High Q^2 Helicity Amplitudes in the hypercentral Constituent Quark Model

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Nucleon Resonances: From Photoproduction to High Photon Virtualities

Trento, 12-16 October 2015

Outline of the talk

- The model

- Some remarks on the spectrum

- The helicity amplitudes
    - Photocouplings
    - Q^2 dependence

- Relativity
    - Elastic form factors
    - N-Δ transition

- Asymptotic behaviour

The model

Hypercentral Constituent Quark Model

$$H_{3q} = T + V_{3q}(\rho,\lambda) + H_{hyp}$$

SU(6)
symmetry

SU(6)
violation

SU(6) configurations L^P_t t=A,M,S (symmetry type)

the quark interaction contains

- a long range spin-independent confinement
- a short range spin dependent term

A separation typical of any CQM

M. Ferraris et al., Phys. Lett. **B364**, 231-238 (1995).

For a review: M.G, E. Santopinto, Chin. J. Phys. **53**, 1-75 (2015).

Hyperspherical coordinates: $\boldsymbol{\rho}, \ \boldsymbol{\lambda} \longrightarrow (x, t, \Omega_\rho, \Omega_\lambda)$

$x = (\rho^2 + \lambda^2)^{1/2}$

hyperradius

(size)

$t = \text{arctg}\ \rho/\lambda$

hyperangle

(form)

In the Schrödinger equation $L^2(\theta,\phi) \longrightarrow L^2(t, \Omega_\rho, \Omega_\lambda)$

$C_2(O(3))$ $C_2(O(6))$

$$L^2(t, \Omega_\rho, \Omega_\lambda)\, Y_{[\gamma]}(t, \Omega_\rho, \Omega_\lambda) = -\gamma\,(\gamma+4)\, Y_{[\gamma]}(t, \Omega_\rho, \Omega_\lambda)$$

$Y_{[\gamma]}(t, \Omega_\rho, \Omega_\lambda)$	hyperspherical harmonics
$\gamma = 2n + l_\rho + l_\lambda$	grand angular quantum number

Hypercentral hypothesis

$$V = V(x)$$

Factorization of the wf

$$\Psi(x,t, \Omega_\rho, \Omega_\lambda) = \psi_{\nu\gamma}(x) \quad Y_{[\gamma]}(\Omega)$$

‘dynamics” “geometry”

ν number of nodes

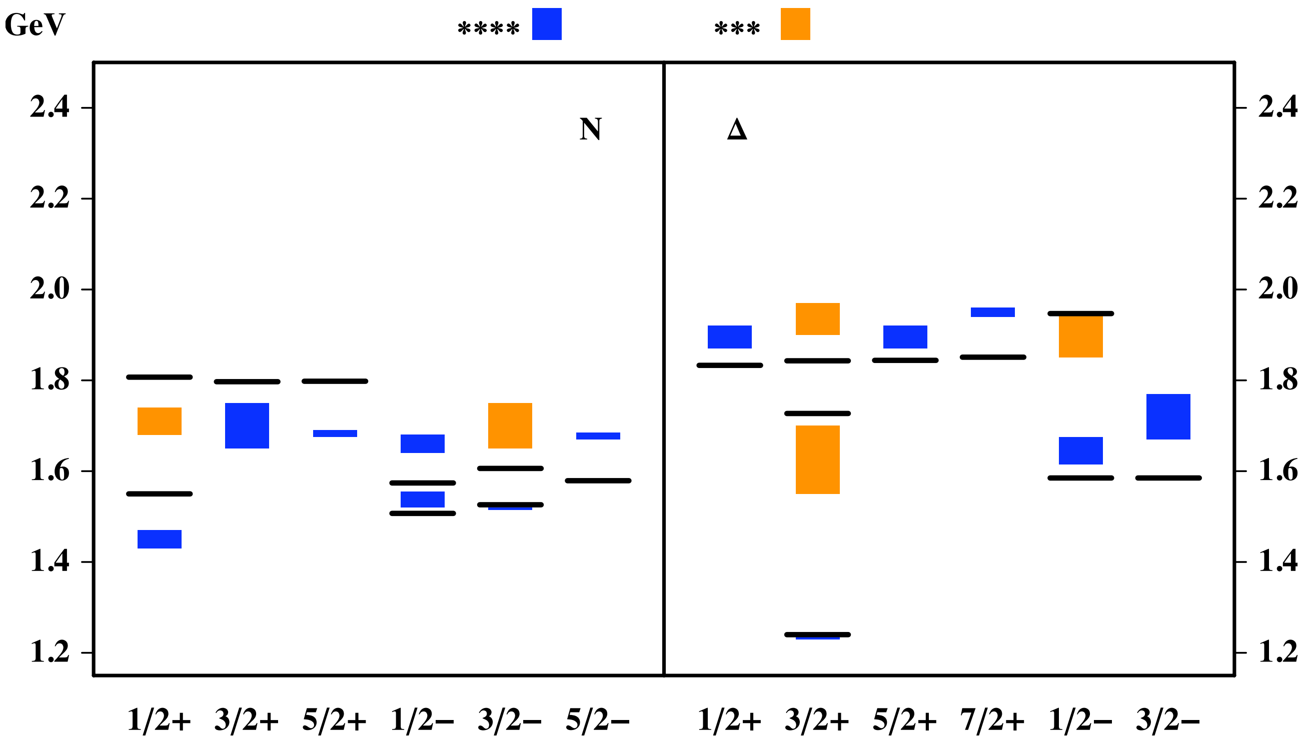
γ grand angular quantum number

Only one “hypercentral” equation for $\psi_{\nu\gamma}(x)$

Hypercentral Model

$$H_{3q} = T + -\tau/x + \alpha x + H_{hyp}$$

three free parameters fitted to the spectrum



Having fixed the parameters => **predictions**

From M. Ferraris et al., Phys. Lett. B364, 231 (1995)

Some remarks on the spectrum

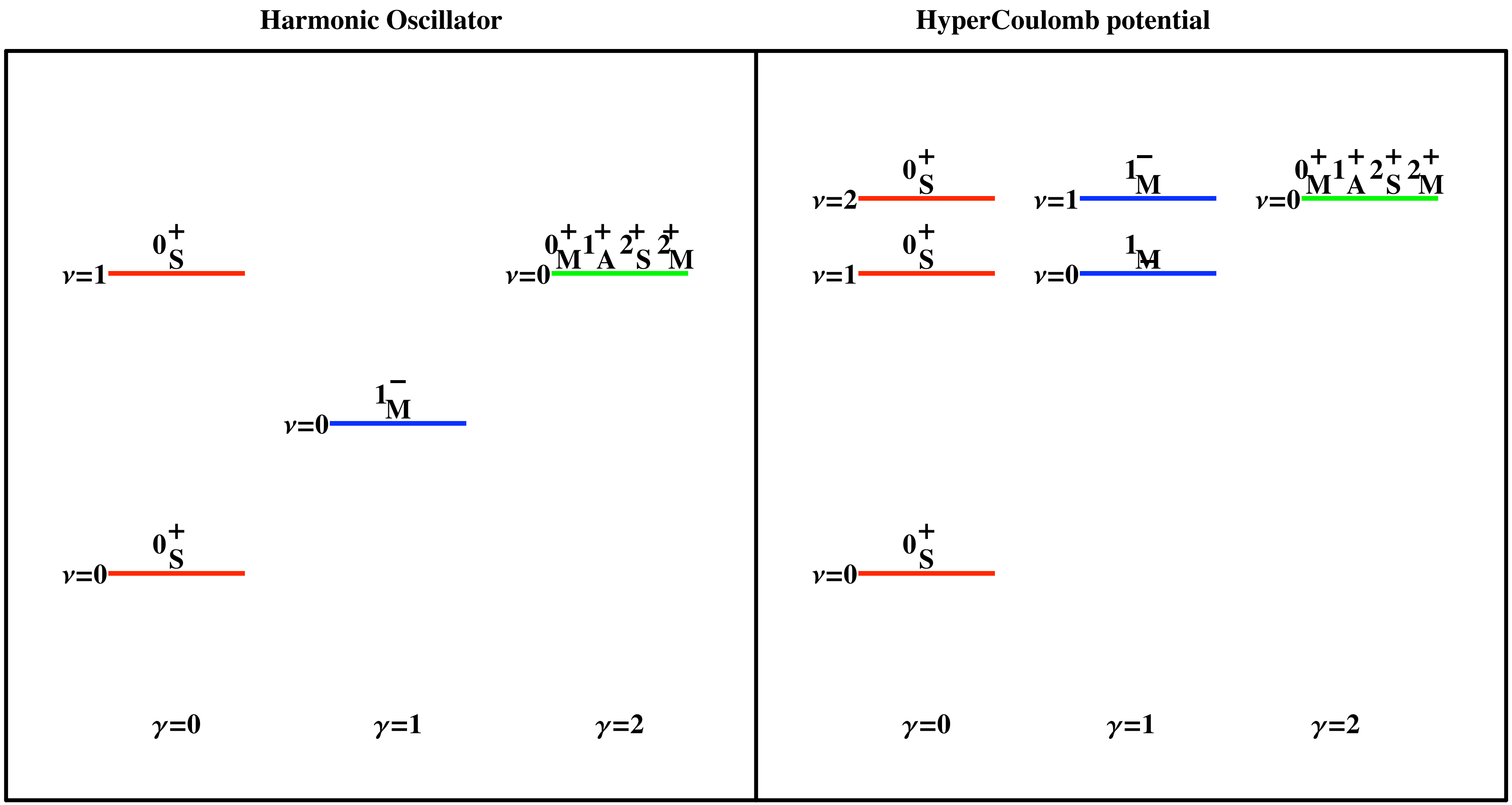
Two analytical solutions of the hypercentral equation

h. o.

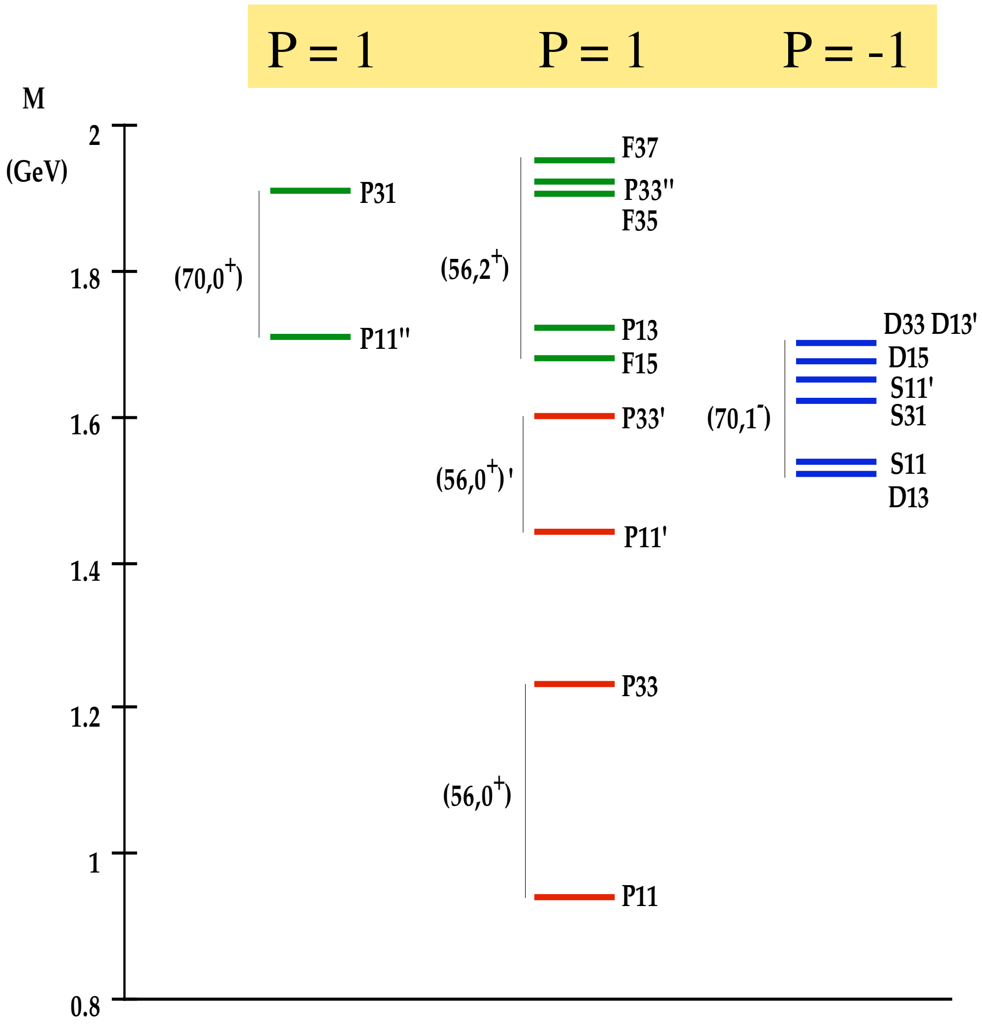
$\Sigma_{i<j} 1/2\ k\ (r_i - r_j)^2 = 3/2\ k\ x^2$

