion relations?

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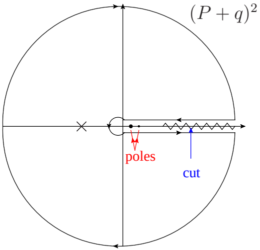


- away from positive real axis it can be expanded around the light cone

$$P_+ z^\mu T_\mu(P,q) = \sum_t \int [dx] (m_N A^t(x_i, Q^2, (P+q)^2) + \not{q}_\perp B^t(x_i, Q^2, (P+q)^2)) \phi_N^t(x_i) N^+$$

- and analytically continued to positive $(P+q)^2$ (modulo subtractions)

$$\int_0^\infty \frac{ds}{s-(P+q)^2} \int [dx] \mathrm{Im} \left(m_N A^t(x_i, Q^2, s) + \not{q} B^t(x_i, Q^2, s) \right) \phi_N^t(x_i) N_\gamma^+ - \text{subtractions}$$

Hadronic Sum

$$P_+ z^\mu T_\mu(P,q) = P_+ \int d^4x\, e^{iqx} \langle 0|T\{\eta(0)\, j_z^{em}(x)\}|N(P)\rangle$$

$$f_N \frac{m_N F_1(Q^2) + \not{q} F_2(Q^2)}{m_N^2 - (P+q)^2} N^+ + \int \frac{ds}{s-(P+q)^2}\rho(s,Q^2) N^+$$

$$\sum_h \int \frac{d^3p}{2p_0(2\pi)^3} |h(\vec{p})\rangle\langle h(\vec{p})|$$

- Combining hadronic sum and OPE result

$$\int_0^\infty \frac{ds}{s-(P+q)^2} \int [dx] \mathrm{Im}\, A^t(x_i, Q^2, s) \phi_N^t(x_i) = \frac{f_N F_1(Q^2)}{m_N^2 - (P+q)^2} + \int_{s_h}^\infty \frac{ds}{s-(P+q)^2} \rho(s, Q^2)$$

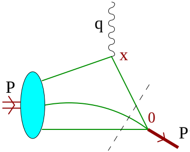
- need some assumption to extract $F_1(Q^2)$ or $F_2(Q^2)$

Quark-Hadron-Duality

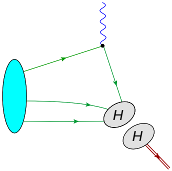
Some details on Duality

- Assume that

$$\int_{s_0}^{\infty} \frac{ds}{s-(p+q)^2}$$

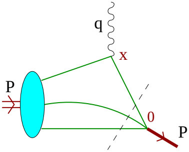


$$\sim \sum_H \int_{s_h}^{\infty} \frac{ds}{s-(p+q)^2}$$

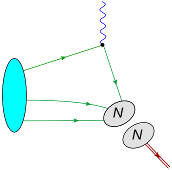


- leads to

$$\int_{0}^{s_0} \frac{ds}{s-(p+q)^2}$$



$$= f_N \frac{1}{m_N-(p+q)^2}$$

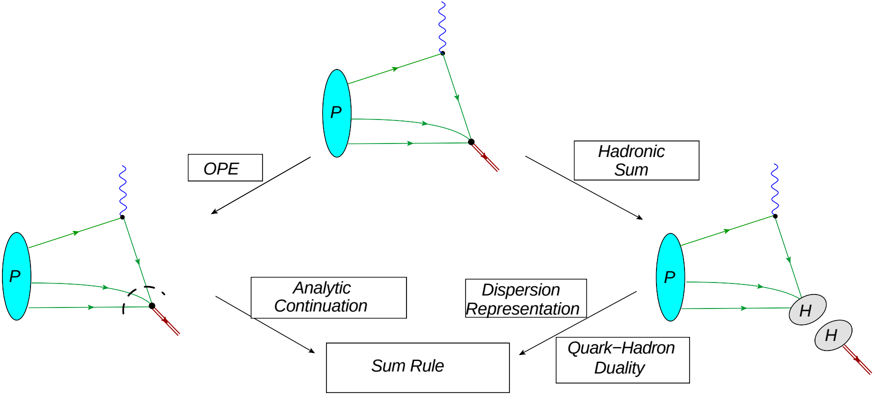


- so that

$$\frac{f_N F_1(Q^2)}{m_N^2-(P+q)^2} = \int_0^{s_0} \frac{ds}{s-(P+q)^2} \int [Dx] \sum_t \mathrm{Im}\, A^t(x_i, s, Q^2, \mu^2) \phi_t(x_i, \mu^2)$$

