Strange and Nonstrange Baryon Spectra in the Interacting qD Model

Jacopo Ferretti
"Sapienza" Università di Roma

NUCLEAR RESONANCES: FROM PHOTOPRODUCTION TO HIGH PHOTON VIRTUALITIES

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Overview

- Three quark QM vs qD Model
- A relativistic Interacting qD Model
  Ferretti, Vassallo and Santopinto, PRC83, 065204 (2011)
- Nonstrange baryon spectrum
- Extension to strange baryons
  Santopinto and Ferretti, PRC92, 025202 (2015)
- A relativistic Interacting qD Model with a spin-isospin transition interaction
  De Sanctis et al., arxiv:1410.0590
- Improved nonstrange spectrum and scalar-axial-vector diquark mixing effects

Three quark QMs

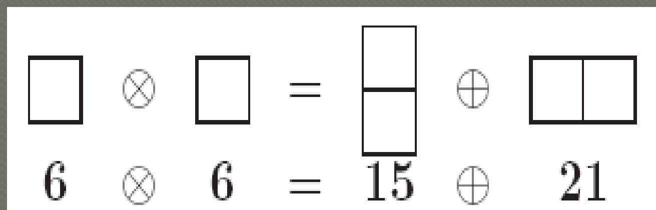
- Several versions: Isgur and Karl, Capstick and Isgur, U(7), Graz, Hypercentral QM …
- Some differences, but share main features:
  1) based on the effective degrees of freedom of three constituent quarks
  2) (linear) confining potential
  3) states classified within $SU_{sf}(6)$
- Reproduce reasonably well many observables: baryon magnetic moments, lower part of baryon spectrum, open-flavor decays …
- They have some problems, including that of the missing resonances

Missing resonances

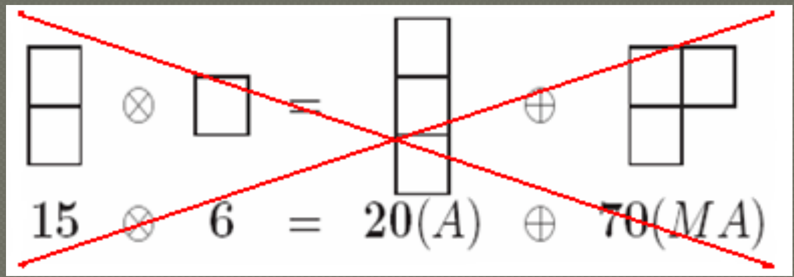
- States predicted by quark models with no corresponding experimental counterparts
- QMs predict eccessive number of states
- Possible explanations:
  1) Some baryon states may be very weakly coupled to single-pion channels. Look for two-pion, three-pion, eta decay channels …
  2) Consider models based on smaller number of effective degrees of freedom (like quark-diquark model): number of missing states decreases notably

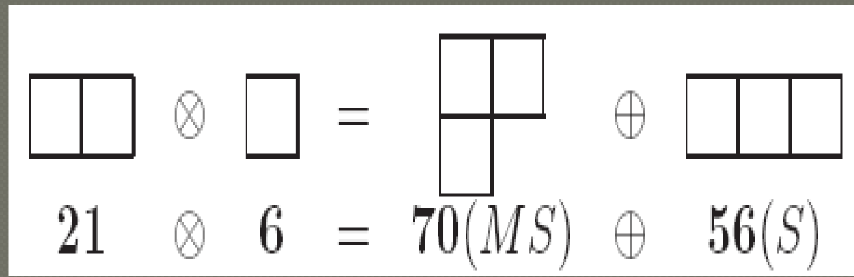
Quark-diquark models

- Diquark: two strongly correlated quarks, with no internal spatial excitations (Ψ_{space} symmetric)
- Diquark as effective bosonic degree of freedom
- Diquark wave function is antisymmetric:
  $\Psi_D = \Psi_{space}\Psi_{color}\Psi_{spin\text{-}flavor}$
- Baryon in color-singlet: Ψ_{color} is antisymmetric
- Diquark spin-flavor wave function is symmetric
  15 spin-flavor representation is neglected



$SU_{sf}(6)$ representations





- **20**(A) and **70**(MA) representations neglected in quark-diquark models
- Thus, the number of states decreases with respect to three quark QMs

Rel. Interacting qD Model

- Model mass formula

$$M = E_0 + \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} + M_{dir} + M_{ex} + M_{cont}$$

