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FACULTY OF SCIENCES

Hunting the Resonances in $p(\gamma, K^+)\Lambda$ Reactions: (Over)Complete Measurements and Partial-Wave Analyses

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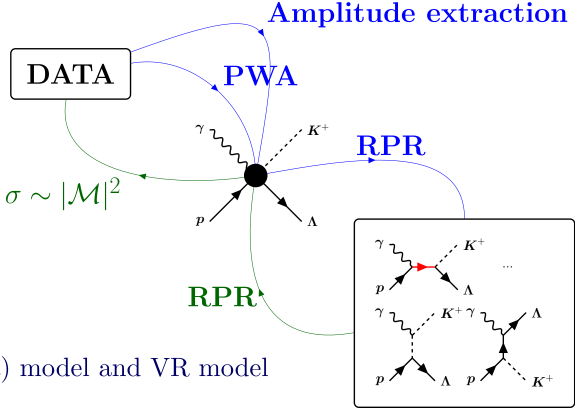
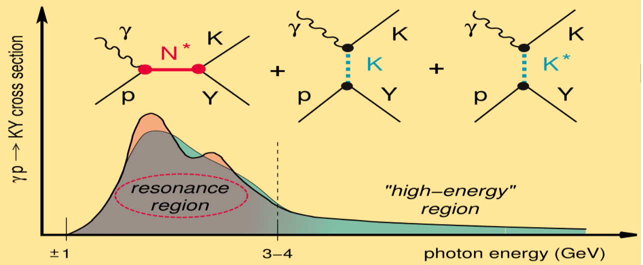
Nucleon Resonances: From Photoproduction to High Photon Virtualities, Trento

OVERVIEW



1. Regge-plus-resonance (RPR) model and VR model
2. Inferring model-independent reaction amplitudes
   - Multipole decomposition (Partial Wave Analysis - PWA)
   - Alternate (complementary) method: amplitude extraction
     - Amplitude extraction using real data
     - From complete to overcomplete sets
     - Amplitude comparison
3. Conclusions



- Regge background: exchange of $K(494)$ and $K^*(892)$ Regge trajectories in t channel
- Enrich Reggeized background with $N^\star$: $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ with $M_{N^\star} \leq 2$ GeV

Bayesian inference of the resonance content of $p(\gamma, K^+)\Lambda$ **[PRL108 (2012) 182002]**

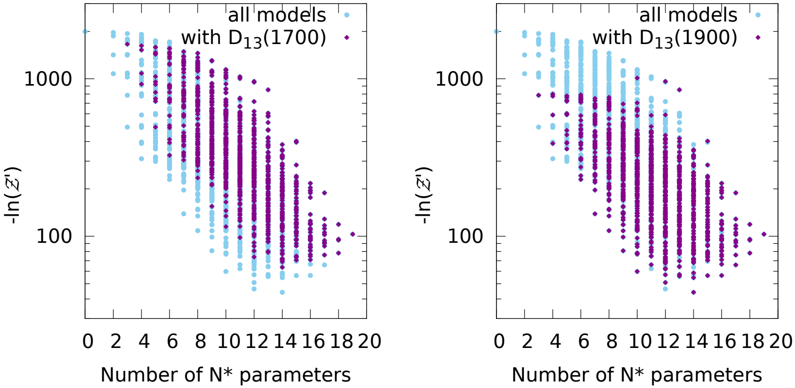
$S_{11}(1535)$, $S_{11}(1650)$, $F_{15}(1680)$, $P_{13}(1720)$, $D_{13}(1875)$, $P_{13}(1900)$, $P_{11}(1900)$, and $F_{15}(2000)$

- **17 parameters**

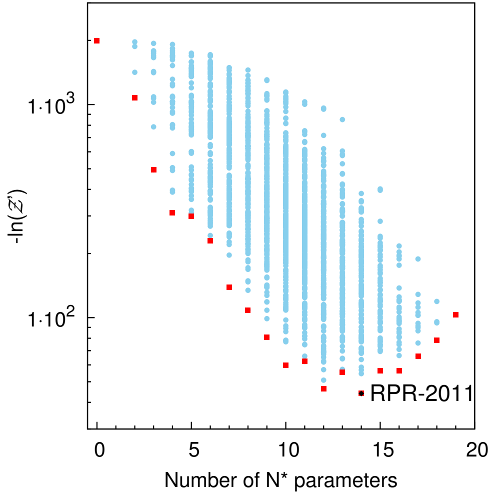
Bayesian analysis: beyond point estimates!

Marginalize over all possible models with resonance R:

$$P(R|D) \to \sum_{M_i|R\in M_i} P(M_i|D) = \sum_{M_i|R\in M_i} \underbrace{P(D|M_i)}_{\mathcal{Z}_i} \frac{P(M_i)}{P(D)}$$



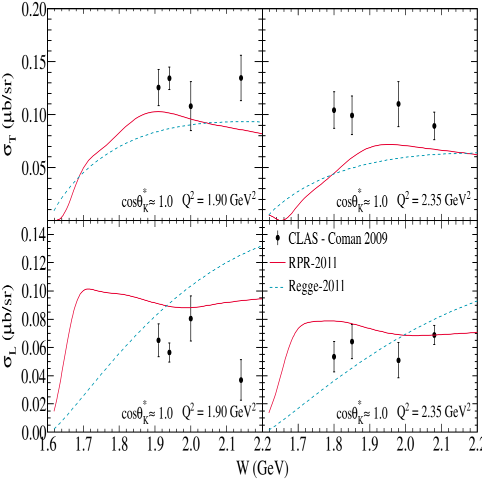
Bayesian evidence map for the 2^{11} model variants



PRL108(2012)182002

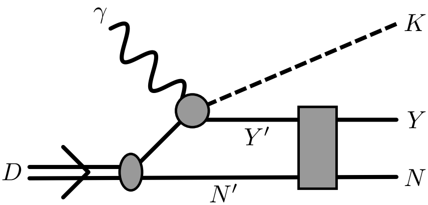
RPR-2011 (PDG-2010)

- $S_{11}(1535)$ ★★★★
- $S_{11}(1650)$ ★★★★
- $D_{15}(1675)$ ★★★★
- $F_{15}(1680)$ ★★★★
- $D_{13}(1700)$ ★★★
- $P_{11}(1710)$ ★★★
- $P_{13}(1720)$ ★★★★
- $D_{13}(1875)$ m
- $P_{13}(1900)$ ★★
- $P_{11}(1900)$ m
- $F_{15}(2000)$ ★★★



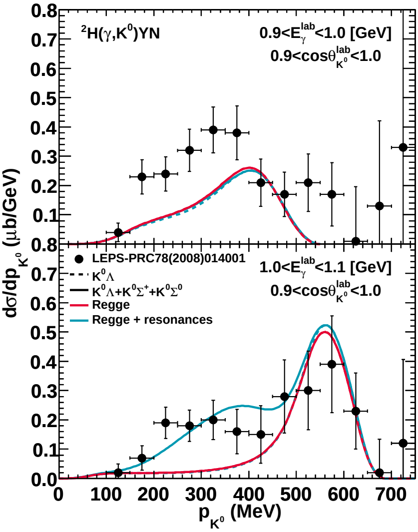
Data: **[PRC81 (2010) 052201]**

1. Test of predictive power: fix against $p(\gamma, K^+)\Lambda$ data, **test** against $p(e, e'K^+)\Lambda$ data (no refitting)
2. EM transition form factors:
   - ***t*-channel**: dipole (also electric *s*-channel Born term),
   - ***s*-channel**: Inferred from Bonn CQM helicity amplitudes using consistent Lagrangians.