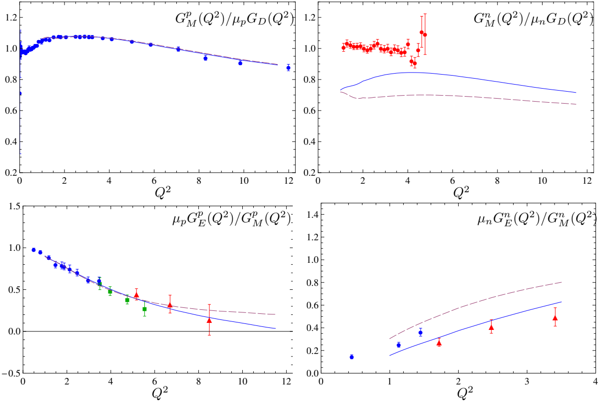
Procedure schematically

$$T_\mu(P,q) = i\int d^4x e^{iq\cdot x}\langle 0|\, T\{\eta(0)\, j_\mu(x)\}|P\rangle$$



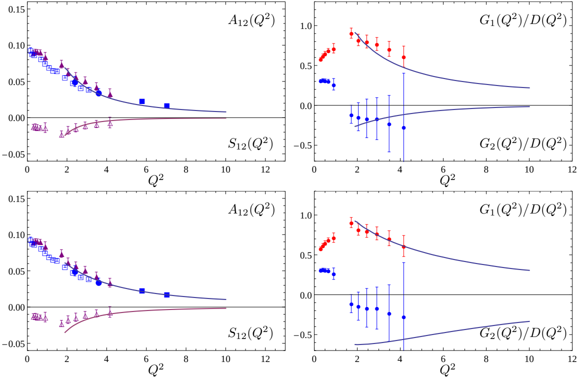
$$F_1(Q^2) = \int_0^{s_0} ds\, e^{\frac{s-m_N^2}{M^2}} \int [Dx] \sum_t \mathrm{Im}\, A^t(x_i, s, Q^2, \mu^2)\, \phi_t(x_i, \mu^2)$$

How does it compare to experiment N?



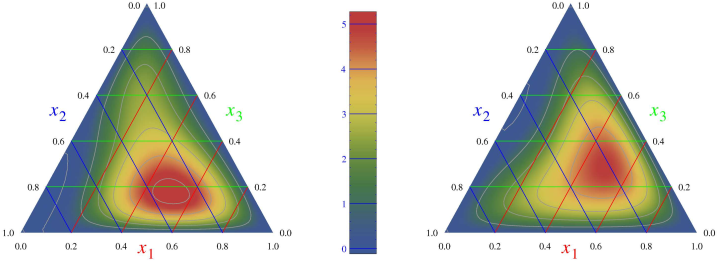
- fits to proton data

How does it compare to experiment N*?



- upper line: fit only to Clas data
- lower line: fit to all available data on helicity amplitudes


What do we learn N?



- leading distribution amplitude can be extracted with 10-20% accuracy
- leading twist distribution amplitude peaks at

$$40\% : 30\% : 30\%$$

- orbital angular momentum contributions are important

What do we learn N*?