- m_1 and m_2: quark and diquark masses
- Direct + exchange + contact terms
- Eigenvalues → numerical variational procedure with h.o. trial wave functions
- Model parameters (14) fitted to data

FERRETTI, VASSALLO AND SANTOPINTO, PRC83, 065204 (2011)

Interactions

- Direct Term

$$M_{dir} = -\frac{\tau}{r}(1 - e^{-\mu r}) + \beta r$$

Smeared Coulomb-like

Linear confining

- Exchange Term

$$M_{ex} = (-1)^{L+1} e^{-\sigma r}[A_s \vec{s}_1 \cdot \vec{s}_2 + A_I \vec{t}_1 \cdot \vec{t}_2 + A_{SI}(\vec{s}_1 \cdot \vec{s}_2)(\vec{t}_1 \cdot \vec{t}_2)]$$

- Contact Term

$$M_{cont} \propto \frac{\eta^3 e^{-\eta^2 r^2}}{\pi^{3/2}}$$

δ simulating function

INTRODUCED TO REPRODUCE Δ-N MASS SPLITTING

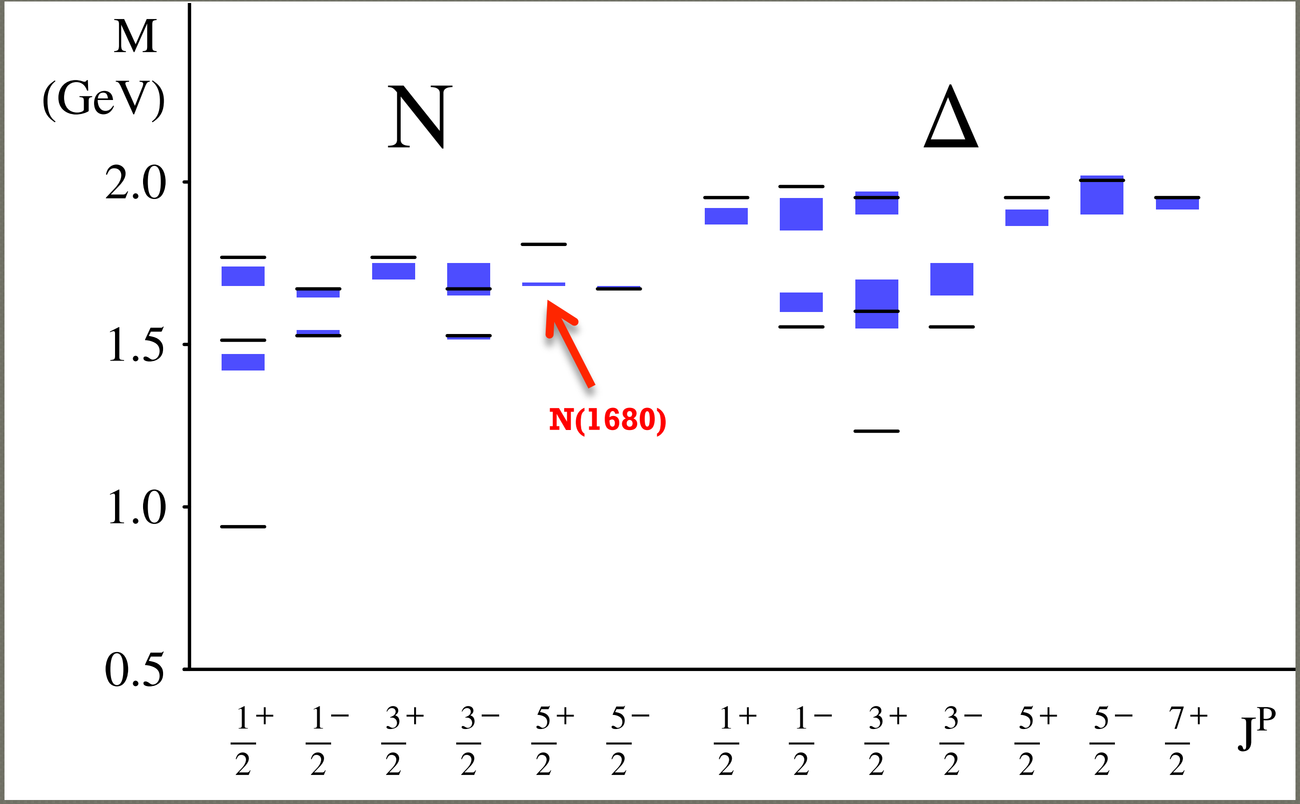
FERRETTI, VASSALLO AND SANTOPINTO, PRC83, 065204 (2011)