hyperCoulomb

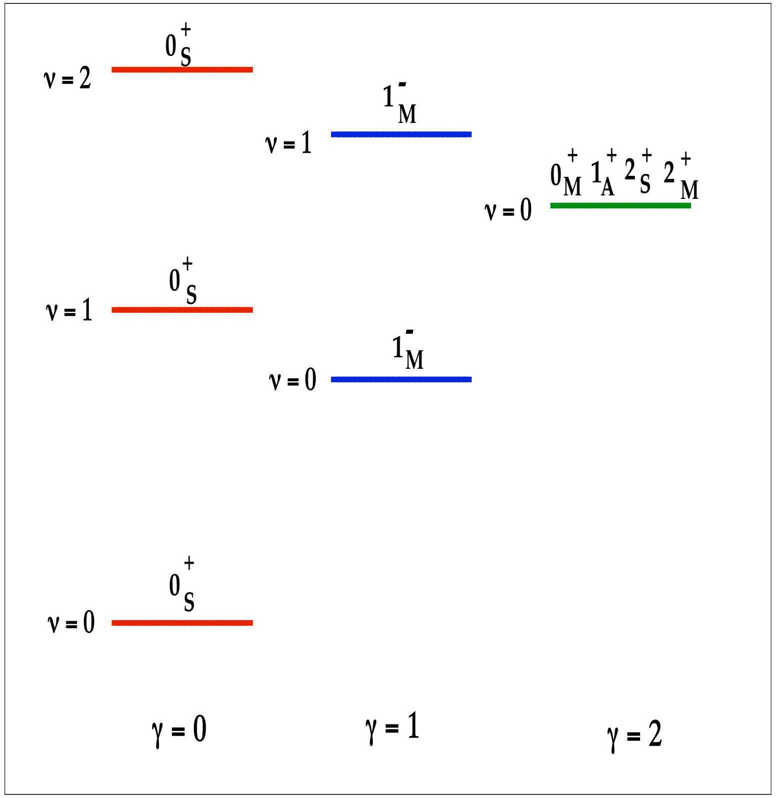
$- \tau/x$



PDG 4* & 3*



$V(x) = - \tau/x + \alpha\ x$



Negative parity states

SU(6) configuration 1^-_M

Possible 3-quark states

obtained combining the orbital angular momentum L=1 with the spin values

Notation $^{2s+1}SU(3)$

$^{2}8$	$^{4}8$	$^{2}10$
S=1/2	S=3/2	S=1/2
N $1/2^-$ N $3/2^-$	N $1/2^-$ N $3/2^-$ N $5/2^-$	Δ $1/2^-$ Δ $3/2^-$

The SU(6) configuration 1^-_M
Contains all the 4* & 3* resonances known prior up to 2010

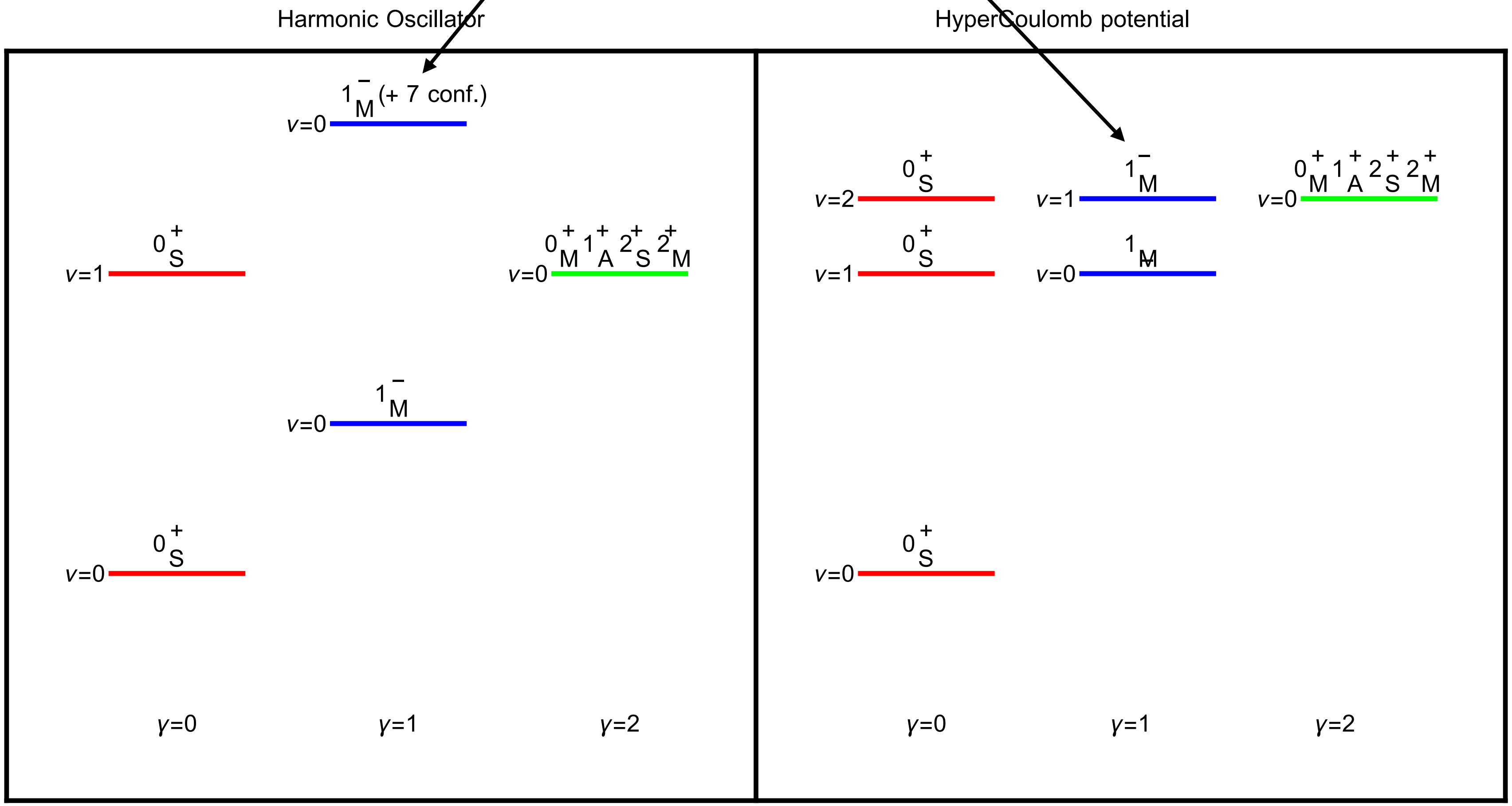
$^{2}8$	$^{4}8$	$^{2}10$
N(1535)1/2$^-$	N(1650)1/2$^-$	Δ(1620)1/2$^-$
N(1520)3/2$^-$	N(1700)3/2$^-$	Δ(1700)3/2$^-$
	N(1675)5/2$^-$	

BUT
in the PDG 2102-2014 there are new entries

3* N(1875) 3/2$^-$ where should it be placed?

(there are also 5 new 2* states!)

N(1875) $3/2^-$

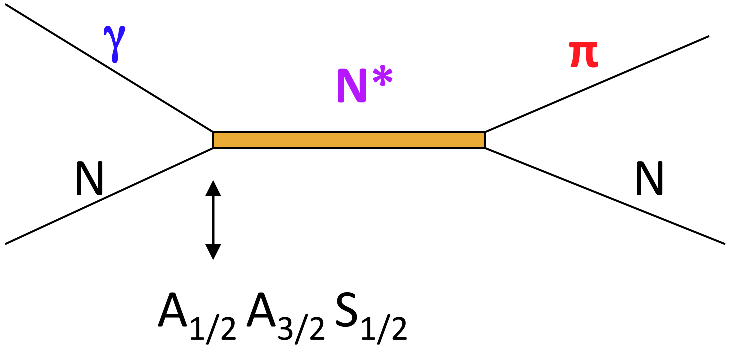


M.G., E. Santopinto, nucl-th 1510.00582

The helicity amplitudes

HELICITY AMPLITUDES

Extracted from electroproduction of mesons



Definition

$A_{1/2} = < N^* \; J_z = 1/2 \mid H^T_{em} \mid N \; J_z = -1/2 >$ §

$A_{3/2} = < N^* \; J_z = 3/2 \mid H^T_{em} \mid N \; J_z = 1/2 >$ §

$S_{1/2} = < N^* \; J_z = 1/2 \mid H^L_{em} \mid N \; J_z = 1/2 >$

N, N* nucleon and resonance as 3q states

H^T_{em} H^l_{em} model transition operator

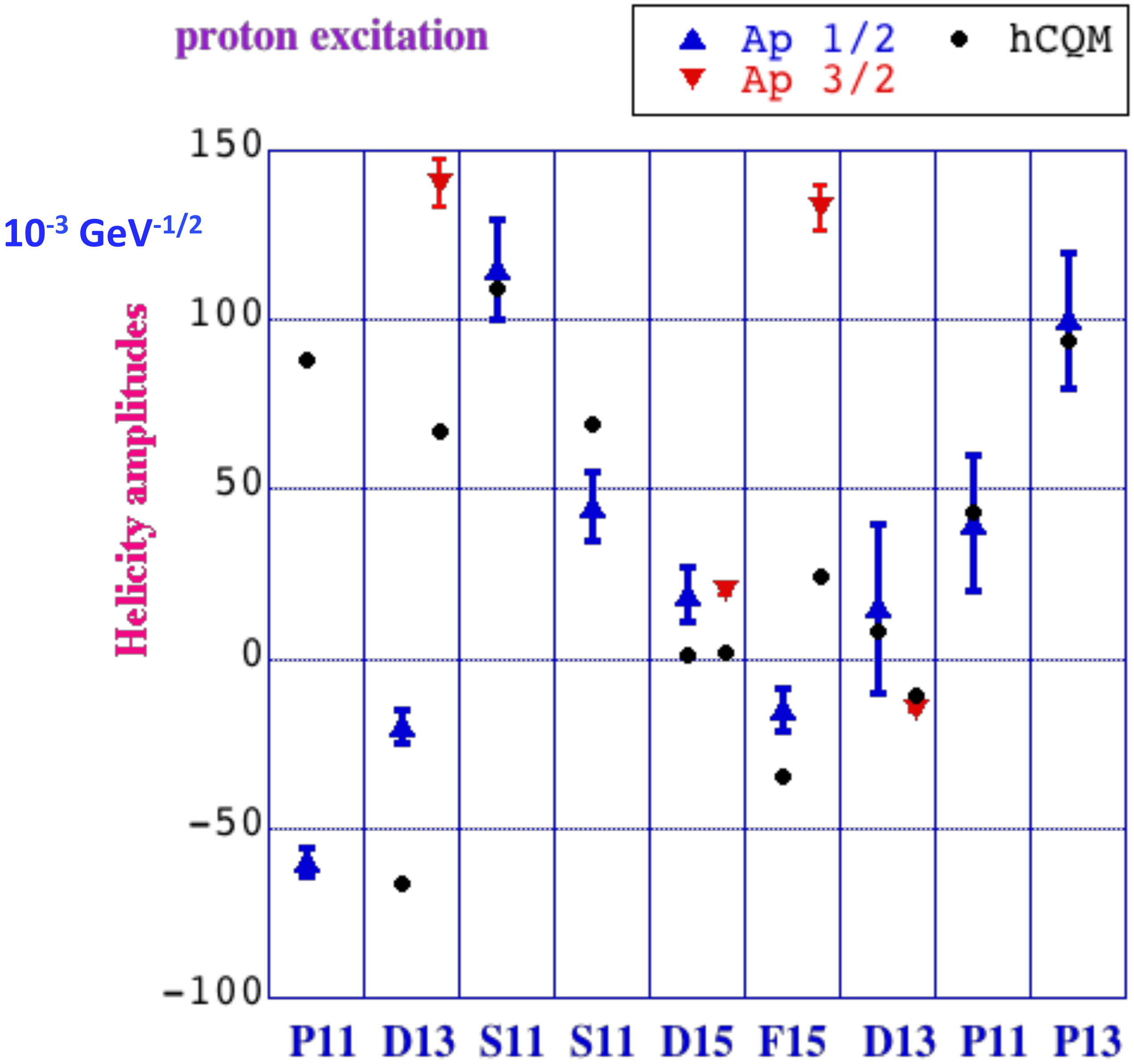
calculated in the Breit System

§ **results for the negative parity resonances:**
M. Aiello, M.G., E. Santopinto J. Phys. G24, 753 (1998)

Systematic predictions for transverse and longitudinal amplitudes
E. Santopinto, M.G., Phys. Rev. C86, 065202 (2012)