- Elementary production operator: RPR model
- Parameters for neutron inferred from the ones of the proton using isospin symmetry
- relativistic *Dnp*-vertex + FSI

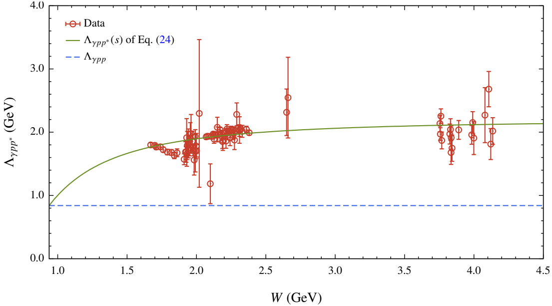
[PLB681 (2009) 428]

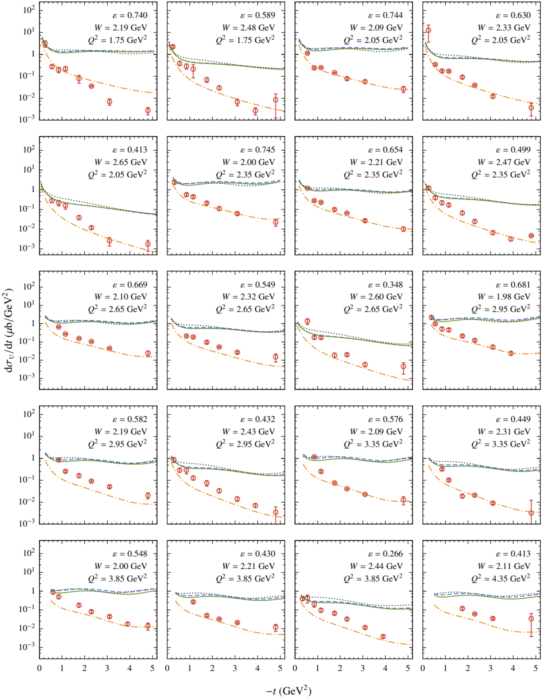


Data: **[PRC78, 014001]**

ABOVE the resonance region: **VR model** ($N(e, e'\pi^{\pm})N'$) **[PRC89 (2014) 025203 (π)]**

- Motivation: models with Reggeized background underestimate σ_T
- Main component: **gauged pion-exchange current** (missing transverse strength provided by residual effects of *nucleon resonances*)
- EM transition FF implements *resonance-parton contributions*
- Running cutoff energy $\Lambda_{\gamma pp^*}(\mathbf{s})$ for the proton EM transition FF
  - **correct on-shell limits**
  - lowers number of free parameters (compare: **[PRC81(2010) 045202]**)
  - simple interpretation: p charge radius asymptotically $\downarrow$ for p virtuality $s \uparrow$





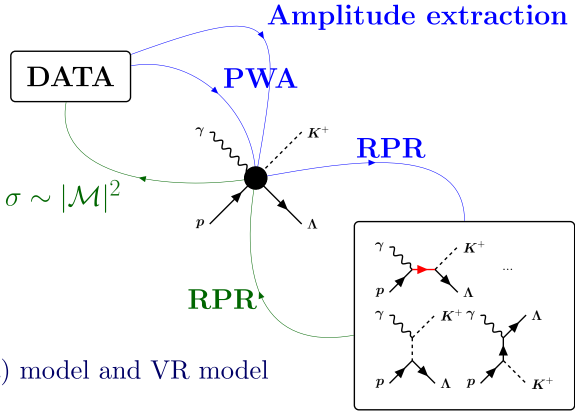
For high $-t > 0.5$ GeV^2 data **[EPJ A49, 16 (2013)]**

- KM and previous VR do not show correct t-dependence
- Additional u-channel trajectories and/or t-dependence for $\Lambda_{\gamma\pi\pi}$ do not considerably improve high $-t$ fit.
- Introduce FF in strong vertex of t-channel Regge amplitudes (monopole in $-t$)

$p(e, e'K^+)\Lambda$

- Use ingredients from π^+ production to ***predict*** **observables of K^+ electroproduction**.

[PRC89 (2014) 065202 (K)]



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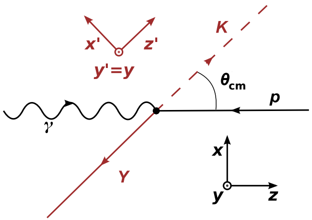
Case study of $p(\gamma, K^+)\Lambda$



Photon: γ	1^-	/
Proton: p	$\frac{1}{2}^+$	uud
Kaon: K^+	0^-	u$\bar{\text{s}}$
Lambda: Λ	$\frac{1}{2}^+$	uds

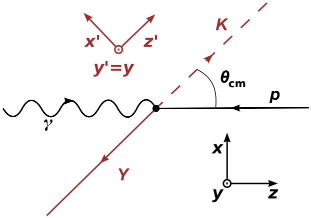
Two independent kinematic variables

- Invariant mass W
- Kaon angle $\theta_{\text{c.m.}}$

Dynamics

- 2 spin-1/2 particles and a real photon $\rightarrow$ 8 combinations
- Symmetries of the reaction
- **4 independent COMPLEX REACTION AMPLITUDES**

$$\mathcal{M}^{\lambda_\gamma}_{\lambda_p,\lambda_\Lambda} \rightarrow \mathcal{M}_{i=1,2,3,4}$$



Transversity Amplitudes (TA) $b_{i=1,\ldots,4}$

$$b_1 \equiv {}_y\langle +|\, J_y \,|+\rangle_y$$
$$b_2 \equiv {}_y\langle -|\, J_y \,|-\rangle_y$$
$$b_3 \equiv {}_y\langle +|\, J_x \,|-\rangle_y$$
$$b_4 \equiv {}_y\langle -|\, J_x \,|+\rangle_y$$

Normalized TA $a_{i=1,\ldots,4}$

$$a_i = \frac{b_i}{\sqrt{|b_1|^2+|b_2|^2+|b_3|^2+|b_4|^2}} = r_i e^{i\alpha_i}$$

CGLN amplitudes and multipole decomposition

$$\mathcal{M} = \langle m_{s_\Lambda}| - iF_1 \boldsymbol{\sigma}.\mathbf{e}_{\mathbf{P}\gamma} - F_2 (\boldsymbol{\sigma}.\mathbf{e}_{\mathbf{p}}) [\boldsymbol{\sigma}.(\mathbf{e}_{\mathbf{k}} \times \mathbf{e}_{\mathbf{P}\gamma})] - iF_3 (\boldsymbol{\sigma}.\mathbf{e}_{\mathbf{k}}) (\mathbf{e}_{\mathbf{p}}.\mathbf{e}_{\mathbf{P}\gamma}) - iF_4 (\boldsymbol{\sigma}.\mathbf{e}_{\mathbf{p}}) (\mathbf{e}_{\mathbf{p}}.\mathbf{e}_{\mathbf{P}\gamma}) |m_{s_p}\rangle$$

$$F_1 = \sum_l P'_{l+1}(\cos\theta_{\text{c.m.}}) [E_{l+} + lM_{l+}] + P'_{l-1}(\cos\theta_{\text{c.m.}}) [E_{l-} + (l+1)M_{l-}]$$
$$F_2 = \sum_l P'_l(\cos\theta_{\text{c.m.}}) [(l+1)M_{l+} + lM_{l-}]$$
$$F_3 = \sum_l P''_{l+1}(\cos\theta_{\text{c.m.}}) [E_{l+} - M_{l+}] + P''_{l-1}(\cos\theta_{\text{c.m.}}) [E_{l-} + M_{l-}]$$
$$F_4 = \sum_l P''_l(\cos\theta_{\text{c.m.}}) [-E_{l-} - M_{l-} - E_{l+} + M_{l+}]$$

Multipoles for $p(\gamma, K^+)\Lambda$ (RPR-2011): BACKGROUND DOMINANCE

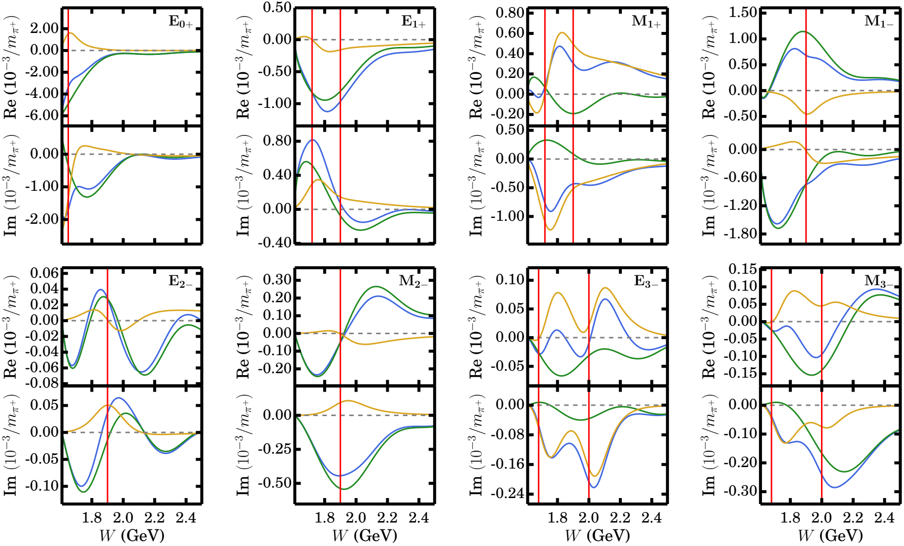


Figure : RPR-2011, RPR-2011 (only background) and RPR-2011 (only $N^\star$).