Model parameters

$m_q = 200$ MeV	$m_S = 600$ MeV	$m_{AV} = 950$ MeV
$\tau = 1.25$	$\mu = 75.0$ fm^{-1}	$\beta = 2.15$ fm^{-2}
$A_S = 375$ MeV	$A_I = 260$ MeV	$A_{SI} = 375$ MeV
$\sigma = 1.71$ fm^{-1}	$E_0 = 154$ MeV	$D = 4.66$ fm^2
$\eta = 10.0$ fm^{-1}	$\epsilon = 0.200$	

FERRETTI, VASSALLO AND SANTOPINTO, PRC83, 065204 (2011)

Nonstrange Spectrum



*** and **** PDG states below 2 GeV

FERRETTI, VASSALLO AND SANTOPINTO, PRC83, 065204 (2011)

Nonstrange Spectrum

Resonance	Status	M^{expt} (MeV)	J^P	L^P	S	s_1	n_r	M^{calc} (MeV)
$N(939)\ P_{11}$	****	939	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	0	939
$N(1440)\ P_{11}$	****	1420–1470	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	1	1513
$N(1520)\ D_{13}$	****	1515–1525	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	0	1527
$N(1535)\ S_{11}$	****	1525–1545	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	0	1527
$N(1650)\ S_{11}$	****	1645–1670	$\frac{1}{2}^-$	1^-	$\frac{1}{2},\frac{3}{2}$	1	0	1671
$N(1675)\ D_{15}$	****	1670–1680	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	1671
$N(1680)\ F_{15}$	****	1680–1690	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	0	1808
$N(1700)\ D_{13}$	***	1650–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2},\frac{3}{2}$	1	0	1671
$N(1710)\ P_{11}$	***	1680–1740	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1768
$N(1720)\ P_{13}$	****	1700–1750	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1768
$\Delta(1232)\ P_{33}$	****	1231–1233	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1233
$\Delta(1600)\ P_{33}$	***	1550–1700	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	1	1602
$\Delta(1620)\ S_{31}$	****	1600–1660	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	0	1554
$\Delta(1700)\ D_{33}$	****	1670–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	0	1554
$\Delta(1900)\ S_{31}$	**	1850–1950	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	1	1986
$\Delta(1905)\ F_{35}$	****	1865–1915	$\frac{5}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$\Delta(1910)\ P_{31}$	****	1870–1920	$\frac{1}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$\Delta(1920)\ P_{33}$	***	1900–1970	$\frac{3}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$\Delta(1930)\ D_{35}$	***	1900–2020	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	2005
$\Delta(1950)\ F_{37}$	****	1915–1950	$\frac{7}{2}^+$	2^+	$\frac{3}{2}$	1	0	1952
$N(2100)\ P_{11}$	*	1855–1915	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	2	1893
$N(2090)\ S_{11}$	*	1869–1987	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	1	1882
$N(1900)\ P_{13}$	**	1820–1974	$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	0	0	1808
$N(2080)\ D_{13}$	**	1740–1940	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	1	1882
$\Delta(1750)\ P_{31}$	*	1708–1780	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1858
$\Delta(1940)\ D_{33}$	*	1947–2167	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	1	1986

No missing states below 2 GeV

FERRETTI, VASSALLO AND SANTOPINTO, PRC83, 065204 (2011)

Extension to strange baryons

- Mass formula

$$M = E_0 + \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} + M_{dir} + M_{ex} + M_{cont}$$

- Exchange potential is generalized to Gürsey-Radicati inspired interaction

$$M_{ex} = (-1)^{L+1} e^{-\sigma r} [A_s \vec{s}_1 \cdot \vec{s}_2 + A_I \vec{t}_1 \cdot \vec{t}_2 + A_F \vec{\lambda}_1 \cdot \vec{\lambda}_2]$$

λ's are SU(3) Gell-Mann matrices

- Results updated to most recent exp. data. Global fit to strange & nonstrange baryons

SANTOPINTO AND FERRETTI, PRC92, 025202 (2015)