Proton and neutron electro-excitation to 14 resonances

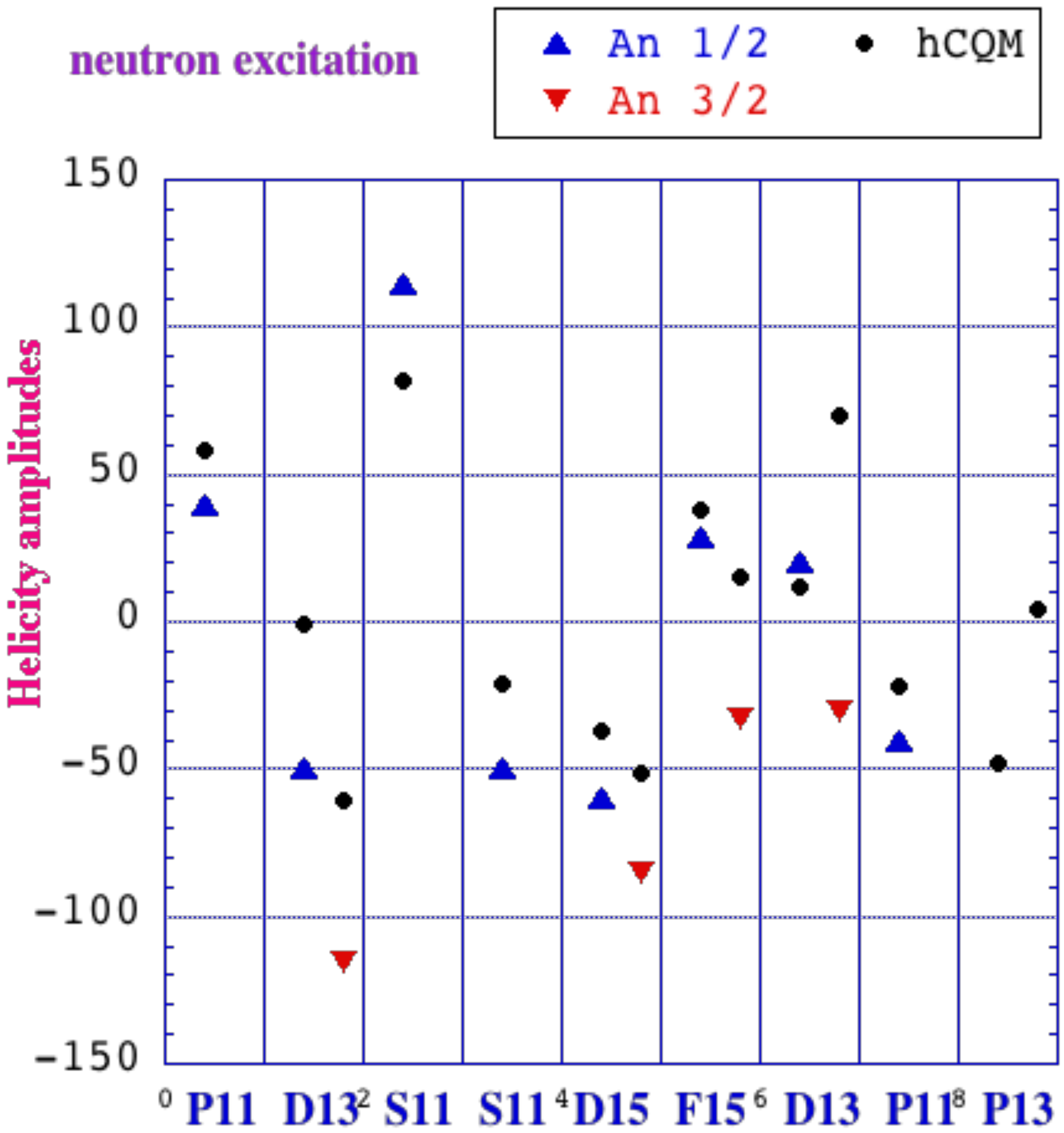
The photocouplings



M. Aiello et al., Phys. Lett. B387, 215 (1996)

E. Santopinto, M.G., Phys. Rev. C86, 065202 (2012)

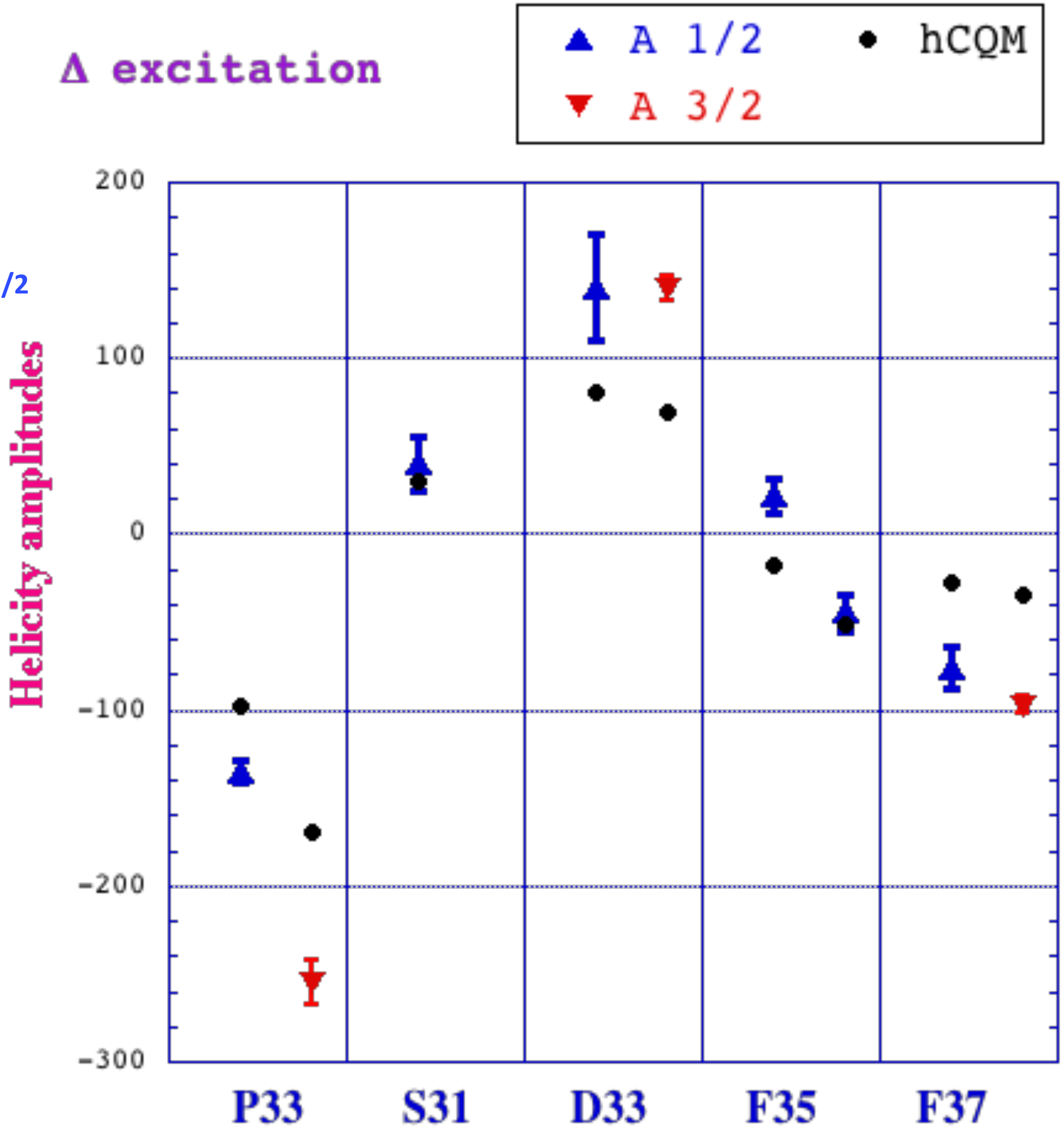
10^{-3} GeV$^{-1/2}$



M. Aiello et al., Phys. Lett. B387, 215 (1996)

E. Santopinto, M.G., Phys. Rev. C86, 065202 (2012)

10^{-3} GeV$^{-1/2}$



M. Aiello et al., Phys. Lett. B387, 215 (1996)

E. Santopinto, M.G., Phys. Rev. C86, 065202 (2012)

Q^2 = 0 values with hCQM

10^{-3} GeV$^{-1/2}$

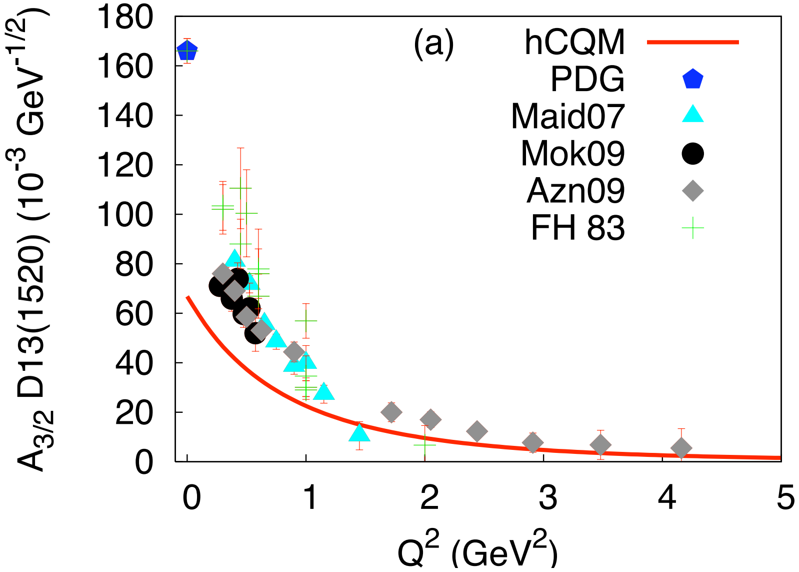
	Ap 1/2	Ap 3/2	Sp 1/2	An 1/2	An 3/2	Sn 1/2
D13 (1520)	-65,7	66,8	78,2	-1,4	-61,1	-79,6
D13 (1700)	🟧 8,0	🟧 -10,9	🟧 -7,9	12	70,1	🟧 8,1
D15 (1675)	🟧 1,4	🟧 1,9	🟩 0	-36,6	-51,1	🟧 -0,2
D33(1700)	80,9	70,2	78,2			
F15 (1680)	-35,4	24,1	27,4	37,7	🟨 14,8	🟧 -0,6
F35(1905)	-16,6	-50,5	🟧 -4,6			
F37(1950)	-28,0	-35,1	🟧 -0,4			
P11(1440)	87,7		65,4	57,9		🟧 -0,9
P11(1710)	42,5		-22,6	-21,7		18,4
P13(1720)	94,1	-17,2	-35,8	-47,6	🟨 3,9	🟧 13,5
P33(1232)	-96,9	-169	🟧 -0,6			
S11(1535)	108		-48,4	-81,7		49,2
S11(1650)	🟧 68,8		🟧 -27,5	-21,0		🟧 28,2
S31(1620)	29,7		-55,3			

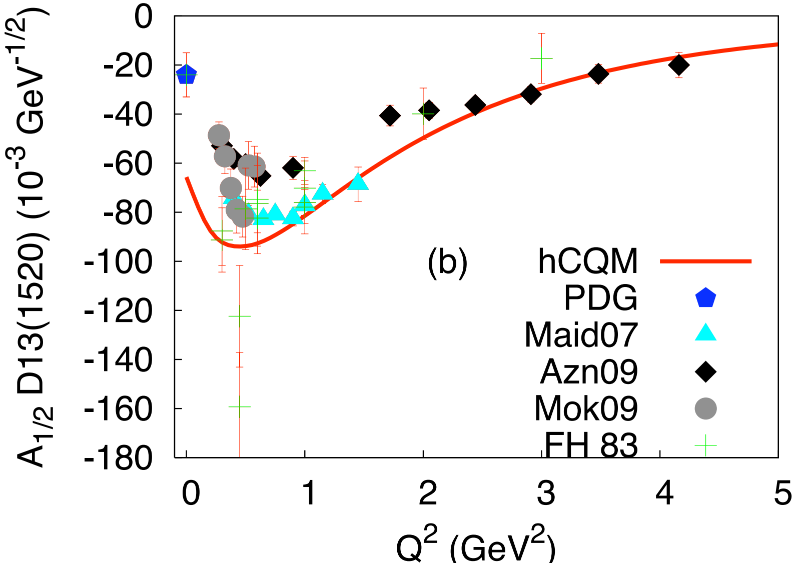
🟧	zero for no Hyp
🟨 10^(-5)	no Hyp
🟩	identically zero

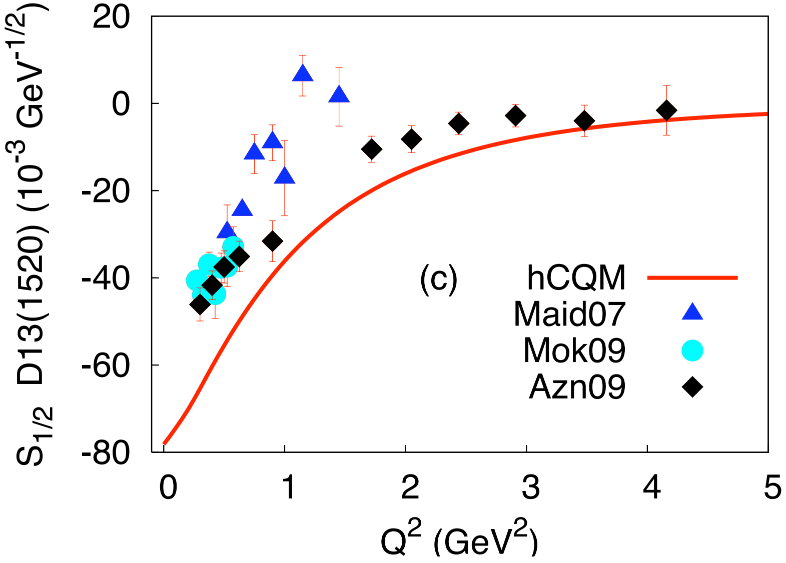
M. Aiello et al., Phys. Lett. B387, 215 (1996)
E. Santopinto, M.G., Phys. Rev. C86, 065202 (2012)