Method	$\lambda_1^{N*}/\lambda_1^N$	f_{N*}/λ_1^{N*}	φ_{10}	φ_{11}	φ_{20}	φ_{21}	φ_{22}	η_{10}	η_{11}
LCSR (1)	0.633	0.027	0.36	-0.95	0	0	0	0.00	0.94
LCSR (2)	0.633	0.027	0.37	-0.96	0	0	0	-0.29	0.23
LATTICE	0.633(43)	0.027(2)	0.28(12)	-0.86(10)	1.7(14)	-2.0(18)	1.7(26)	-	-

Parameters of the N* distribution amplitude at $\mu^2 = 2$ GeV^2

- results nearly insensitive to leading twist 3 distribution amplitude
  - related to small value of $f_{N*} \sim 0.7 \cdot 10^{-3}$ GeV^2
- dominated by twist 4 DAs related to orbital angular momentum
- mass corrections very important at low Q^2
- *qqqG*-Fock states not taking into account
- need larger Q^2 fit region to reliably extract shape parameters of DAs

What has been done?

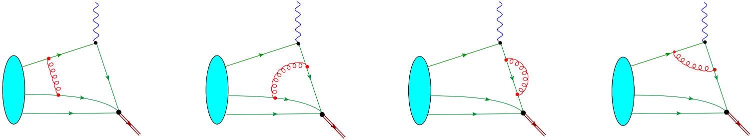
- consistent renormalization scheme for three-quark operators

  Krankl, Manashov

- light-cone expansion of three-quark operators with generic coordinates

  Anikin, Braun, Offen

- NLO coefficient functions for twist 3 and 4



  Anikin, Braun, Offen

- nucleon mass corrections for all twists

  Anikin, Manashov

- nucleon electromagnetic form factors
- N^* form factors

  Anikin, Braun, Offen

What else can be done/improved?

What can be done?

- Roper resonance and $N^*(1650)$ in principle no problem
  - but constraints from lattice helpful
- axial form factors
- threshold pion production at large momentum transfer
- $\Lambda_{b,c} \to N^*$ or other heavy baryon decays first estimate gives $\text{BR}(\Lambda_b \to N^* l\nu) \sim \frac{1}{6}\,\text{BR}(\Lambda_b \to N l\nu)$

What improvements are possible?

- m_N^2-corrections (complete resummation of m_N corrections?)
- inclusion of $qqqG$-Fock state distribution amplitudes

Conclusions

- light cone sum rules give most direct connection of experimental data and distribution amplitudes
- only minimal model assumption
  - Quark-Hadron-duality
- large contribution of angular momentum
  - important for N(940)
  - dominant for $N^*(1535)$
- qualitatively different distribution amplitudes for N(940) and $N^*(1535)$
- extension to heavier resonances possible
- more data from experiment and lattice are needed

Example of Coefficient functions

$$
\begin{aligned}
&x_2 C_d^{\mathcal{V}_1}(x_i) = \\
&= 2x_2x_3\Big[3(L-2)g_1(x_3) + 2(L-1)g_{11}(x_3,x_3) + g_{21}(x_3,x_3)\Big] + \Big[2x_2 + (4L-3)x_3\Big]h_{11}(x_3) + (3-4L)\bar{x}_1 h_{11}(\bar{x}_1) \\
&\quad + 2x_3 h_{21}(x_3) - 2\bar{x}_1 h_{21}(\bar{x}_1) - 2\Big[3(x_2/x_3)(2L-3) + 5L - 7\Big]h_{12}(x_3) + 2(5L-7)h_{12}(\bar{x}_1) - \Big[6(x_2/x_3)+5\Big]h_{22}(x_3) \\
&\quad + 5h_{22}(\bar{x}_1) + (6/x_3)(L-2)h_{13}(x_3) - (6/\bar{x}_1)(L-2)h_{13}(\bar{x}_1) + (3/x_3)h_{23}(x_3) - (3/\bar{x}_1)h_{23}(\bar{x}_1)\,, \qquad \text{(E.1)}
\end{aligned}
$$