Model parameters

Parameter	Value (fit 1)	Value (fit 2)	Parameter	Value (fit 1)	Value (fit 2)
m_n	200 MeV	159 MeV	m_s	550 MeV	213 Mev
$m_{[n,n]}$	600 MeV	607 MeV	$m_{[n,s]}$	900 MeV	856 MeV
$m_{\{n,n\}}$	950 MeV	963 MeV	$m_{\{n,s\}}$	1200 MeV	1216 MeV
$m_{\{s,s\}}$	1580 MeV	1352 MeV	τ	1.20	1.02
μ	75.0 fm^{-1}	28.4 fm^{-1}	β	2.15 fm^{-2}	2.36 fm^{-2}
A_S	350 MeV	–436 MeV	A_F	100 MeV	193 MeV
A_I	250 MeV	791 MeV	σ	2.30 fm^{-1}	2.25 fm^{-1}
E_0	141 MeV	150 MeV	ϵ	0.37	
D	6.13 fm^2		η	11.0 fm^{-1}	

SANTOPINTO AND FERRETTI, PRC92, 025202 (2015)

N spectrum and N(1900)P_{13}

Resonance	Status	$M^{exp.}$ (MeV)	J^P	L^P	S	s_1	n_r	$M^{calc.}$ (fit 1) (MeV)
$N(939)\ P_{11}$	****	939	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	0	939
$N(1440)\ P_{11}$	****	1420–1470	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	1	1511
$N(1520)\ D_{13}$	****	1515–1525	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	0	1537
$N(1535)\ S_{11}$	****	1525–1545	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	0	1537
$N(1650)\ S_{11}$	****	1645–1670	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	0	1625
$N(1675)\ D_{15}$	****	1670–1680	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	1746
$N(1680)\ F_{15}$	****	1680–1690	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	0	1799
$N(1700)\ D_{13}$	***	1650–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	0	1625
$N(1710)\ P_{11}$	***	1680–1740	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1776
$N(1720)\ P_{13}$	****	1700–1750	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1648
Missing			$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	0	1746
Missing			$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	0	1746
Missing			$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	0	0	1799
$N(1875)\ D_{13}$	***	1820–1920	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	1	1888
$N(1880)\ P_{11}$	**	1835–1905	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	2	1890
$N(1895)\ S_{11}$	**	1880–1910	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	1	1888
$N(1900)\ P_{13}$	***	1875–1935	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	1	1947

3 missing states

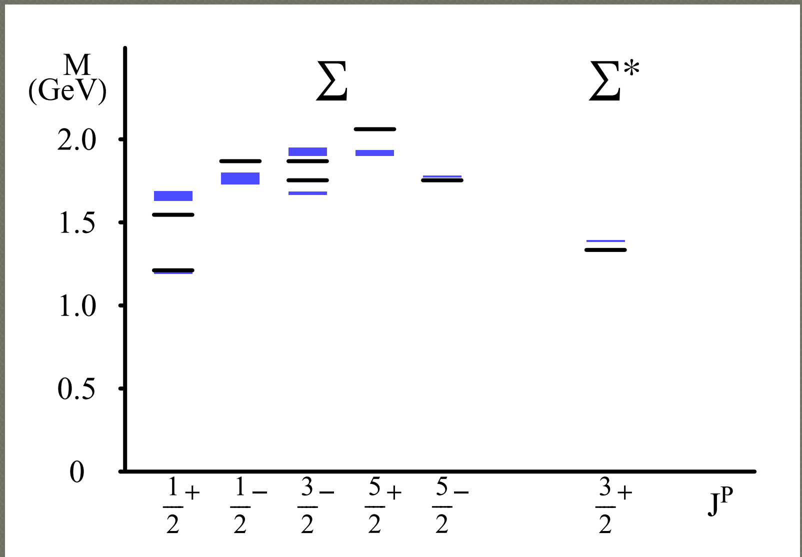
SANTOPINTO AND FERRETTI, PRC92, 025202 (2015)