Q^2 dependence

N(1520) 3/2$^-$ transition amplitudes

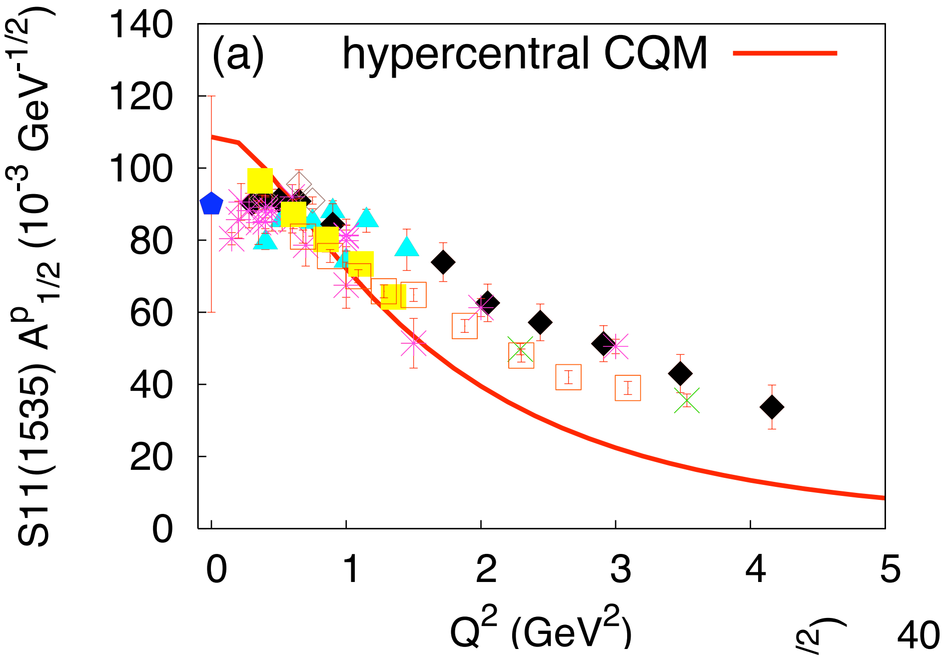


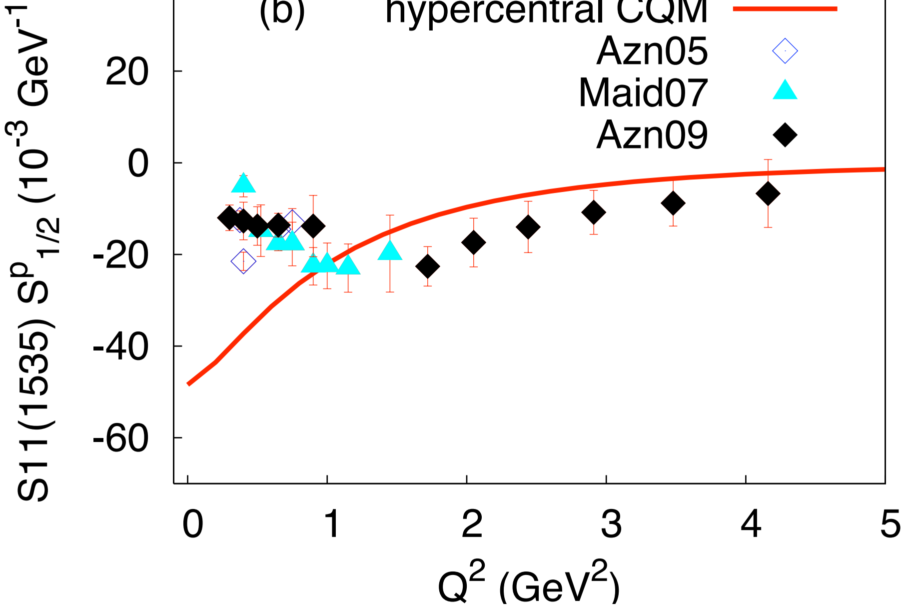




M. Aiello et al.,
Phys. Lett. B387,
215 (1996)
E. Santopinto, M.G.
Phys. Rev. C86,
065202 (2012)

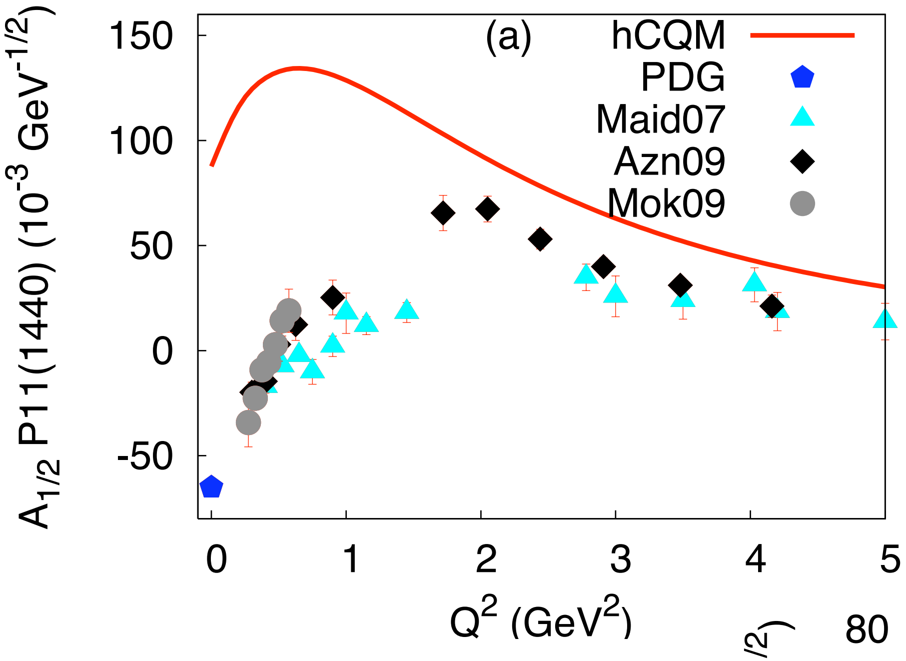
N(1535) ½ $^-$ transition amplitudes

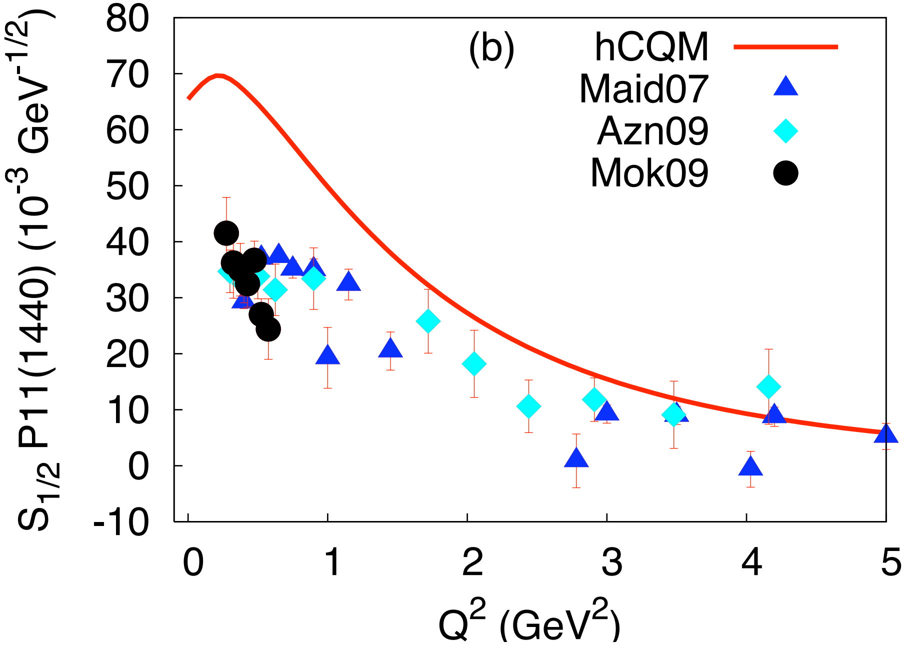




M. Aiello et al.,
Phys. Lett. B387,
215 (1996)
E. Santopinto, M.G.
Phys. Rev. C86,
065202 (2012)

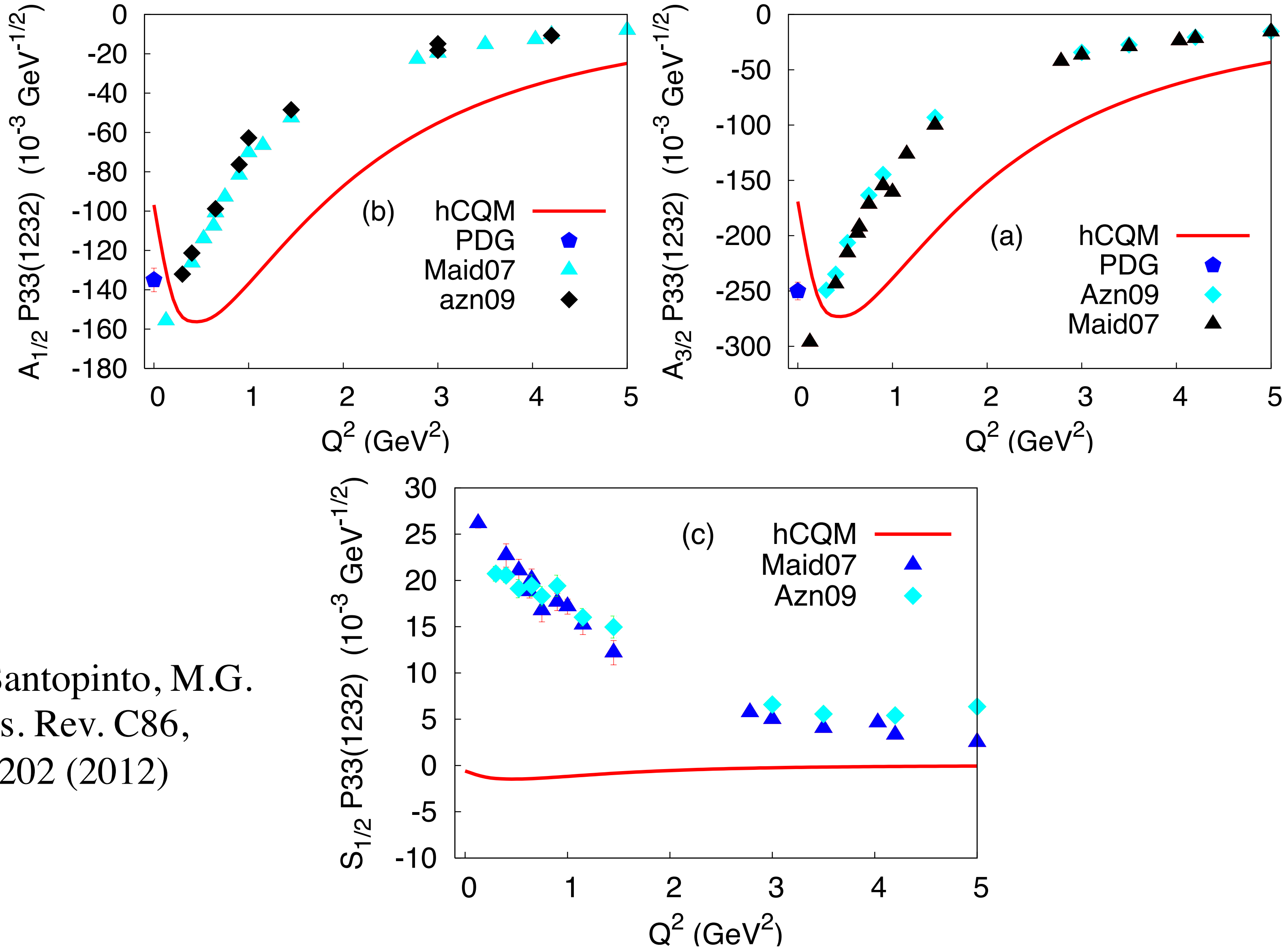
N(1440) ½ $^{+}$
(Roper)
transition amplitudes