Polarization observables in pseudoscalar-meson photoproduction

	$(\mathcal{B}_1, \mathcal{T}_1, \mathcal{R}_1)$	$(\mathcal{B}_2, \mathcal{T}_2, \mathcal{R}_2)$	Transversity expression
Σ	$(y, 0, 0)$	$(x, 0, 0)$	$r_1^2 + r_2^2 - r_3^2 - r_4^2$
T	$(0, +y, 0)$	$(0, -y, 0)$	$r_1^2 - r_2^2 - r_3^2 + r_4^2$
P	$(0, 0, +y)$	$(0, 0, -y)$	$r_1^2 - r_2^2 + r_3^2 - r_4^2$
C_x	$(+, 0, +x)$	$(+, 0, -x)$	$-2\,\mathrm{Im}(a_1 a_4^* + a_2 a_3^*)$
C_z	$(+, 0, +z)$	$(+, 0, -z)$	$+2\,\mathrm{Re}(a_1 a_4^* - a_2 a_3^*)$
O_x	$(+\frac{\pi}{4}, 0, +x)$	$(+\frac{\pi}{4}, 0, -x)$	$+2\,\mathrm{Re}(a_1 a_4^* + a_2 a_3^*)$
O_z	$(+\frac{\pi}{4}, 0, +z)$	$(+\frac{\pi}{4}, 0, -z)$	$+2\,\mathrm{Im}(a_1 a_4^* - a_2 a_3^*)$
E	$(+, -z, 0)$	$(+, +z, 0)$	$+2\,\mathrm{Re}(a_1 a_3^* - a_2 a_4^*)$
F	$(+, +x, 0)$	$(+, -x, 0)$	$-2\,\mathrm{Im}(a_1 a_3^* + a_2 a_4^*)$
G	$(+\frac{\pi}{4}, +z, 0)$	$(+\frac{\pi}{4}, -z, 0)$	$-2\,\mathrm{Im}(a_1 a_3^* - a_2 a_4^*)$
H	$(+\frac{\pi}{4}, +x, 0)$	$(+\frac{\pi}{4}, -x, 0)$	$+2\,\mathrm{Re}(a_1 a_3^* + a_2 a_4^*)$
T_x	$(0, +x, +x)$	$(0, +x, -x)$	$+2\,\mathrm{Re}(a_1 a_2^* + a_3 a_4^*)$
T_z	$(0, +x, +z)$	$(0, +x, -z)$	$+2\,\mathrm{Im}(a_1 a_2^* + a_3 a_4^*)$
L_x	$(0, +z, +x)$	$(0, +z, -x)$	$-2\,\mathrm{Im}(a_1 a_2^* - a_3 a_4^*)$
L_z	$(0, +z, +z)$	$(0, +z, -z)$	$+2\,\mathrm{Re}(a_1 a_2^* - a_3 a_4^*)$

- $\frac{d\sigma}{d\Omega}^{(\mathcal{B},\mathcal{T},\mathcal{R})}$: cross section for given beam ($\mathcal{B}$), target ($\mathcal{T}$), recoil ($\mathcal{R}$) polarization
- Asymmetries $\mathcal{A} = \frac{\frac{d\sigma}{d\Omega}^{(\mathcal{B}_1,\mathcal{T}_1,\mathcal{R}_1)} - \frac{d\sigma}{d\Omega}^{(\mathcal{B}_2,\mathcal{T}_2,\mathcal{R}_2)}}{\frac{d\sigma}{d\Omega}^{(\mathcal{B}_1,\mathcal{T}_1,\mathcal{R}_1)} + \frac{d\sigma}{d\Omega}^{(\mathcal{B}_2,\mathcal{T}_2,\mathcal{R}_2)}}$
- $\frac{d\sigma}{d\Omega}^{(0,0,0)} = \frac{\rho}{4} \sum_{i=1}^{4} |b_i|^2$

SINGLE asymmetries: MODULI

DOUBLE asymmetries: PHASES

4 complex amplitudes, or 8 real variables

- There is one arbitrary global phase

$$\delta_i^{\alpha_4} = \alpha_i - \alpha_4 \,.$$

- Take $\alpha_4 = 0$ and use normalized transversity amplitudes

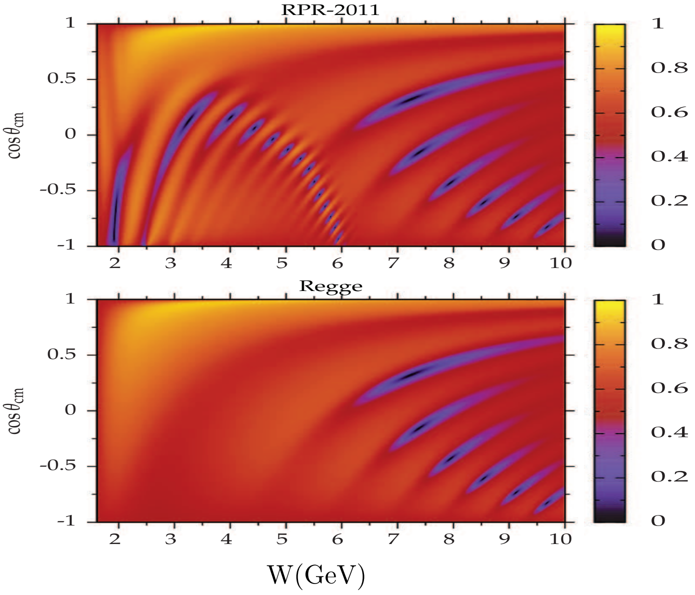
$$1 = |a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2$$

We need 6 real variables and an independent scaling factor

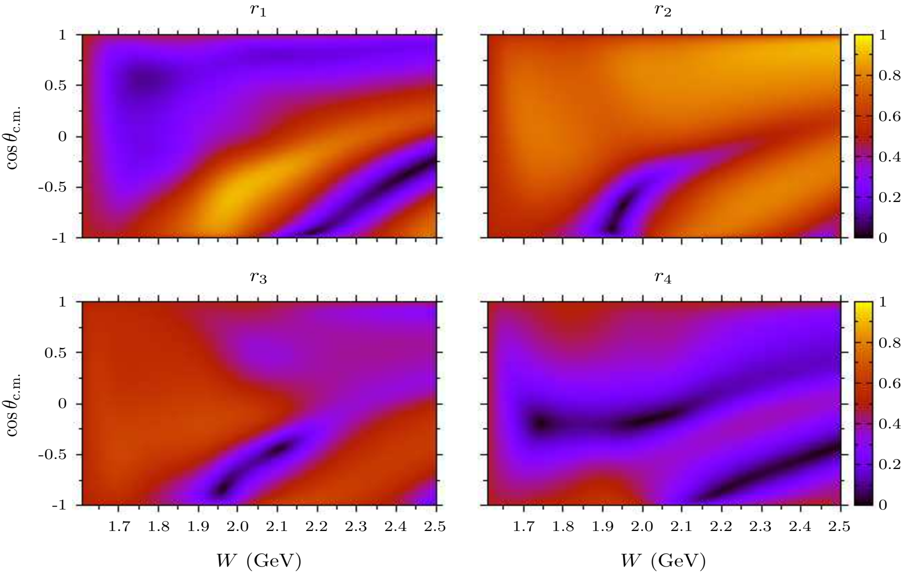
Definition **COMPLETE SET**

A complete set is a minimum set of observables from which one can determine the underlying reaction amplitudes unambiguously.
[Chiang & Tabakin PRC55 (1997) 2054]: 8 observables