$$
\begin{aligned}
&x_1x_3 C_u^{\mathcal{V}_1}(x_i) = \\
&= x_1x_2x_3\Big[(17-7L)g_1(x_2) + (1+2L)g_{11}(\bar{x}_1,x_2) + 2(2L-3)g_{11}(\bar{x}_3,x_2) + 2(5-7L)g_{11}(x_2,x_2) + g_{21}(\bar{x}_1,x_2) \\
&\quad + 2g_{21}(\bar{x}_3,x_2) - 7g_{21}(x_2,x_2)\Big] - x_1x_3\Big[(1+2L)h_{11}(\bar{x}_1) + 2(2L-3)h_{11}(\bar{x}_3) + 2(5-7L)h_{11}(x_2)\Big] \\
&\quad + (1+2L)x_1h_{12}(\bar{x}_1) + 4x_3h_{12}(\bar{x}_3) - \Big[4x_3 + (x_1/x_2)[x_2(1+2L) + 4x_3(4-L)]\Big]h_{12}(x_2) - 2(L-2)(x_1/\bar{x}_1)h_{13}(\bar{x}_1) \\
&\quad + 2(2L-7)(x_3/\bar{x}_3)h_{13}(\bar{x}_3) + (1/x_2)\Big[2(L-2)x_1 + 2(7-2L)x_3\Big]h_{13}(x_2) - x_1x_3\Big[h_{21}(\bar{x}_1) + 2h_{21}(\bar{x}_3) - 7h_{21}(x_2)\Big] \\
&\quad + x_1h_{22}(\bar{x}_1) + x_1\Big[2(x_3/x_2) - 1\Big]h_{22}(x_2) - (x_1/\bar{x}_1)h_{23}(\bar{x}_1) + 2(x_3/\bar{x}_3)h_{23}(\bar{x}_3) + \Big[(x_1 - 2x_3)/x_2\Big]h_{23}(x_2)\,, \qquad \text{(E.2)}
\end{aligned}
$$

$$
\begin{aligned}
&x_2 C_d^{\mathcal{V}_3}(x_i) = \\
&= 2x_2x_3\Big[(5-3L)g_1(x_3) + (3-4L)g_{11}(\bar{x}_1,x_3) + 2(2L-1)g_{11}(x_3,x_3) - 2g_{21}(\bar{x}_1,x_3) + 2g_{21}(x_3,x_3)\Big] \\
&\quad + 2(4L-3)(2x_2+x_3)h_{11}(\bar{x}_1) + \Big[8(1-2L)x_2 + 2(3-4L)x_3\Big]h_{11}(x_3) + 4(2x_2+x_3)\Big[h_{21}(\bar{x}_1) - h_{21}(x_3)\Big] \\
&\quad + 6(3-4L)h_{12}(\bar{x}_1) + 6\Big[4L - 3 + 4(x_2/x_3)(L-1)\Big]h_{12}(x_3) - 12h_{22}(\bar{x}_1) + 12(\bar{x}_1/x_3)h_{22}(x_3) \\
&\quad + (4/\bar{x}_1)(2L-1)h_{13}(\bar{x}_1) - (4/x_3)(2L-1)h_{13}(x_3) + (4/\bar{x}_1)h_{23}(\bar{x}_1) - (4/x_3)h_{23}(x_3)\,, \qquad \text{(E.3)}
\end{aligned}
$$