E. Santopinto, M.G.
Phys. Rev. C86,
065202 (2012)

Δ(1232) 3/2⁺ transition amplitudes



E. Santopinto, M.G.
Phys. Rev. C86,
065202 (2012)

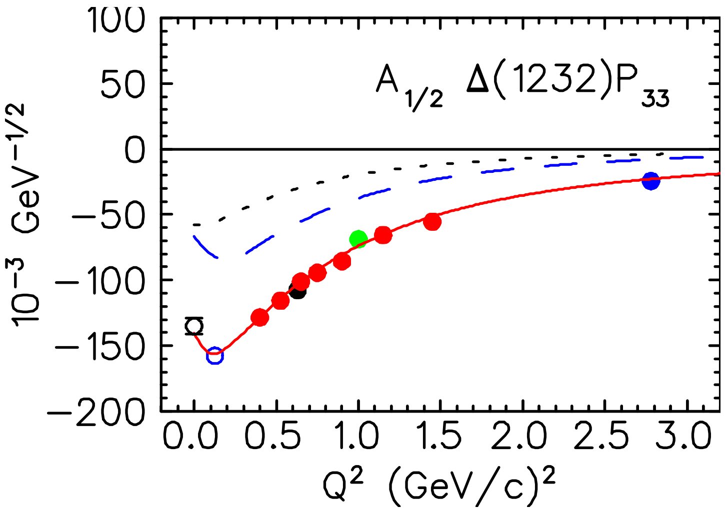
There is missing strength at low Q^2

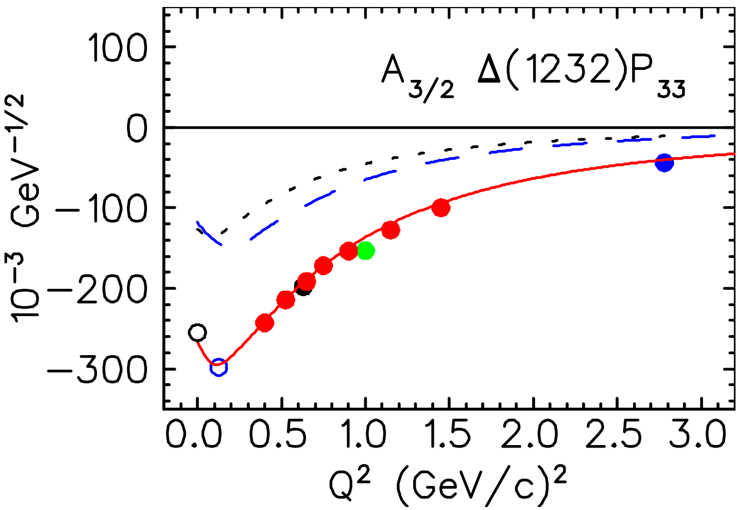
The reason is attributed to the lack of

Quark-antiquark pair mechanisms

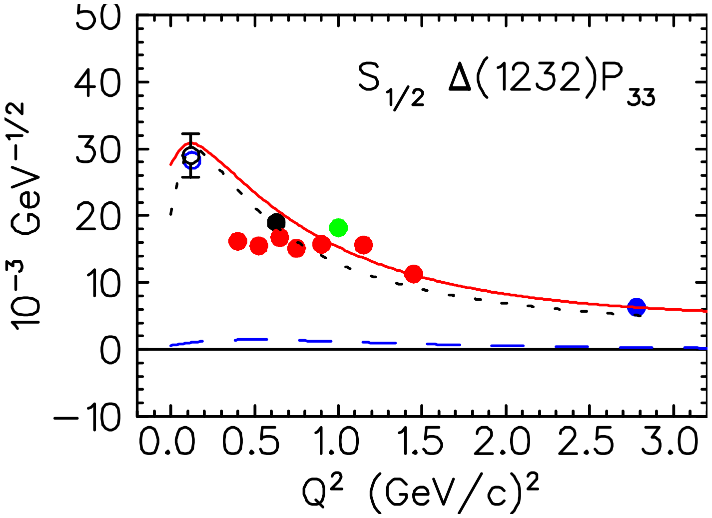
not present in a three-quark model

E. Santopinto, Ph. D. Thesis (Genova 1996).
M. Aiello et al., Phys. Lett. B387, 215 (1996)





solid: MAID
dotted: dynamical model
dashed: hCQM predictions



L. Tiatot et al., Eur. Phys. J. A19}, 55 (2004).

Relativity

Various levels

- Lorentz boosts
- Relativistic dynamics

- quark-antiquark pair effects (meson cloud)
- [relativistic equations (BS, DS)]

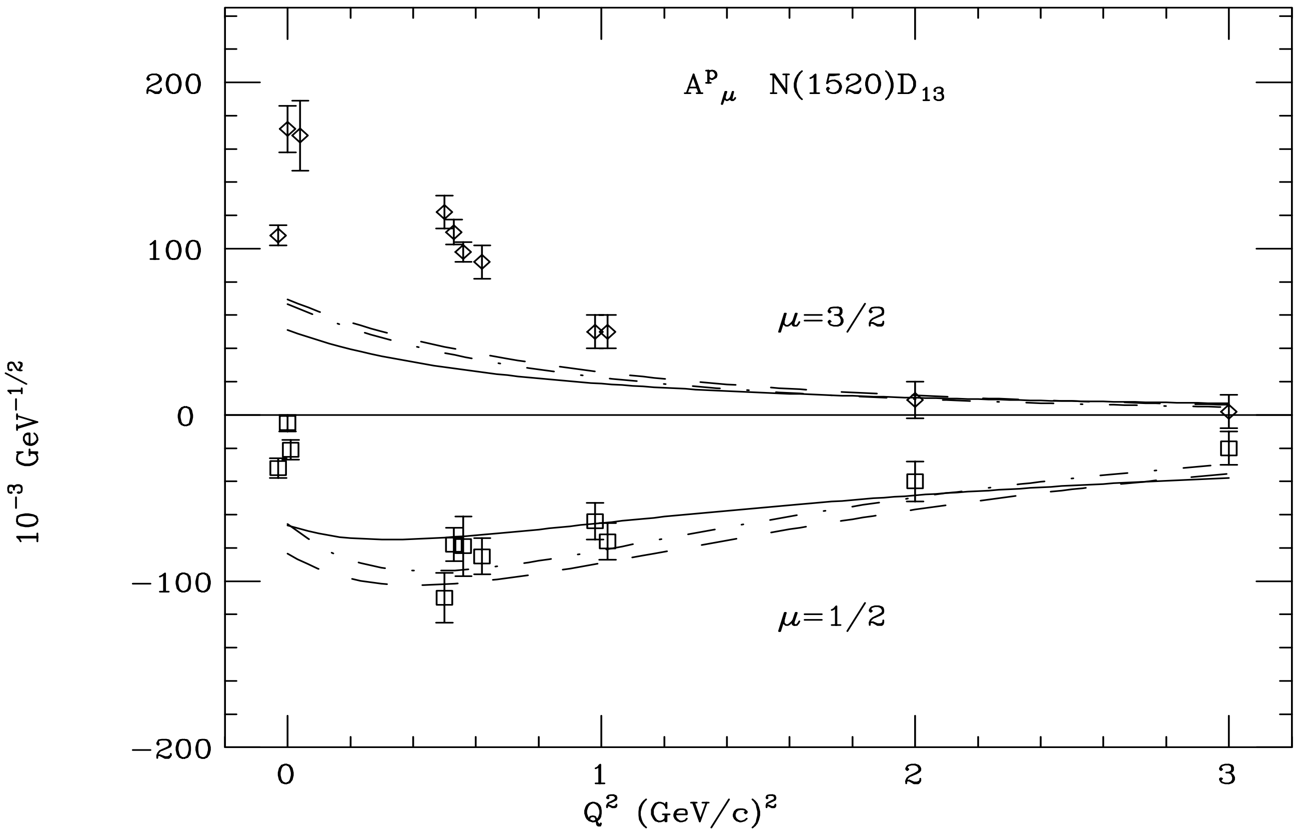
Relativistic corrections to form factors

- Lorentz boosts applied to the initial and final state
- Expansion of current matrix elements up to first order in quark momentum