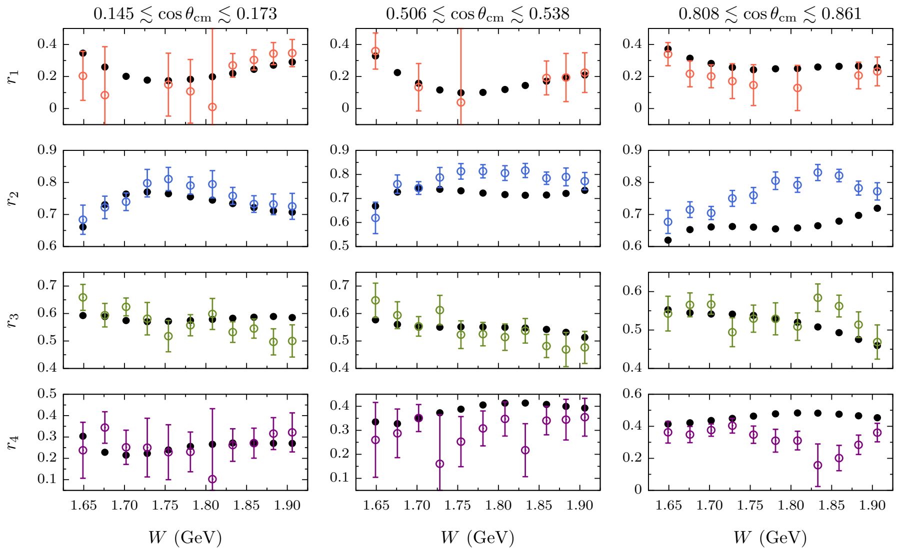
Role of resonances for the NTA moduli (r_2)



RPR-2011 predictions for $(W, \cos\theta_{c.m.})$ dependence of NTA moduli for $p(\gamma, K^+)\Lambda$

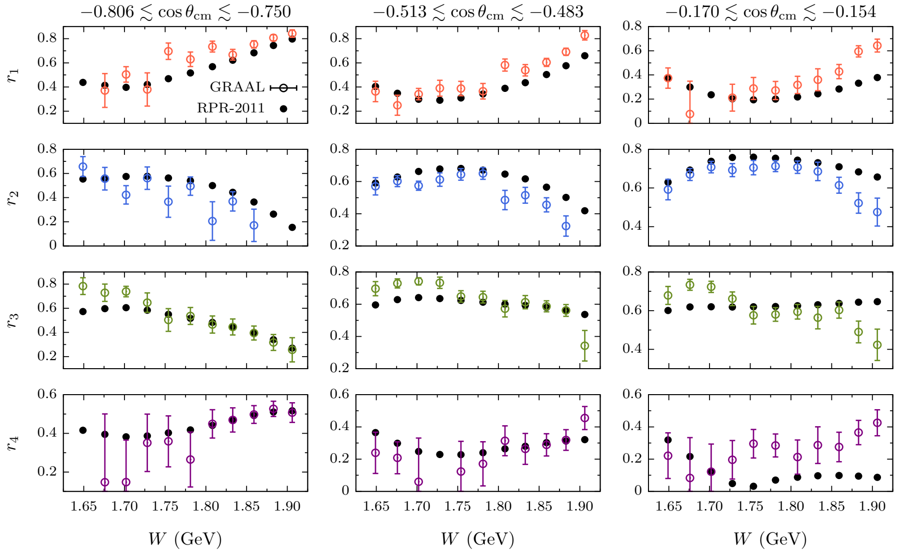


Extracted NTA moduli for $p(\gamma, K^+)\Lambda$: FORWARD [PRC87 (2013) 055205]



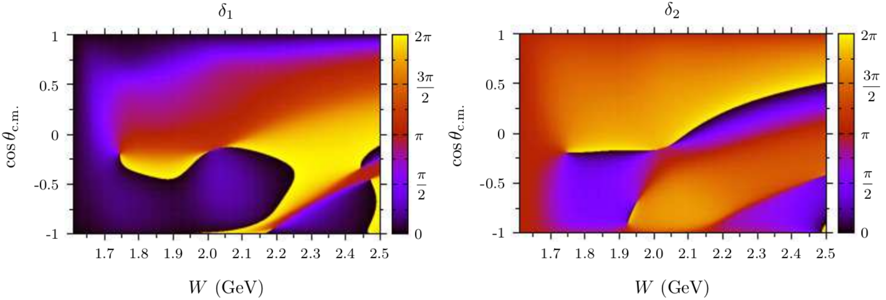
$r_2 : b_2 = {}_y \langle - | J_y | - \rangle_y$

Extracted NTA moduli for $p(\gamma, K^+)\Lambda$: BACKWARD [PRC87 (2013) 055205]



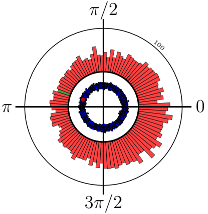
$r_2 : b_2 = {}_y \langle - | \, J_y \, | - \rangle_y$

RPR-2011 predictions for $(W, \cos\theta_{\mathrm{c.m.}})$ dependence of NTA relative phases $\delta_i = \alpha_i - \alpha_4$ for $p(\gamma, K^+)\Lambda$

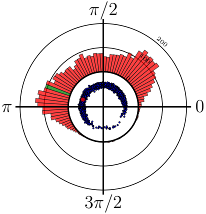


- at forward angles the background dominates and the W-dependence of δ_i is mild
- at backward angles large $N^\star$ contributions and the W-dependence of δ_i is wild

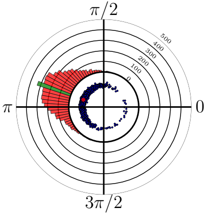
Full amplitude extraction $(r_i, \delta_i^{\alpha_4})$ at single (s,t) [JPG42 (2015) 034016]



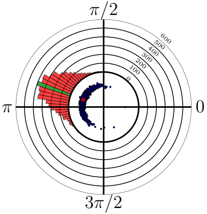
(a) "Complete"



(b) Discrete amb.



(c) Unimodal

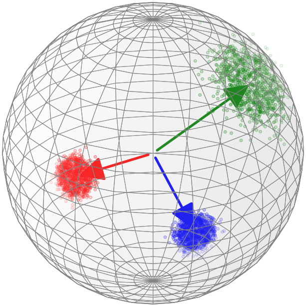


(d) Improved

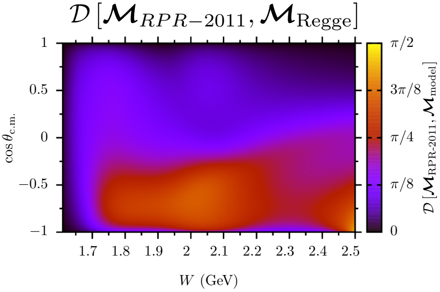
$$\boldsymbol{\mathcal{M}}_a(s,t) \equiv \begin{pmatrix} a_1(s,t) \\ a_2(s,t) \\ a_3(s,t) \\ a_4(s,t) \end{pmatrix}$$
$$\boldsymbol{\mathcal{M}}_a^\dagger \boldsymbol{\mathcal{M}}_a = 1$$