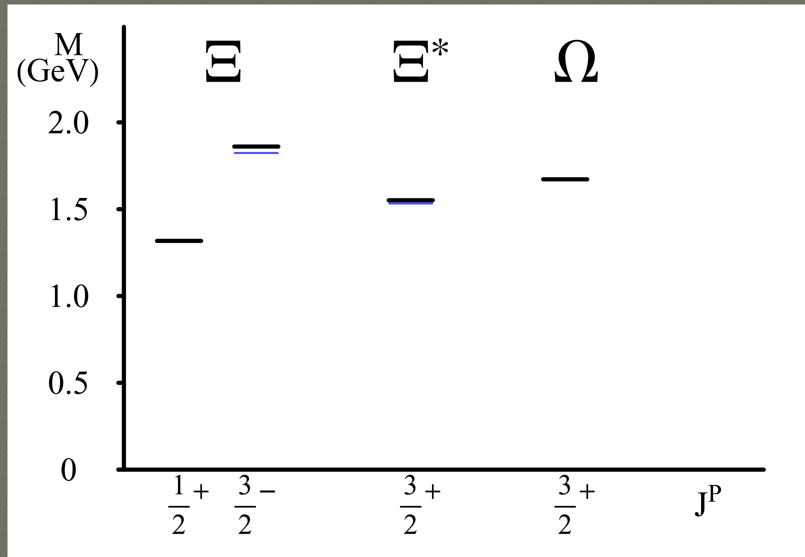
Δ spectrum

Resonance	Status	$M^{\text{exp.}}$ (MeV)	J^P	L^P	S	s_1	n_r	$M^{\text{calc.}}$ (fit 1) (MeV)
$\Delta(1232)\ P_{33}$	****	1230–1234	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1247
$\Delta(1600)\ P_{33}$	***	1500–1700	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	1	1689
$\Delta(1620)\ S_{31}$	****	1600–1660	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	0	1830
$\Delta(1700)\ D_{33}$	****	1670–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	0	1830
$\Delta(1750)\ P_{31}$	*	1708–1780	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1489
$\Delta(1900)\ S_{31}$	**	1840–1920	$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	0	1910
$\Delta(1905)\ F_{35}$	****	1855–1910	$\frac{5}{2}^+$	2^+	$\frac{3}{2}$	1	0	2042
$\Delta(1910)\ P_{31}$	****	1860–1920	$\frac{1}{2}^+$	2^+	$\frac{3}{2}$	1	0	1827
$\Delta(1920)\ P_{33}$	***	1900–1970	$\frac{3}{2}^+$	2^+	$\frac{3}{2}$	1	0	2042
$\Delta(1930)\ D_{35}$	***	1900–2000	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	1910
$\Delta(1940)\ D_{33}$	**	1940–2060	$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	0	1910
$\Delta(1950)\ F_{37}$	****	1915–1950	$\frac{7}{2}^+$	2^+	$\frac{3}{2}$	1	0	2042

No missing states below 2 GeV

Σ, Σ*, Ξ, Ξ* and Ω spectrum





*** and **** PDG states below 2 GeV

SANTOPINTO AND FERRETTI, PRC92, 025202 (2015)