- Results

$A_{rel}\,(Q^2) = F\ \ A_{n.rel}(Q^2{}_{eff})$

F = kin factor $\qquad Q^2{}_{eff} = Q^2\,(M_N/E_N)^2$

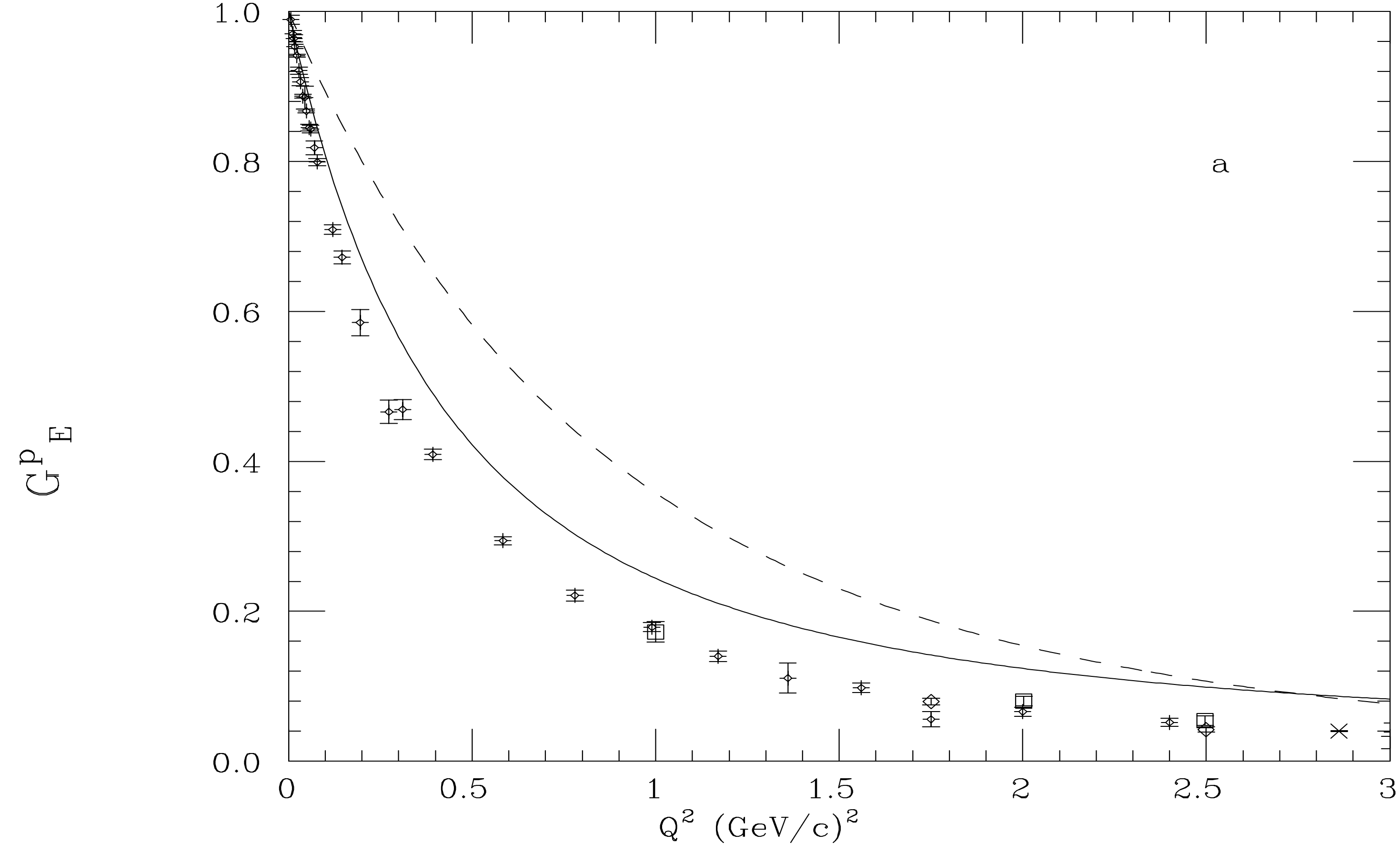


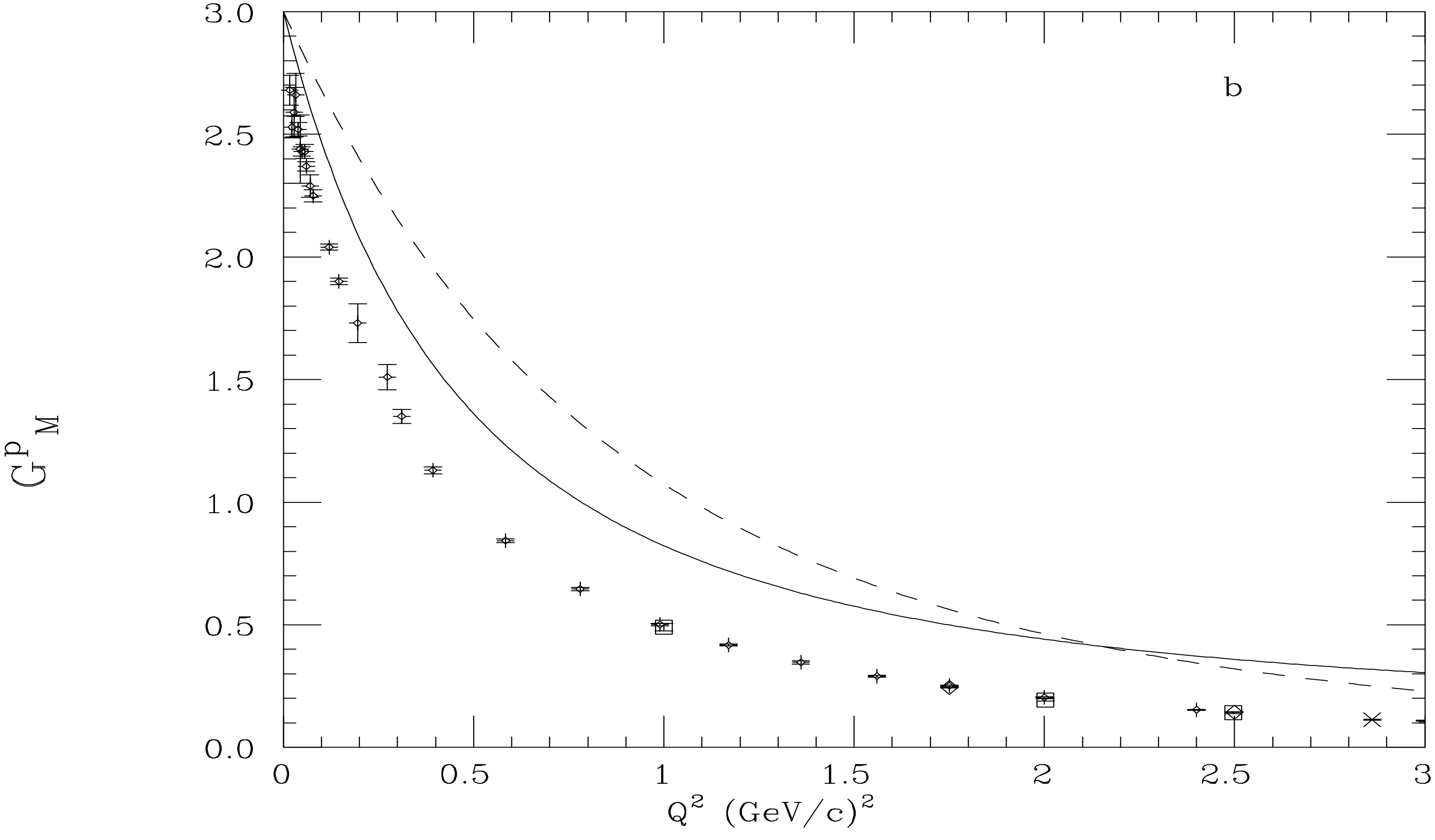
A^p_μ N(1520)D_{13}
μ=3/2
μ=1/2
200
100
0
−100
−200
10^{-3} GeV$^{-1/2}$
0
1
2
3
Q^2 (GeV/c)^2

In the case of the helicity amplitudes the application of Lorentz boosts does not alter the results

BUT

for the elastic form factors the situation is different

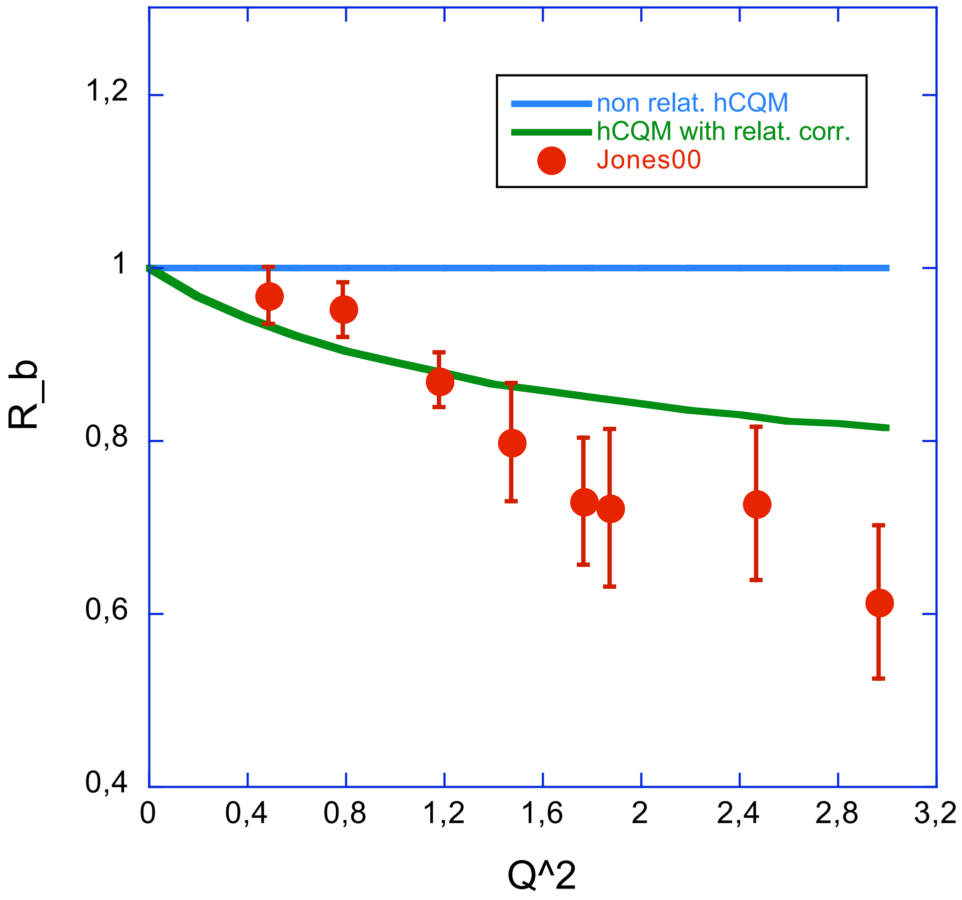

1.0
0.8
0.6
0.4
0.2
0.0
0
0.5
1
1.5
2
2.5
3
a
G^p_E
Q^2 (GeV/c)^2


3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
0.5
1
1.5
2
2.5
3
b
$G^p{}_M$
Q^2 (GeV/c)^2

With Lorentz boosts:

improvement of the elastic f.f.

depletion of the ratio G_E^P/G_M^P



A fully relativistic treatment is necessary

But

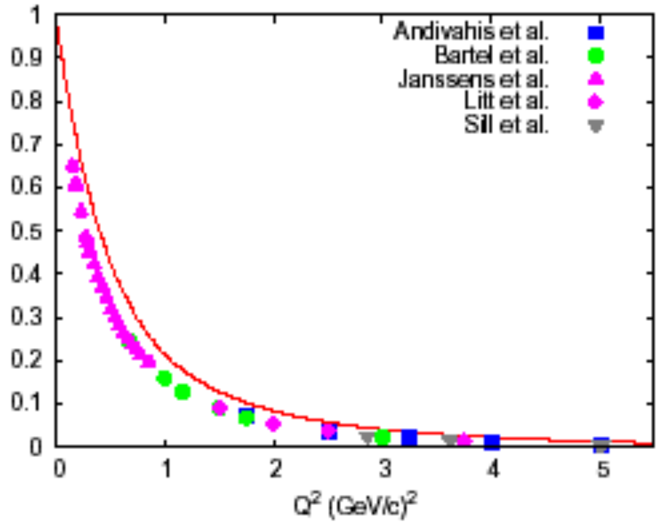
The relativistic effects are expected to be more important for the elastic form factors

(the ground state)

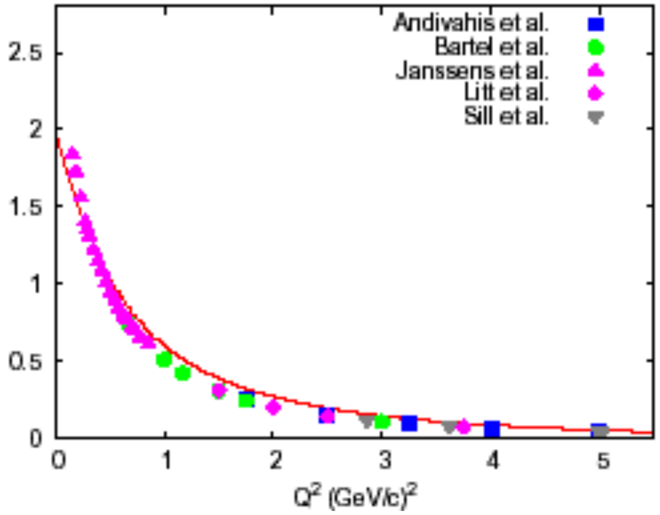
Calculated values
Point Form

- Boosts to initial and final states
- Expansion of current to any order
- Conserved current