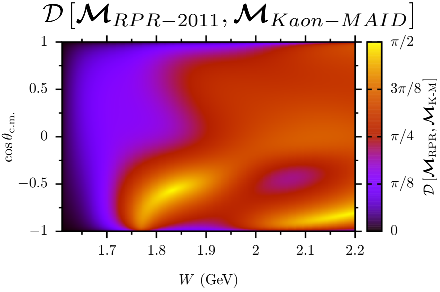
Q: What is the distance between $\boldsymbol{\mathcal{M}}_1$ and $\boldsymbol{\mathcal{M}}_2$?
A: $\mathcal{D}\left[\boldsymbol{\mathcal{M}}_1, \boldsymbol{\mathcal{M}}_2\right] = \arccos \mathrm{Re}\, \boldsymbol{\mathcal{M}}_1^\dagger \boldsymbol{\mathcal{M}}_2$

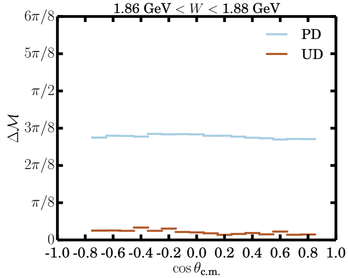
Both $\boldsymbol{\mathcal{M}}_{i=1,2}$ have an unknown α_4.
Q: How to calculate $\mathcal{D}\left[\boldsymbol{\mathcal{M}}_1, \boldsymbol{\mathcal{M}}_2\right]$ independent of choice α_4?
A: $\alpha_4 = \underset{\alpha_4}{\mathrm{argmin}} \left(\mathcal{D}\left[\boldsymbol{\mathcal{M}}_1(\alpha_4), \boldsymbol{\mathcal{M}}_2(\alpha_4' = 0)\right]\right)$



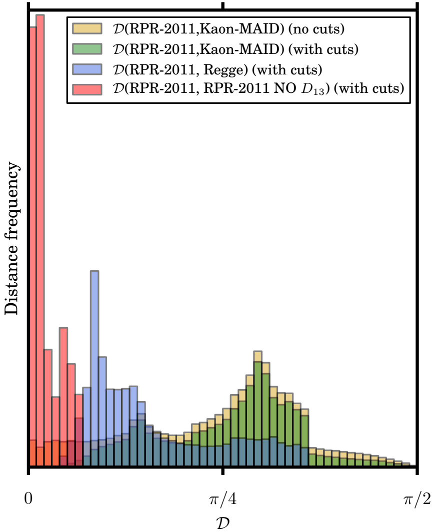
Model comparison in amplitude space





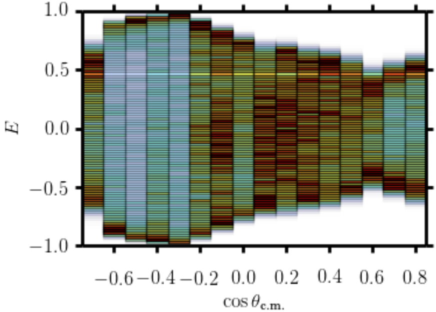


Resolution of the data



To obtain "sensible" results:

- Are the models **falsifiable**?
- Project information in *amplitude* space onto *observable* space (observables are not independent)
- Clear effect of measurements of individual observables by comparing posterior to prior distributions



SUMMARY

Available models

- RPR model for $Q^2 = 0$ over large W range.
- VR for high virtuality $Q^2 \neq 0$ above the resonance region.

Model-independent amplitude inference

- Obtaining resonance information in background-dominated reactions requires background-subtraction schemes, such as RPR-2011.
- Hierarchy in the quality/quantity of the data!
- Quadratic equations connect $\{\Sigma, P, T\}$ to the moduli $\{r_1, r_2, r_3, r_4\}$ of the normalized transversity amplitudes
  1. Analysis of $\gamma p \rightarrow K^+\Lambda$ with $\{\Sigma, T, P\}$ from GRAAL ($1.65 \lesssim W \lesssim 1.91$ GeV) allowed to extract $\{r_1, r_2, r_3, r_4\}$ in $\approx 95\%$ of considered $(W, \cos\theta_{c.m.})$
  2. RPR-2011 is in reasonable agreement with the extracted r_i
- Extracting the NTA independent phases $\{\delta_1, \delta_2, \delta_3\}$ is far more challenging (connected to asymmetries by means of non-linear equations)
- **Mathematical Completeness does not imply Practical Completeness!!**
- Overcomplete sets provide a solution!

- J. Nys, T. Vrancx and J. Ryckebusch
  Amplitude extraction in pseudoscalar-meson photoproduction: towards a situation of complete information
  J. Phys. G **42** (2015) 3, 034016
- D. G. Ireland
  Information Content of Polarization Measurements
  Phys. Rev. C **82** (2010) 025204
- T. Vrancx, J. Ryckebusch, T. Van Cuyck T, P. Vancraeyveld
  Incompleteness of complete pseudoscalar-meson photoproduction
  Phys. Rev. C **87** (2013) 055205.
- L. De Cruz, J. Ryckebusch, T. Vrancx, P. Vancraeyveld
  A Bayesian analysis of kaon photoproduction with the Regge-plus-resonance model
  Phys. Rev. C **86** (2012) 015212
- L. De Cruz, T. Vrancx, P. Vancraeyveld, J. Ryckebusch
  Bayesian inference of the resonance content of $p(\gamma, K^+)\Lambda$
  Phys. Rev. Lett. **108** (2012) 182002

Backup slides

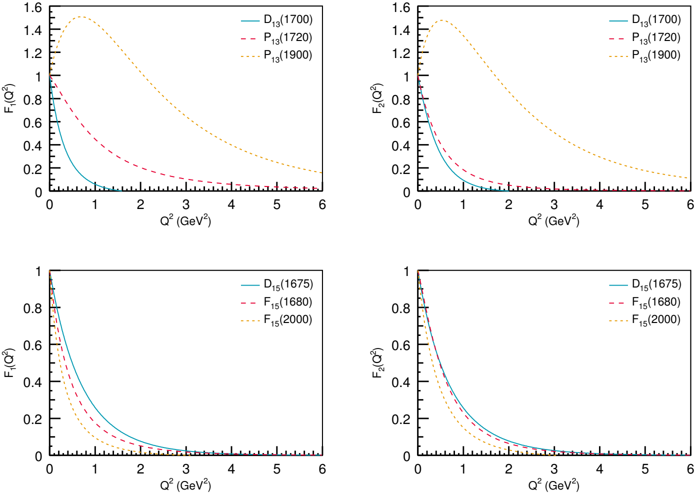


Figure 5.21 – The EM transition form factors $F_1(Q^2)$ (left panels) and $F_2(Q^2)$ (right panels) derived from helicity amplitudes calculated by the Bonn CQM [176]. The top panels show the results for the spin-3/2 resonances $D_{13}(1700)$, $P_{13}(1720)$ and $P_{13}(1900)$, while the lower panels display the results for the spin-5/2 resonances $D_{15}(1675)$, $F_{15}(1680)$ and $F_{15}(2000)$.