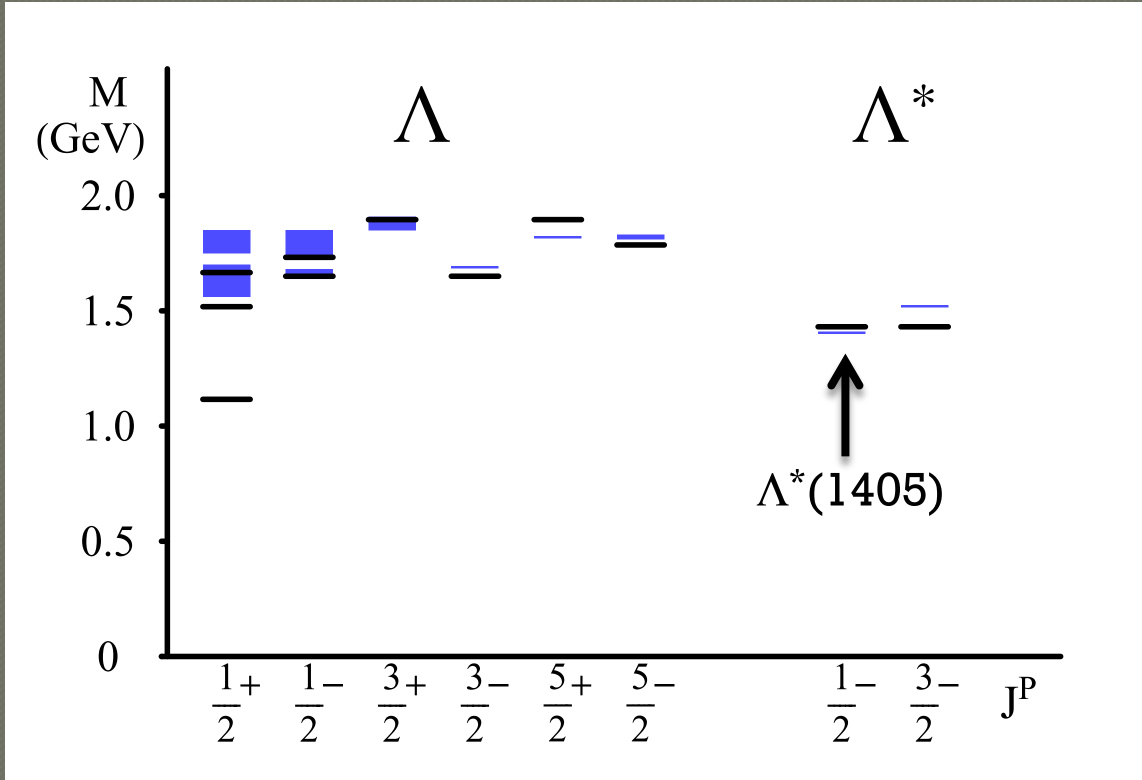
Σ and Σ^* spectrum

Resonance	Status	$M^{exp.}$ (MeV)	J^P	L^P	S	s_1	Q^2q	**F**	**F₁**	I	t_1	n_r	$M^{calc.}$ (fit 2) (MeV)
$\Sigma(1193)$ P_{11}	****	1189—1197	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	1211
$\Sigma(1620)$ S_{11}	**	≈1620	$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1753
$\Sigma(1660)$ P_{11}	***	1630–1690	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1546
$\Sigma(1670)$ D_{13}	****	1665–1685	$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1753
$\Sigma(1750)$ S_{11}	***	1730–1800	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	1868
$\Sigma(1770)$ P_{11}	*	≈1770	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,s\}n$	**8**	**6**	1	$\frac{1}{2}$	0	1668
$\Sigma(1775)$ D_{15}	****	1770–1780	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1753
$\Sigma(1880)$ P_{11}	**	≈1880	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	1	1801
$\Sigma(1915)$ F_{15}	****	1900–1935	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	2061
$\Sigma(1940)$ D_{13}	***	1900–1950	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	1	$\frac{1}{2}$	0	1868
Missing	1 missing state		$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1895
$\Sigma(2000)$ S_{11}	*	≈2000	$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,n\}s$	**8**	**6**	1	1	0	1895
$\Sigma^*(1385)$ P_{13}	****	1382–1388	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,n\}s$	**10**	**6**	1	1	0	1334
$\Sigma^*(1840)$ P_{13}	*	≈1840	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,s\}n$	**10**	**6**	1	$\frac{1}{2}$	0	1439
$\Sigma^*(2080)$ P_{13}	**	≈2080	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,n\}s$	**10**	**6**	1	1	1	1924

Ξ, Ξ^* and Ω spectrum

Resonance	Status	$M^{\text{exp.}}$ (MeV)	J^P	L^P	S	s_1	Q^2q	**F**	**F$_1$**	I	t_1	n_r	$M^{\text{calc.}}$ (fit 2) (MeV)
$\Xi(1318)\ P_{11}$	****	1315–1322	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]s$	**8**	$\bar{\mathbf{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1317
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,s\}s$	**8**	**6**	$\frac{1}{2}$	$\frac{1}{2}$	0	1772
$\Xi(1820)\ D_{13}$	***	1818–1828	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]s$	**8**	$\bar{\mathbf{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1861
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]s$	**8**	$\bar{\mathbf{3}}$	$\frac{1}{2}$	$\frac{1}{2}$	1	1868
Missing	5 missing states		$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{s,s\}n$	**8**	**6**	$\frac{1}{2}$	0	0	1874
Missing			$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}s$	**8**	**6**	$\frac{1}{2}$	$\frac{1}{2}$	0	1971
$\Xi^*(1530)\ P_{13}$	****	1531–1532	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,s\}s$	**10**	**6**	$\frac{1}{2}$	$\frac{1}{2}$	0	1552
Missing			$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{s,s\}n$	**10**	**6**	$\frac{1}{2}$	0	0	1653
$\Omega(1672)\ P_{03}$	****	1672–1673	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{s,s\}s$	**10**	**6**	0	0	0	1672

Λ and Λ* spectrum



*** and **** PDG states below 2 GeV

SANTOPINTO AND FERRETTI, PRC92, 025202 (2015)