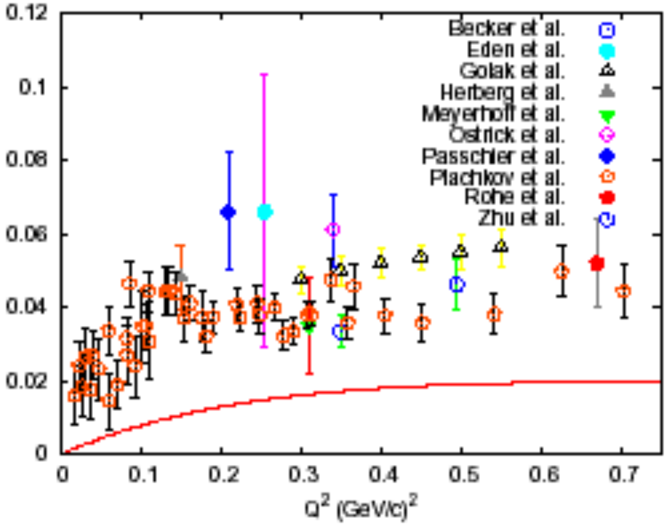
G_E^p



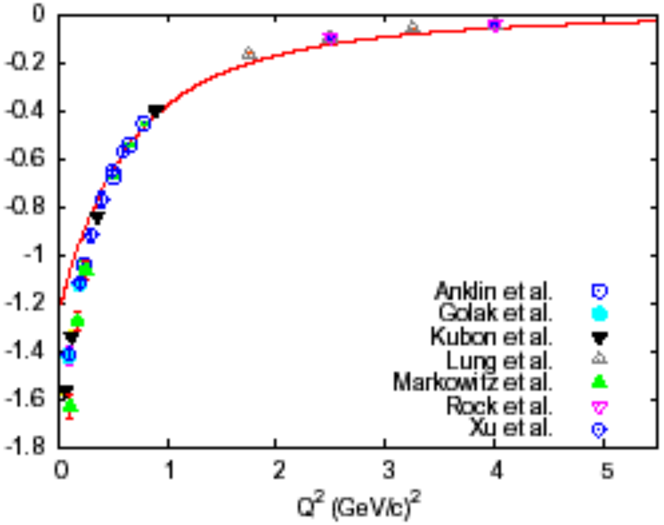
G_M^p

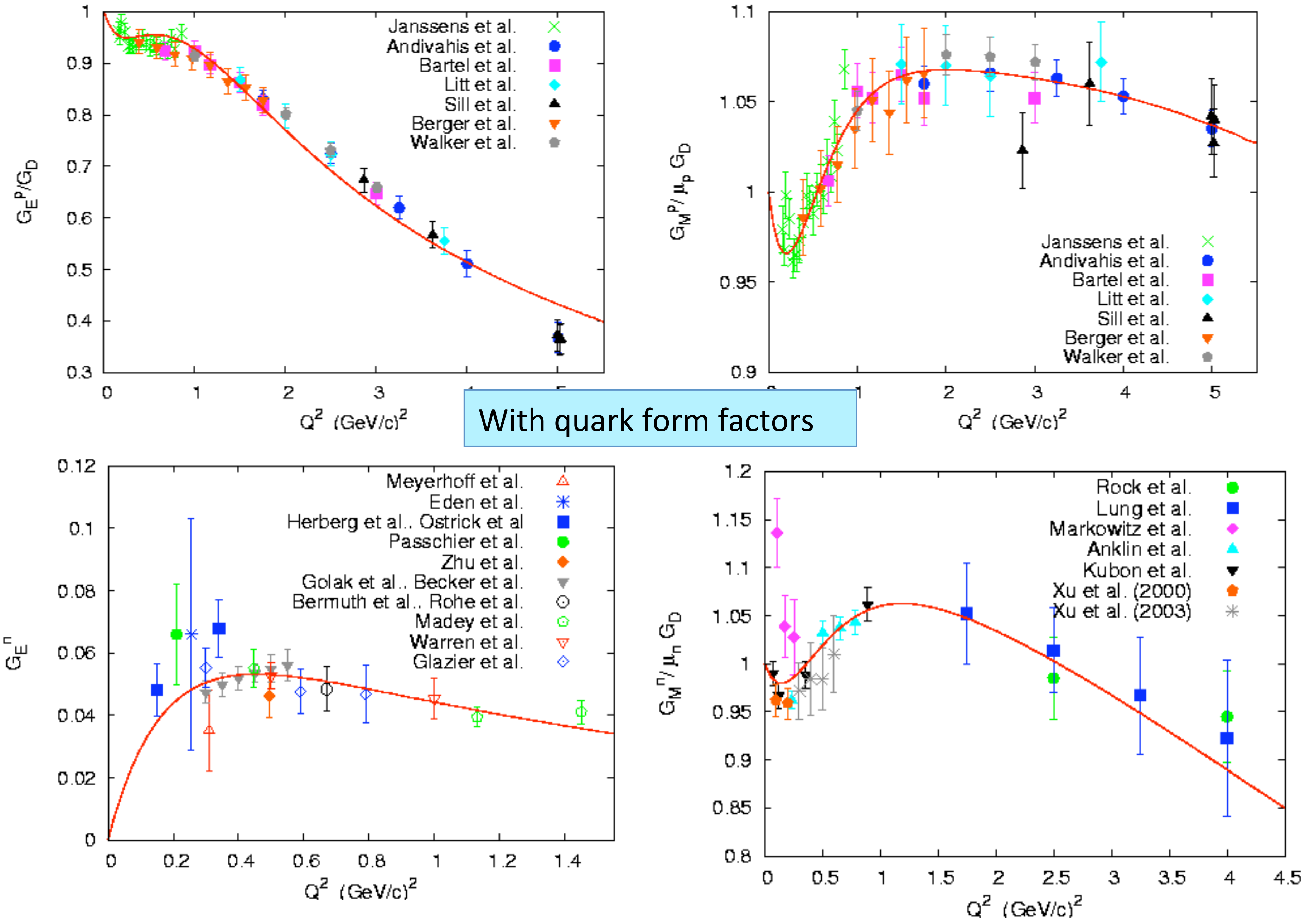


G_E^n



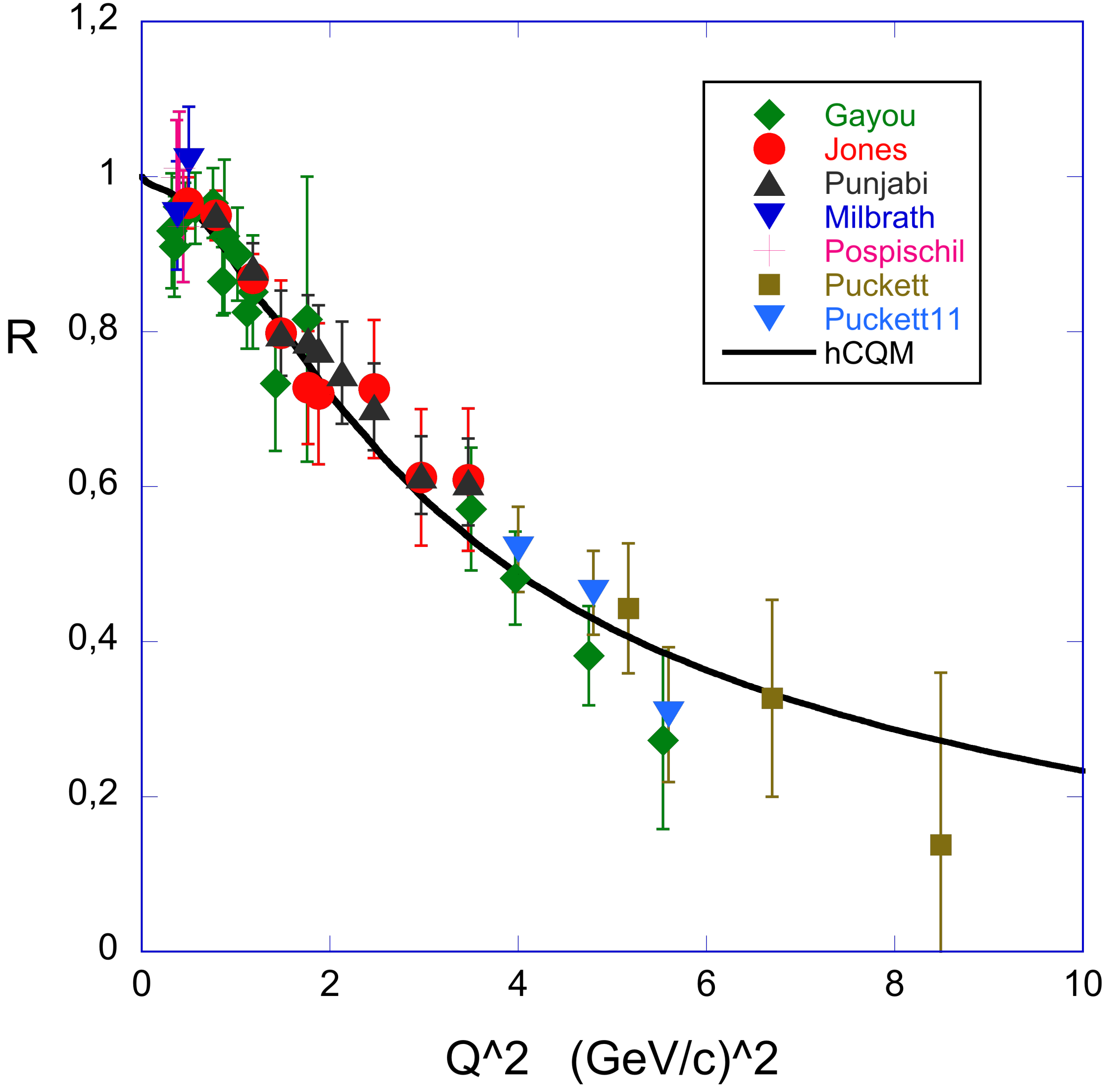
G_M^n





With quark form factors

Genoa group, Phys. Rev. C76, 062201 (2007)

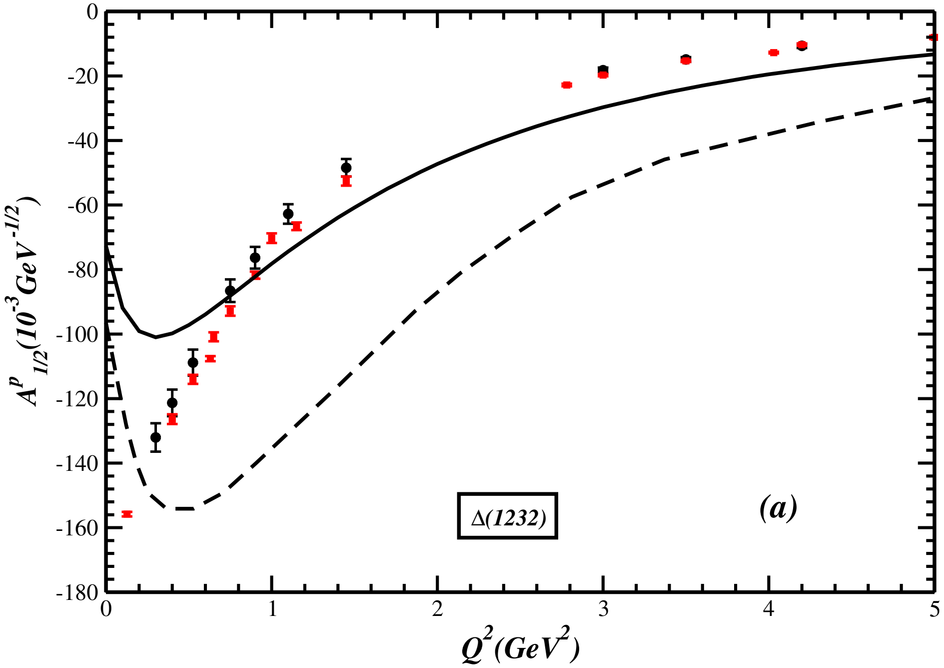

1,2
1
0,8
0,6
0,4
0,2
0
R
0
2
4
6
8
10
Q^2 (GeV/c)^2
Gayou
Jones
Punjabi
Milbrath
Pospischil
Puckett
Puckett11
hCQM

Δ(1232)

Structure similar to the nucleon

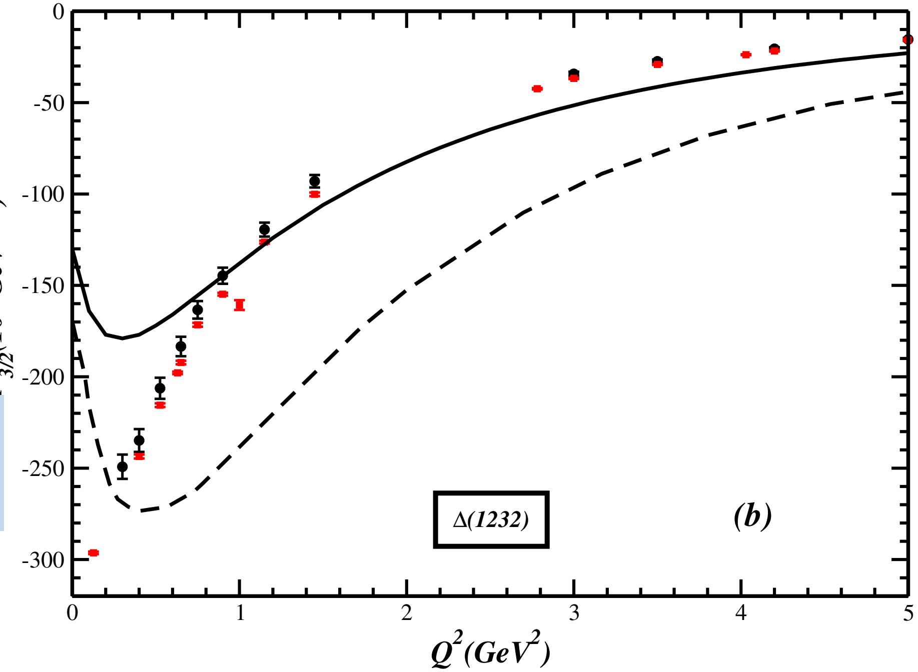
Spin-isospin splitting of the ground state

Relativistic effects important also for the excitation?



Relativistic hCQM Point Form

Dash non relativistic
Full point form



Y. B. Dong, M.G., E. Santopinto, A. Vassallo, Few Body Syst. **55**, 873-876 (2014).

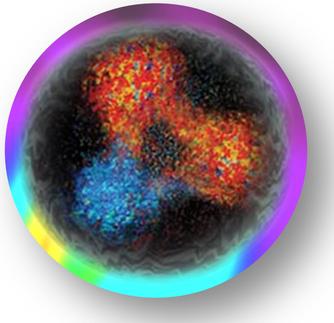
Relativity is an important issue for the description of elastic and inelastic form factors

but it is not the only important issue

the medium Q^2 behaviour is fairly well reproduced (1/x potential)

there is lack of strength at low Q^2 (outer region) in the e.m. transitions

specially for the A 3/2 amplitudes



3-quark core (about 0.5 fm)
+
quark-antiquark pairs
(Meson cloud)

0.5 fm
is the value
predicted
by hCQM

How to introduce it?

Two main approaches

- the physical nucleon N is made of a bare nucleon dressed by a surrounding meson cloud

$$| \tilde{N}\rangle = \Psi^{N}_{(3q)} | N(qqq)\rangle + \sum_{B,M} \Psi^{(BM)}_{(3q)(q\bar{q})} | B(qqq)M(q\bar{q})\rangle + \cdots$$

Problems of inconsistency

- Introducing higher Fock components

$$| \Psi\rangle = \Psi_{3q} | qqq\rangle + \Psi_{3q\,q\bar{q}} | 3q\, q\bar{q}\rangle$$

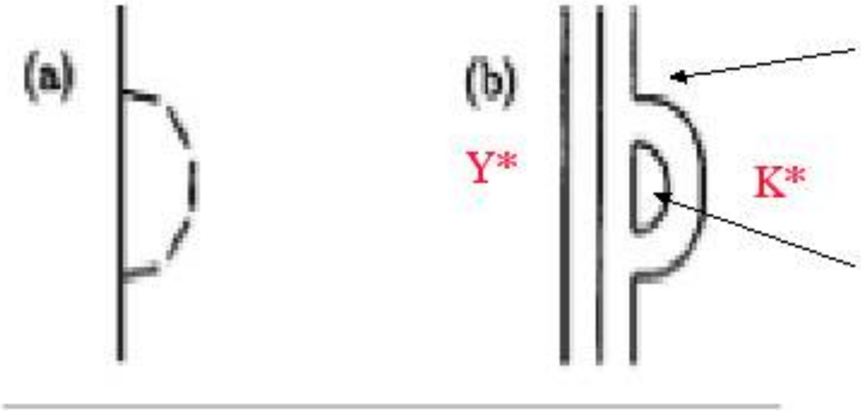
Consistency ok
But: how many components?