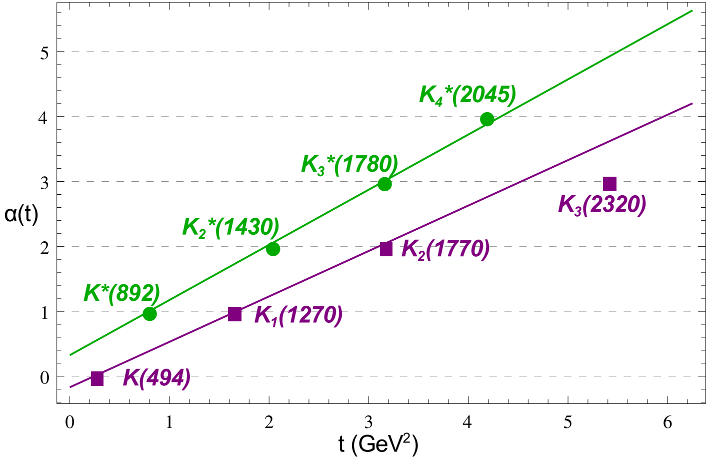
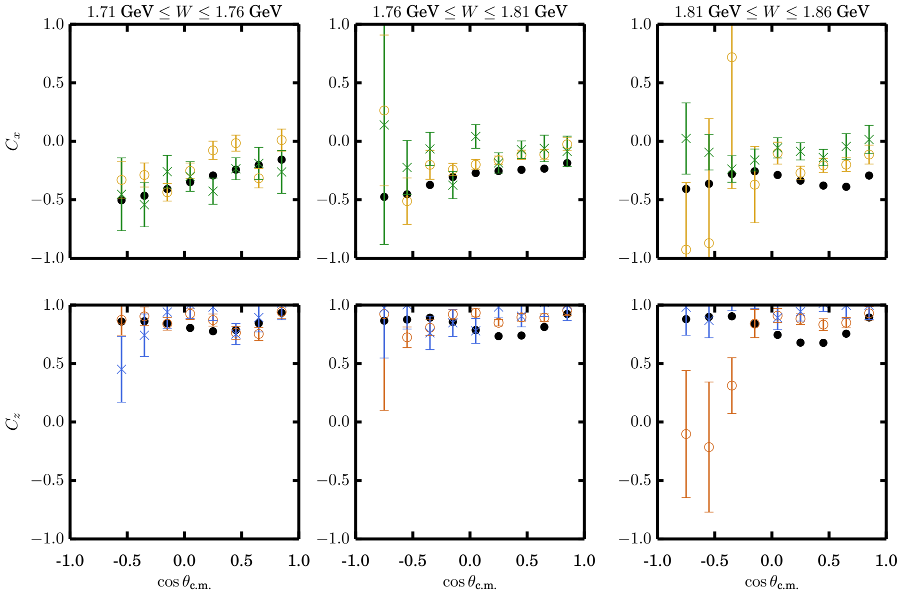
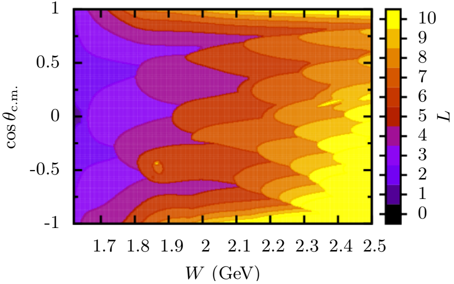


Figure 2.6 – Chew-Frautschi plots for the two lightest kaon trajectories. Meson masses as listed by the PDG were used [25]. Note that each line represents two trajectories, corresponding with the odd- and even-parity states.

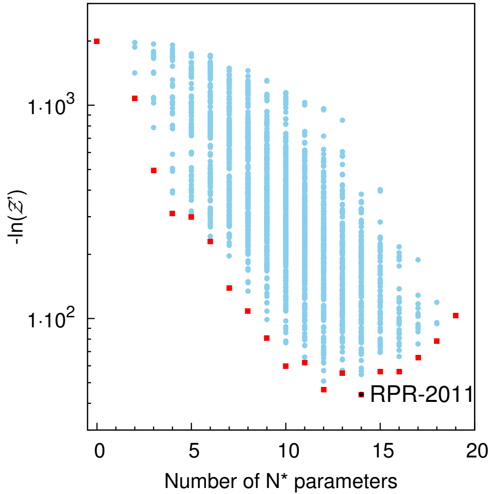
Reconstructed observables: CLAS check



$L_{\max}$ for 5% accuracy in RPR-2011



Bayesian evidence map for the 2^{11} model variants



PRL108(2012)182002

RPR-2011 (PDG-2010)

- $S_{11}(1535)$ ★★★★
- $S_{11}(1650)$ ★★★★
- $D_{15}(1675)$ ★★★★
- $F_{15}(1680)$ ★★★★
- $D_{13}(1700)$ ★★★
- $P_{11}(1710)$ ★★★
- $P_{13}(1720)$ ★★★★
- $D_{13}(1875)$ m
- $P_{13}(1900)$ ★★
- $P_{11}(1900)$ m
- $F_{15}(2000)$ ★★★

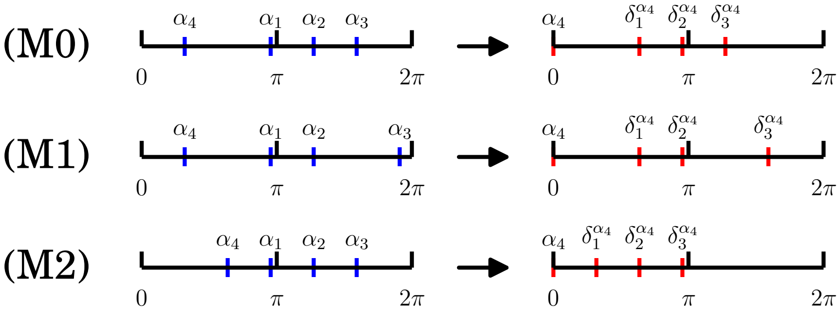


Figure : Example of a situation where a global phase transformation, followed by $\alpha_4 = 0$ can give a distorted picture of the degree of compatibility of two models.

Observable	# data	Experiment	Year	Reference
$d\sigma_0$	56	SLAC	1969	Boyarski *et al.* [38]
	720	SAPHIR	2004	Glander *et al.* [39]
	1377	CLAS	2006	Bradford *et al.* [40]
	12	LEPS	2007	Hicks *et al.* [41]
	2066	CLAS	2010	McCracken *et al.* [32]
Σ	9	SLAC	1979	Quinn *et al.* [42]
	45	LEPS	2003	Zegers *et al.* [43]
	54	LEPS	2006	Sumihama *et al.* [44]
	4	LEPS	2007	Hicks *et al.* [41]
	66	GRAAL	2007	Lleres *et al.* [30]
T	3	BONN	1978	Althoff *et al.* [45]
	66	GRAAL	2009	Lleres *et al.* [31]
P	7	DESY	1972	Vogel *et al.* [46]
	233	CLAS	2004	McNabb *et al.* [33]
	66	GRAAL	2007	Lleres *et al.* [30]
	1707	CLAS	2010	McCracken *et al.* [32]
$C_x,\ C_z$	320	CLAS	2007	Bradford *et al.* [34]
$O_x,\ O_z$	132	GRAAL	2009	Lleres *et al.* [31]

- The Search for Missing Resonances in $\gamma p \to K^+ + \Lambda$ and $K^+ + \Sigma^0$ Using Circularly Polarized Photons on a **Transversely** Polarized Frozen Spin Target (N. Walford, g9b, T, F, T_x, T_z).
- The Search for Missing Resonances in $\gamma p \to K^+ + \Lambda$ Using Circularly Polarized Photons on a **Longitudinally** Polarized Frozen Spin Target (L. Casey, g9a, $E, L_{x'}, L_{z'}$).

N* photoproduction program at CLAS

	σ	Σ	T	P	E	F	G	H	T_x	T_z	L_x	L_z	O_x	O_z	C_x	C_z
Proton targets																
$p\pi^0$	✔	✔	✓		✓	✓	✓	✓								
$n\pi^+$	✔	✔	✓		✓	✓	✓	✓								
$p\eta$	✔	✓	✓		✓	✓	✓	✓								
$p\eta'$	✔	✓	✓		✓	✓	✓	✓								
$p\omega/\varphi$	✔	✓	✓		✓	✓	✓	✓								
$N\pi\pi$	✓	✓														
$K^+\Lambda$	✔	✓	✓	✔	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✔	✔
$K^+\Sigma^0$	✔	✓	✓	✔	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✔	✔
$K^{0*}\Sigma^+$	✔	✓									✓	✓				
$K^{+*}\Sigma^0$	✔	✓														
Neutron targets																
$p\pi^-$	✔	✓			✓	✓	✓									
$p\rho^-$	✓	✓			✓	✓	✓									
$K^-\Sigma^+$	✓	✓			✓	✓	✓									
$K^0\Lambda$	✓	✓		✓	✓	✓	✓				✓	✓	✓	✓	✓	✓
$K^0\Sigma^0$	✓	✓		✓	✓	✓	✓				✓	✓	✓	✓	✓	✓
$K^{0*}\Sigma^0$	✓	✓														