$$
\begin{aligned}
&x_1x_3 C_u^{\mathcal{V}_3}(x_i) = \\
&= 2x_1x_2x_3\Big[5(L-3)g_1(x_2) + 2(1-2L)g_{11}(\bar{x}_1,x_2) + (5-4L)g_{11}(\bar{x}_3,x_2) + 2(8L-5)g_{11}(x_2,x_2) \\
&\quad - 2g_{21}(\bar{x}_1,x_2) - 2g_{21}(\bar{x}_3,x_2) + 8g_{21}(x_2,x_2)\Big] + 2x_3\Big[(6L-8)x_1 + (2L-3)x_2\Big]h_{11}(\bar{x}_3) \\
&\quad + 4x_1\Big[Lx_2 + (3L-1)x_3\Big]h_{11}(\bar{x}_1) - 2\Big[4x_1x_3(5L-3) + x_2x_3(2L-3) + 2x_1x_2L\Big]h_{11}(x_2) \\
&\quad + 2x_1(x_2+3x_3)h_{21}(\bar{x}_1) + 2x_3(3x_1+x_2)h_{21}(\bar{x}_3) - 2\Big[10x_1x_3 + x_2\bar{x}_2\Big]h_{21}(x_2) - 4(3+2L)x_1h_{12}(\bar{x}_1) \\
&\quad + 2x_3(15-8L)h_{12}(\bar{x}_3) + 2(x_3/x_2)\Big[4(L-1)x_1 + (8L-15)x_2\Big]h_{12}(x_2) + 4(x_1/x_2)\Big[(3+2L)x_2 + 6x_3\Big]h_{12}(x_2) \\
&\quad - 4x_1h_{22}(\bar{x}_1) - 8x_3h_{22}(\bar{x}_3) + (4/x_2)\Big[2x_2x_3 + x_1\bar{x}_1\Big]h_{22}(x_2) + 12(x_1/\bar{x}_1)h_{13}(\bar{x}_1) + 8(x_3/\bar{x}_3)(L-2)h_{13}(\bar{x}_3) \\
&\quad - (4/x_2)[3x_1 + 2x_3(L-2)\Big]h_{13}(x_2) + 4(x_3/\bar{x}_3)h_{23}(\bar{x}_3) - 4(x_3/x_2)h_{23}(x_2)\,, \qquad \text{(E.4)}
\end{aligned}
$$

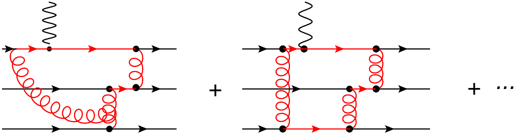
$$
\begin{aligned}
&x_1x_2 C_u^{\mathcal{V}_2^{(1)}}(x_i) = \\
&= -8\bar{x}_3x_2g_1(\bar{x}_3) + 2x_2^2\Big[4(L-2) + (2L-5)x_1\Big]g_1(x_2) - 4x_2^2\Big[(L-3) + (2L-3)x_1\Big]g_{11}(\bar{x}_3,x_2) \\
&\quad + 2x_2^2(1+2x_1)\Big[2(L-1)g_{11}(x_2,x_2) - g_{21}(\bar{x}_3,x_2) + g_{21}(x_2,x_2)\Big] + 4x_2\Big[(L-3) + (2L-3)x_1\Big]h_{11}(\bar{x}_3) \\
&\quad - 2x_2(1+2x_1)\Big[2(L-1)h_{11}(x_2) - h_{21}(\bar{x}_3) + 2h_{21}(x_2)\Big] + 2x_2\Big[(4L-12)/\bar{x}_3 + (4L-13)/x_1 - 4\Big]h_{12}(\bar{x}_3) \\
&\quad + 2\Big[4(1+x_2-L) + (x_2/x_1)(13-4L)\Big]h_{12}(x_2) + 4\Big[1 - (x_1/\bar{x}_3) + (x_2/x_1)\Big]h_{22}(\bar{x}_3) - 4\Big[1 + (x_2/x_1)\Big]h_{22}(x_2) \\
&\quad + 2\Big[(x_1/\bar{x}_3^2)(8L-25) - 2(x_2/\bar{x}_3)(2L-7) - (1/x_1)(8L-25)\Big]h_{13}(\bar{x}_3) + 2\Big[2(2L-7) + (1/x_1)(8L-25)\Big]h_{13}(x_2) \\
&\quad + 4\Big[2(x_1/\bar{x}_3^2) - (x_2/\bar{x}_3) - (2/x_1)\Big]h_{23}(\bar{x}_3) + 4\Big[1 + (2/x_1)\Big]h_{23}(x_2)\,, \qquad \text{(E.5)}
\end{aligned}
$$

Perturbative part of F_1

- next to leading order calculation in progress

$$F_1(Q^2) = \frac{1}{Q^4} \int [dx_i] \int [dy_i] \phi_N(x_i;\mu)\, T_H(x_i, y_i;\mu)\, \phi_N(y_i;\mu)$$

- 1890 diagrams with up to seven legs needed



- renormalization for asymptotic distribution amplitude done
- some tricks to avoid overlapping phase space divergences still needed