Λ and Λ^* spectrum

Resonance	Status	$M^{\text{exp.}}$ (MeV)	J^P	L^P	S	s_1	Q^2q	**F**	$\mathbf{F}_1$	I	t_1	n_r	$M^{\text{calc.}}$ (fit 2) (MeV)
$\Lambda(1116)\ P_{01}$	****	1116	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	0	1116
$\Lambda(1600)\ P_{01}$	***	1560–1700	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1518
$\Lambda(1670)\ S_{01}$	****	1660–1680	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	0	1650
$\Lambda(1690)\ D_{03}$	****	1685–1695	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	0	1650
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1732
Missing			$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1785
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	1	1785
$\Lambda(1800)\ S_{01}$	***	1720–1850	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1732
$\Lambda(1810)\ P_{01}$	***	1750–1850	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	1	1666
$\Lambda(1820)\ F_{05}$	****	1815–1825	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	$[n,n]s$	**8**	$\bar{\mathbf{3}}$	0	0	0	1896
$\Lambda(1830)\ D_{05}$	****	1810–1830	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1785
$\Lambda(1890)\ P_{03}$	****	1850–1910	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1896
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1955
Missing			$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	$[n,s]n$	**8**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	1	1960
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1969
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	$\{n,s\}n$	**8**	**6**	0	$\frac{1}{2}$	0	1969
$\Lambda^*(1405)\ S_{01}$	****	1402–1410	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	**1**	$\bar{\mathbf{3}}$	0	0	0	1431
$\Lambda^*(1520)\ D_{03}$	****	1519–1521	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	**1**	$\bar{\mathbf{3}}$	0	0	0	1431
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**1**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1443
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**1**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	0	1443
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	**1**	$\bar{\mathbf{3}}$	0	0	1	1854
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,n]s$	**1**	$\bar{\mathbf{3}}$	0	0	1	1854
Missing			$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**1**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	1	1928
Missing			$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	$[n,s]n$	**1**	$\bar{\mathbf{3}}$	0	$\frac{1}{2}$	1	1928

13 missing states

SANTOPINTO AND FERRETTI, PRC92, 025202 (2015)

Relativistic qD Model with Spin-Isospin (SI) transition interaction

- SI transition interaction mixes scalar and axial-vector diquark components
- Motivations:
  1. Improve reproduction of nonstrange baryon spectrum
  2. Introduce axial-vector diquark component in nucleon WF
- Better reproduction of nucleon e.m. form factors expected De Sanctis et al. PRC84, 055201 (2011)
- Other observables can also be computed

Model Hamiltonian

- $$H = E_0 + \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} + M_{dir} + M_{ex} + M_{cont} + M_{tr}$$

- $$M_{tr} = V_0 e^{-\frac{1}{2}\nu^2 r^2} (\vec{s}_2 \cdot \vec{S})(\vec{t}_2 \cdot \vec{T})$$

- S and T are spin and isospin transition operators

DE SANCTIS ET AL., ARXIV:1410.0590

Model parameters

m_q	=	140 MeV	m_S	=	150 MeV	m_{AV}	=	360 MeV
τ	=	1.23	μ	=	125 fm^{-1}	β	=	1.57 fm^{-2}
A_S	=	125 MeV	A_I	=	85 MeV	A_{SI}	=	350 MeV
σ	=	0.60 fm^{-1}	E_0	=	826 MeV	D	=	2.00 fm^2
η	=	10.0 fm^{-1}	V_0	=	1450 MeV	ν	=	0.35 fm^{-1}