Proton targets

Data taking completed May 18, 2012

✔-published, ✓-acquired

Neutron targets

Observables for particular experimental setups

Configuration			$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}(\text{conf.})/\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega}^{(0,0,0)}$
$\mathcal{B}$	$\mathcal{T}$	$\mathcal{R}$	
0	0	N	1
0	0	Y	$1 + PP^R_{y'}$
0	L	N	1
0	L	Y	$1 + PP^R_{y'} + P^T_z(L_{x'}P^R_{x'} + L_{z'}P^R_{z'})$
0	T	N	$1 + TP^T_y$
0	T	Y	$1 + \Sigma P^T_y P^R_{y'} + TP^T_y + PP^R_{y'} + P^T_x(T_{x'}P^R_{x'} + T_{z'}P^R_{z'})$
c	0	N	1
c	0	Y	$1 + PP^R_{y'} + P^\gamma_c(C_{x'}P^R_{x'} + C_{z'}P^R_{z'})$
c	L	N	$1 - EP^\gamma_c P^T_z$
c	L	Y	$1 + PP^R_{y'} - EP^\gamma_c P^T_z - HP^\gamma_c P^T_z P^R_{y'} + P^\gamma_c(C_{x'}P^R_{x'} + C_{z'}P^R_{z'}) + P^T_z(L_{x'}P^R_{x'} + L_{z'}P^R_{z'})$
c	T	N	$1 + TP^T_y + FP^\gamma_c P^T_x$
c	T	Y	$1 + \Sigma P^T_y P^R_{y'} + TP^T_y + PP^R_{y'} + GP^\gamma_c P^R_{y'} P^T_x + FP^\gamma_c P^T_x + P^\gamma_c P^T_y(C_{x'}P^R_{z'} + C_{z'}P^R_{x'})$
			$+P^\gamma_c P^T_y(O_{x'}P^R_{z'} + O_{z'}P^R_{x'}) + P^T_x(T_{x'}P^R_{x'} + T_{z'}P^R_{z'})$
l	0	N	$1 - \Sigma P^\gamma_l \cos(2\phi_\gamma)$
l	0	Y	$1 - \Sigma P^\gamma_l \cos(2\phi_\gamma) - TP^\gamma_l P^R_{y'} \cos(2\phi_\gamma) + PP^R_{y'} + P^\gamma_l \sin(2\phi_\gamma)(O_{x'}P^R_{x'} + O_{z'}P^R_{z'})$
l	L	N	$1 - \Sigma P^\gamma_l \cos(2\phi_\gamma) + GP^\gamma_l P^T_z \sin(2\phi_\gamma)$
l	L	Y	$1 + PP^R_{y'} - P^\gamma_l \cos(2\phi_\gamma)\left(TP^R_{y'} + \Sigma + P^T_x(T_{x'}P^R_{z'} - T_{z'}P^R_{x'})\right)$
			$+P^\gamma_l \sin(2\phi_\gamma)\left(GP^T_z + FP^R_{y'}P^T_z + O_{x'}P^R_{x'} + O_{z'}P^R_{z'}\right) + P^T_z(L_{x'}P^R_{x'} + L_{z'}P^R_{z'})$
l	T	N	$1 + TP^T_y - P^\gamma_l \cos(2\phi_\gamma)(PP^T_y + \Sigma) + HP^\gamma_l P^T_x \sin(2\phi_\gamma)$
l	T	Y	$1 - P^\gamma_l P^T_y P^R_{y'} \cos(2\phi_\gamma) + \Sigma P^T_y P^R_{y'} + TP^T_y + PP^R_{y'} + P^T_x(T_{x'}P^R_{x'} + T_{z'}P^R_{z'})$
			$-P^\gamma_l \cos(2\phi_\gamma)\left(-P^T_x(L_{x'}P^R_{z'} - L_{z'}P^R_{x'}) + PP^T_y + \Sigma + TP^R_{y'}\right)$
			$+P^\gamma_l \sin(2\phi_\gamma)\left((O_{x'}P^R_{x'} + O_{z'}P^R_{z'}) + HP^T_x + EP^R_{y'}P^T_x - P^T_y(C_{x'}P^R_{z'} - C_{z'}P^R_{x'})\right)$

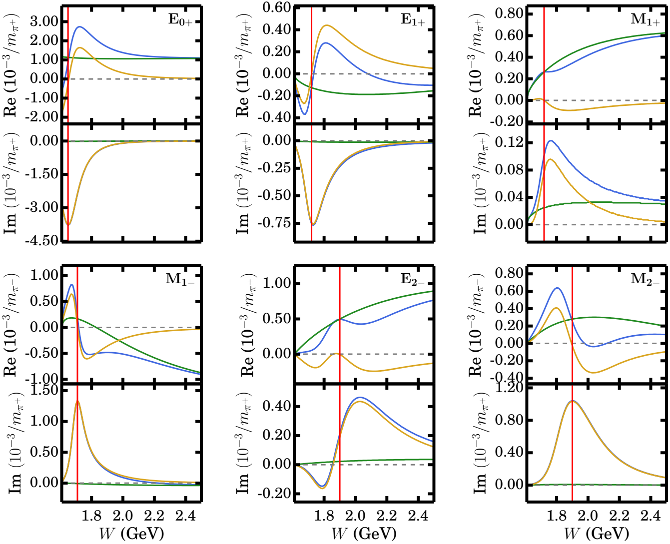
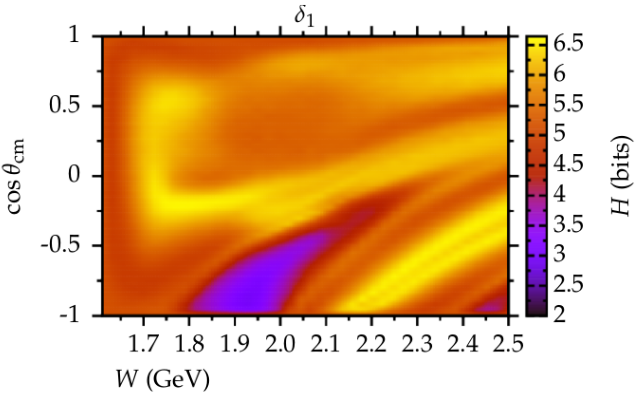
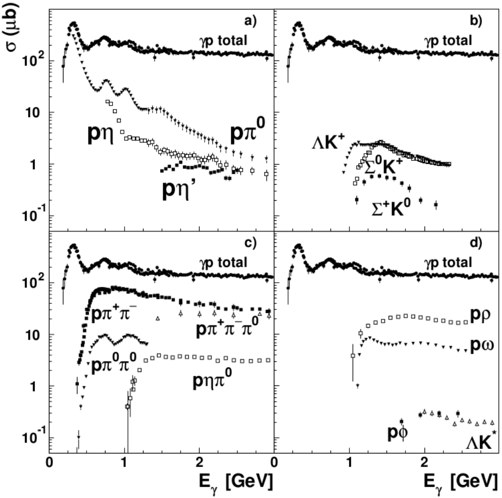


Figure : Kaon-MAID, Kaon-MAID \Resonances and Kaon-MAID \Bg.