Necessity of unquenching the quark model

baryons

Unquenching the quark model

The qq-pair creation mechanism is introduced at the microscopical level
→ string-like qq pair creation mechanism



Pair creation vertex

Σ over all the big towers of intermediate states

Construction of the formalism (group theory)
Problems that have been solved
- sum over all intermediate states
- permutational symmetry for all identical quarks
- determination of the pair creation vertex

R. Bijker, E. Santopinto,
Phys.Rev.C80:065210,2009

High Q^2 behaviour

High Q^2 behaviour

- Helicity ratio

$$Z = \frac{|A_{1/2}|^2 - |A_{3/2}|^2}{|A_{1/2}|^2 + |A_{3/2}|^2}$$

goes to 1 for increasing Q^2

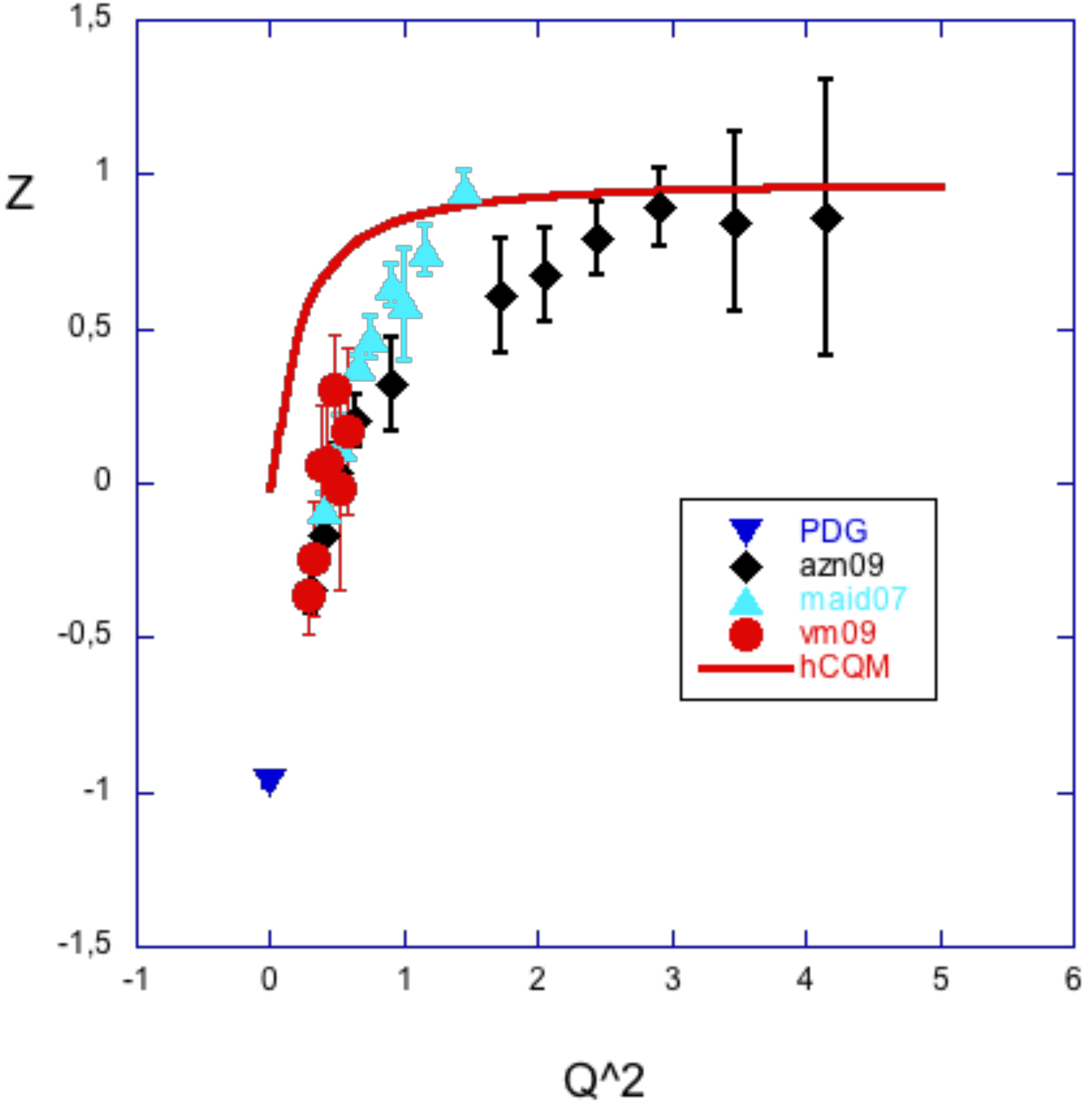
(helicity conservation, Carlson 1986)

Helicity ratio

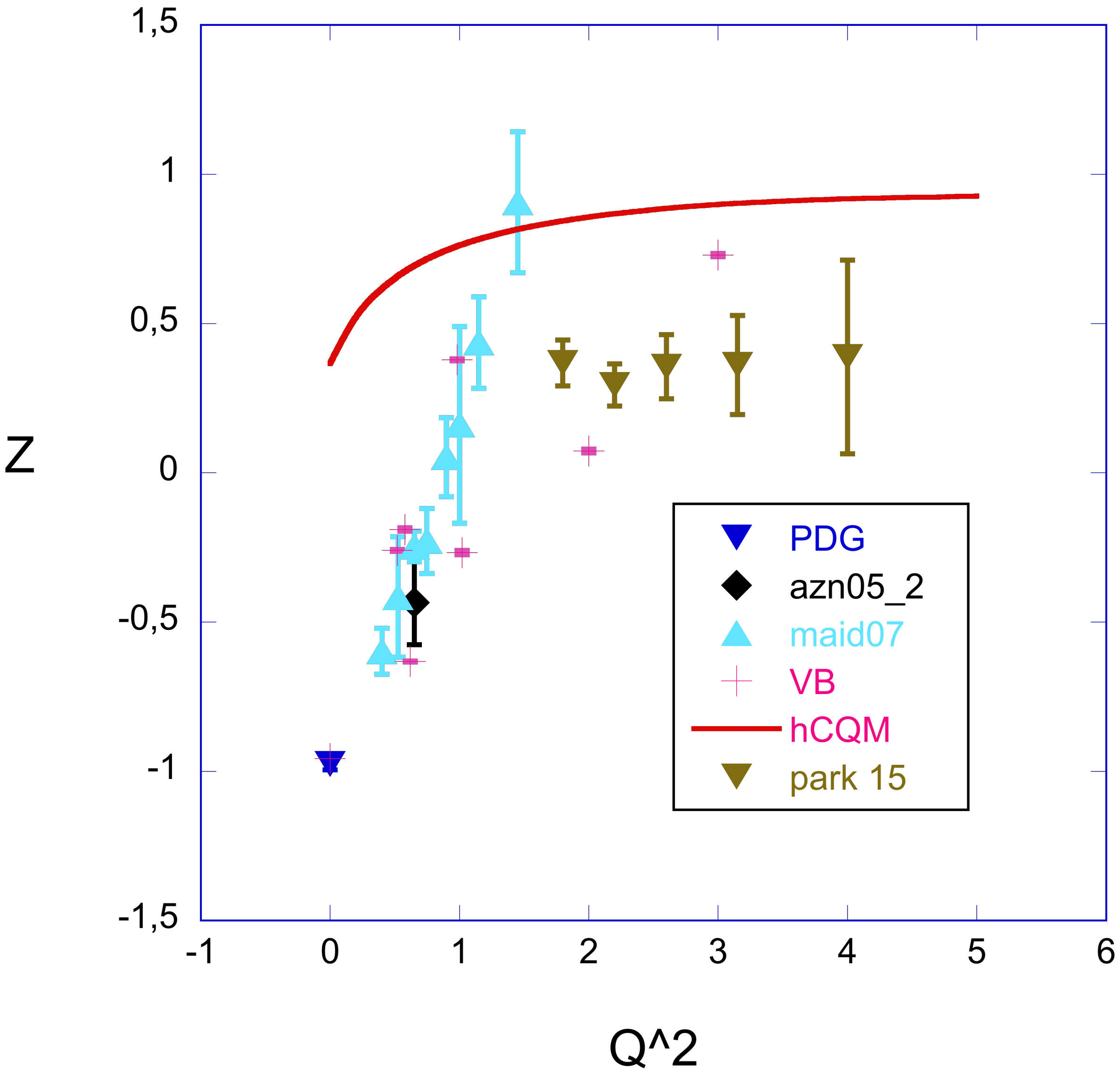
	proton	neutron
P33	≈ -0.5	
D13	ok	ok
F15	ok	≈ 0.7
D13*	ok	0.96
D33	ok	
D15	1/3	≈ 0.32
F35	-0.82	
F37	-0.32	
P13	ok	ok

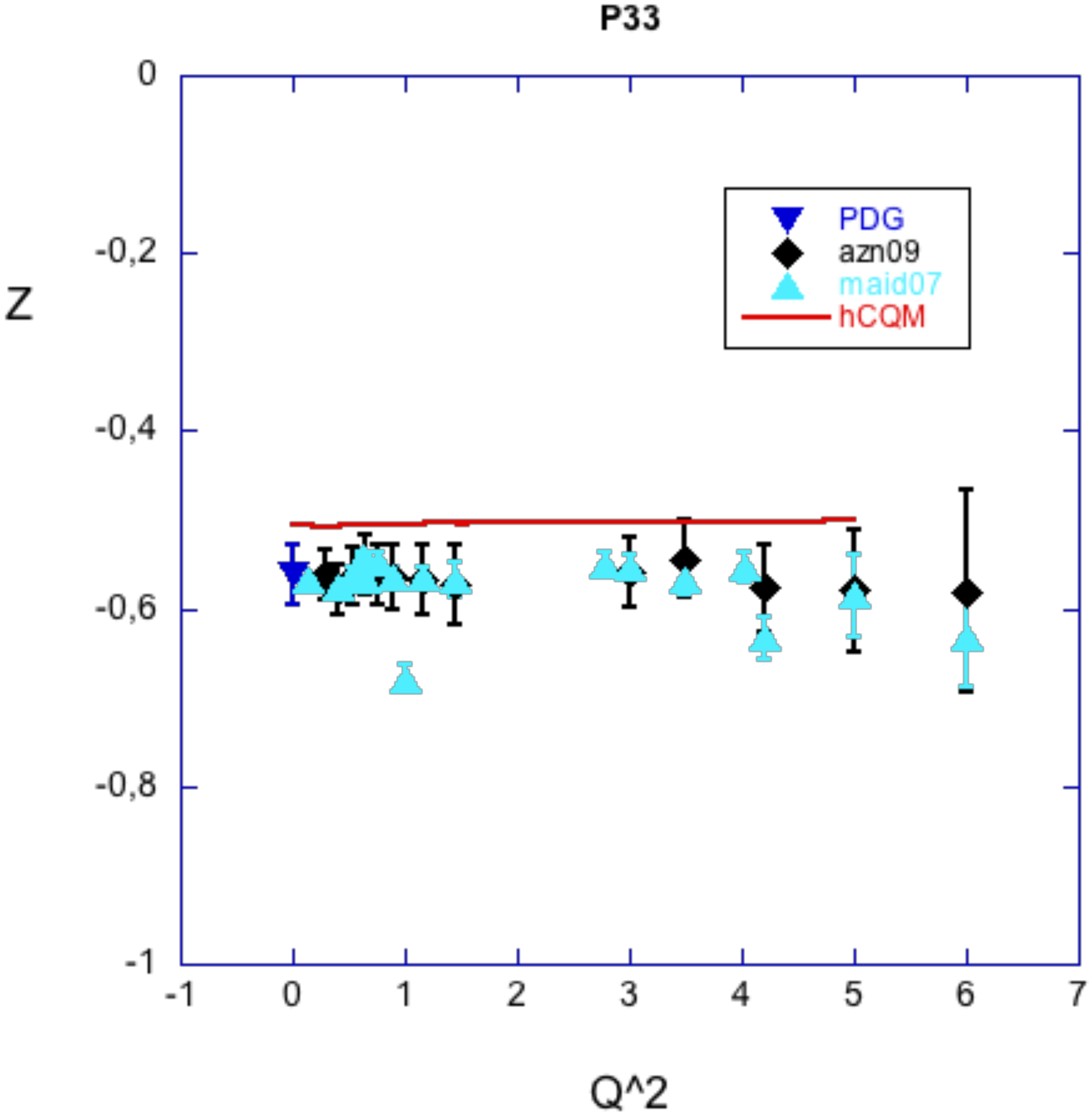
Structure effects ?

D13



F15




P33
Z
Q^2
0
-0,2
-0,4
-0,6
-0,8
-1
-1
0
1
2
3
4
5
6
7
PDG
azn09
maid07
hCQM

Asymptotic behaviour of Δ excitation

$A_{1/2} \approx G_M - 3\, G_E$

$A_{3/2} \approx 3^{1/2}\, (G_M + G_E)$

$Z \rightarrow 1 \quad \text{if} \quad G_E \rightarrow -\, G_M$

Simplified h.o. model for N and Δ states

$|N\rangle = a_s\, |0^+{}_S\rangle + a_m\, |0^+{}_M\rangle$

$|\Delta\rangle = b_s\, |0^+{}_S\rangle + b_d\, |2^+{}_M\rangle$

D-wave

$Z = 1 \quad \text{if} \quad b_d \approx 98\%$!

Not possible in models with three quarks

higher L components?

Conclusions

- hCQM provides a simple and systematic approach to baryon properties
  (spectrum, helicity amplitudes, elastic ff)
- the hCQM structure of levels allows to describe all the new negative parity resonances without invoking
  higher shells
- relativity is important for the elastic ff and the Δ-excitation
- The missing strength at low Q^2 is due to the lack of
  quark-antiquark pair mechanisms
- Such mechanisms may be important also for the high Q^2 behaviour of elastic ff and resonance excitation, but also for the spectrum and the strong decays