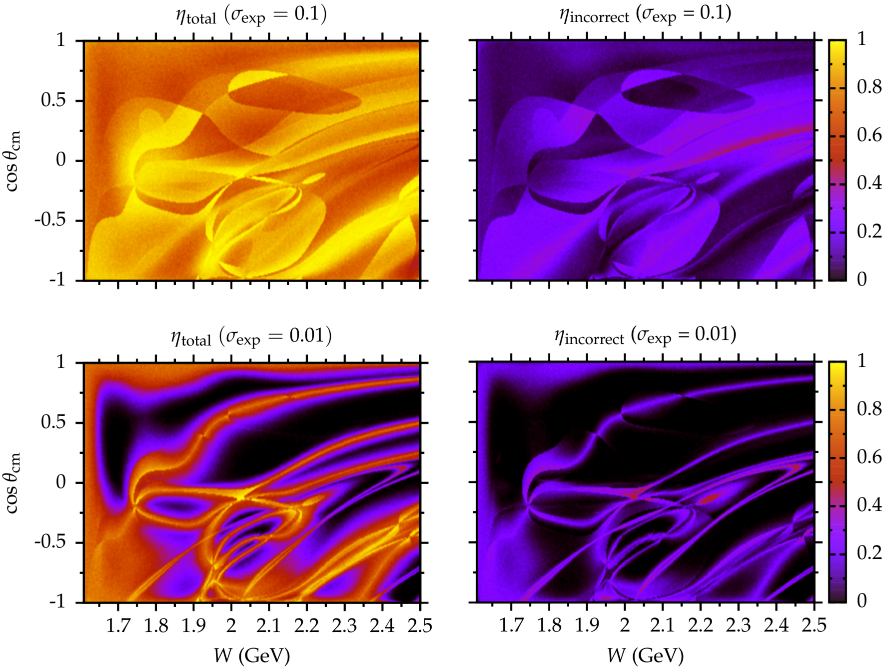

δ_1
$\cos\theta_{cm}$
1
0.5
0
-0.5
-1
1.7 1.8 1.9 2.0 2.1 2.2 2.3 2.4 2.5
W (GeV)
6.5
6
5.5
5
4.5
4
3.5
3
2.5
2
H (bits)

In the following, we study the effect of additional observables on the precision of the extracted amplitudes.

Set number	Observables
1	$\{C_x, O_x, E, F\}$
2	$\{C_x, O_x, E, F, \boldsymbol{C_z}\}$
3	$\{C_x, O_x, E, F, C_z, \boldsymbol{O_z}\}$
4	$\{C_x, O_x, E, F, C_z, O_z, \boldsymbol{G}\}$
5	$\{C_x, O_x, E, F, C_z, O_z, G, \boldsymbol{H}\}$
6	$\{C_x, O_x, E, F, C_z, O_z, G, H, \boldsymbol{T_x}\}$
7	$\{C_x, O_x, E, F, C_z, O_z, G, H, T_x, \boldsymbol{L_x}\}$
8	$\{C_x, O_x, E, F, C_z, O_z, G, H, T_x, L_x, \boldsymbol{T_z}\}$
9	$\{C_x, O_x, E, F, C_z, O_z, G, H, T_x, L_x, T_z, \boldsymbol{L_z}\}$

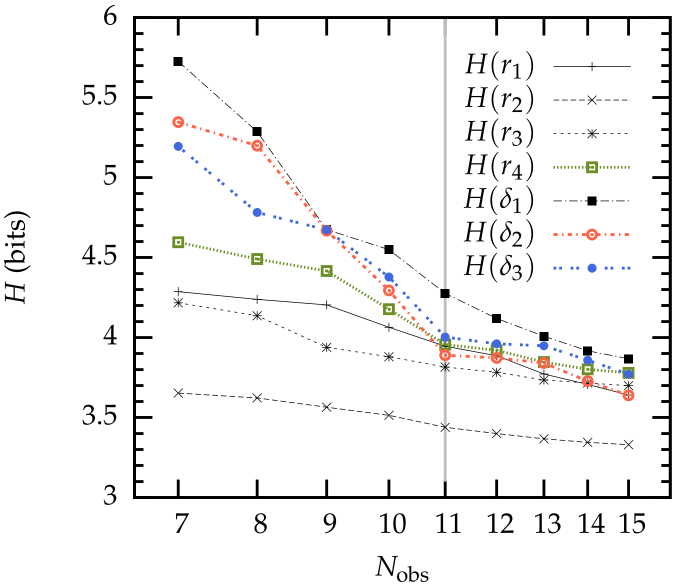

σ (μb)
a)
b)
c)
d)
γp total
pη
pπ0
pη'
ΛK+
Σ0K+
Σ+K0
pπ+π−
pπ+π−π0
pπ0π0
pηπ0
pρ
pω
pφ
ΛK*
Eγ [GeV]
Eγ [GeV]

Σ	$(R_1^2 + R_2^2 - R_3^2 - R_4^2)/\mathcal{N}$
T	$(R_1^2 - R_2^2 - R_3^2 + R_4^2)/\mathcal{N}$
P	$(R_1^2 - R_2^2 + R_3^2 - R_4^2)/\mathcal{N}$
C_x	$-2\left(R_1 R_4 \sin\delta_1 + R_2 R_3 \sin(\delta_2 - \delta_3)\right)/\mathcal{N}$
C_z	$+2\left(R_1 R_4 \cos\delta_1 - R_2 R_3 \cos(\delta_2 - \delta_3)\right)/\mathcal{N}$
O_x	$+2\left(R_1 R_4 \cos\delta_1 + R_2 R_3 \cos(\delta_2 - \delta_3)\right)/\mathcal{N}$
O_z	$+2\left(R_1 R_4 \sin\delta_1 - R_2 R_3 \sin(\delta_2 - \delta_3)\right)/\mathcal{N}$
E	$+2\left(R_1 R_3 \cos(\delta_1 - \delta_3) - R_2 R_4 \cos\delta_2\right)/\mathcal{N}$
F	$-2\left(R_1 R_3 \sin(\delta_1 - \delta_3) + R_2 R_4 \sin\delta_2\right)/\mathcal{N}$
G	$-2\left(R_1 R_3 \sin(\delta_1 - \delta_3) - R_2 R_4 \sin\delta_2\right)/\mathcal{N}$
H	$+2\left(R_1 R_3 \cos(\delta_1 - \delta_3) + R_2 R_4 \cos\delta_2\right)/\mathcal{N}$
T_x	$+2\left(R_1 R_2 \cos(\delta_1 - \delta_2) + R_3 R_4 \cos\delta_3\right)/\mathcal{N}$
T_z	$+2\left(R_1 R_2 \sin(\delta_1 - \delta_2) + R_3 R_4 \sin\delta_3\right)/\mathcal{N}$
L_x	$-2\left(R_1 R_2 \sin(\delta_1 - \delta_2) - R_3 R_4 \sin\delta_3\right)/\mathcal{N}$
L_z	$+2\left(R_1 R_2 \cos(\delta_1 - \delta_2) - R_3 R_4 \cos\delta_3\right)/\mathcal{N}$



η_{total} ($\sigma_{\text{exp}} = 0.1$)
$\eta_{\text{incorrect}}$ ($\sigma_{\text{exp}} = 0.1$)
η_{total} ($\sigma_{\text{exp}} = 0.01$)
$\eta_{\text{incorrect}}$ ($\sigma_{\text{exp}} = 0.01$)
$\cos\theta_{\text{cm}}$
1
0.5
0
-0.5
-1
1.7 1.8 1.9 2 2.1 2.2 2.3 2.4 2.5
W (GeV)
1
0.8
0.6
0.4
0.2
0

			Kinematics nr.			
			1	2	3	4
Single		S	0.21	0.43	0.21	0.47
		T	-0.89	-0.57	-0.52	**0**
		P	-0.15	-0.54	0.25	**0.03**
Double	$\mathcal{BR}$	C_x	-0.28	-0.51	-0.32	-0.16
		C_z	0.84	0.28	0.62	0.85
		O_x	-0.92	-0.64	-0.74	**0.02**
		O_z	-0.33	-0.31	-0.37	-0.19
	$\mathcal{BT}$	E	**0.03**	**0.02**	0.44	0.22
		F	**−0.09**	0.56	-0.27	0.83
		G	-0.30	-0.55	-0.37	**0.08**
		H	0.29	0.41	0.58	-0.17
	$\mathcal{TR}$	T_x	-0.24	-0.59	-0.39	-0.30
		T_z	-0.24	-0.16	-0.49	0.93
		L_x	0.33	0.43	0.52	-0.40
		L_z	**0.02**	**−0.09**	-0.21	-0.34



$H(r_1)$
$H(r_2)$
$H(r_3)$
$H(r_4)$
$H(\delta_1)$
$H(\delta_2)$
$H(\delta_3)$
H (bits)
N_{obs}
6
5.5
5
4.5
4
3.5
3
7
8
9
10
11
12
13
14
15