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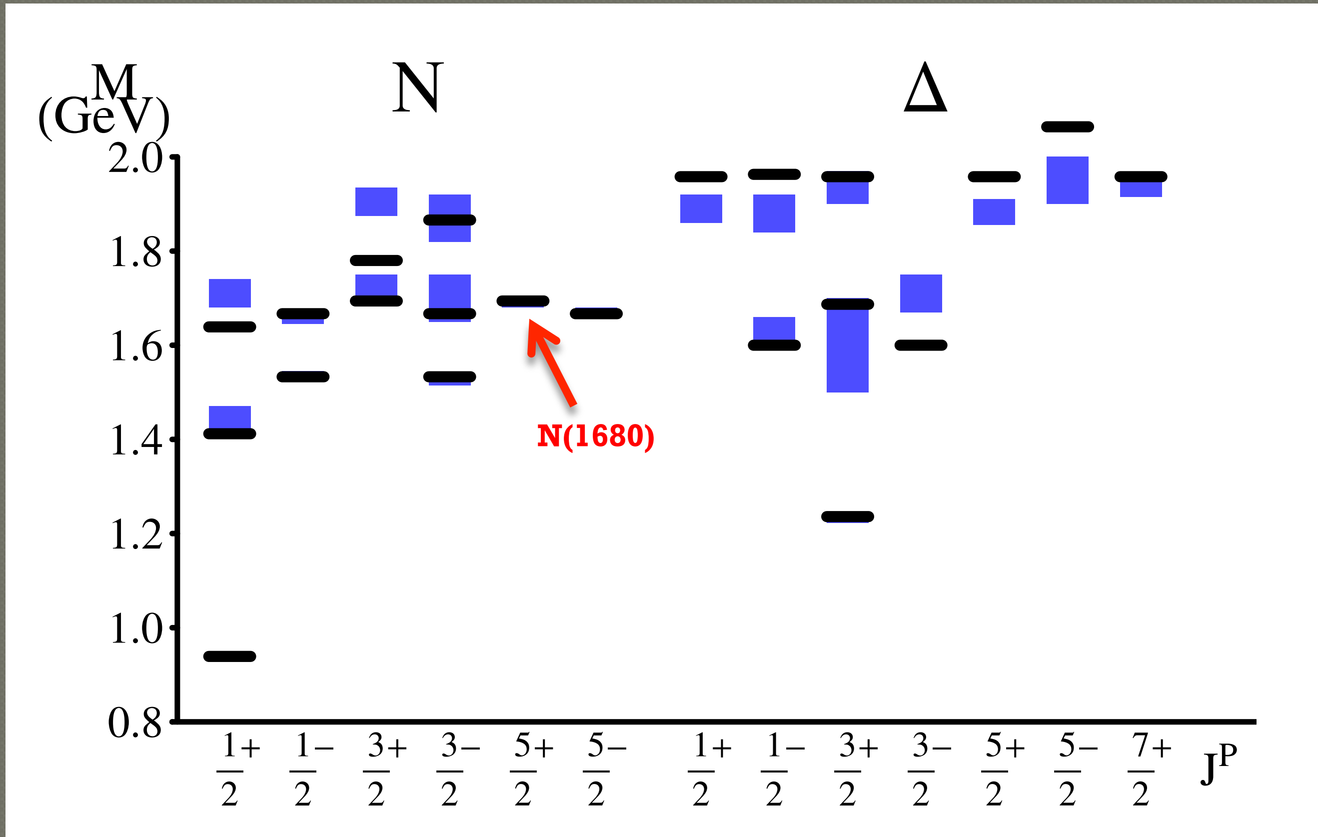
Nonstrange spectrum



DE SANCTIS ET AL., ARXIV:1410.0590

Nonstrange spectrum

Resonance	Status	$M^{\text{exp.}}$ (MeV)	J^P	L^P	S	s_1	n_r	$M^{\text{calc.}}$ (MeV)
$N(939)\ P_{11}$	****	939	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0,1	0	939
$N(1440)\ P_{11}$	****	1420 - 1470	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0,1	1	1412
$N(1520)\ D_{13}$	****	1515 - 1525	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0,1	0	1533
$N(1535)\ S_{11}$	****	1525 - 1545	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0,1	0	1533
$N(1650)\ S_{11}$	****	1645 - 1670	$\frac{1}{2}^-$	1^-	$\frac{3}{2}$	1	0	1667
$N(1675)\ D_{15}$	****	1670 - 1680	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	1667
$N(1680)\ F_{15}$	****	1680 - 1690	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0,1	0	1694
$N(1700)\ D_{13}$	***	1650 - 1750	$\frac{3}{2}^-$	1^-	$\frac{3}{2}$	1	0	1667
$N(1710)\ P_{11}$	***	1680 - 1740	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0,1	2	1639
$N(1720)\ P_{13}$	****	1700 - 1750	$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	0,1	0	1694
$N(1875)\ D_{13}$	***	1820 - 1920	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0,1	1	1866
$N(1880)\ P_{11}$	**	1835 - 1905	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0,1	3	1786
$N(1895)\ S_{11}$	**	1880 - 1910	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0,1	1	1866
$N(1900)\ P_{13}$	***	1875 - 1935	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	0	0	1780
missing	1 missing state		$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	0,1	1	1990
$N(2000)\ F_{15}$	**	1950 - 2150	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0,1	1	1990

Resonance	Status	$M^{\text{exp.}}$ (MeV)	J^P	L^P	S	s_1	n_r	$M^{\text{calc.}}$ (MeV)
$\Delta(1232)\ P_{33}$	****	1230 - 1234	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	0	1236
$\Delta(1600)\ P_{33}$	***	1500 - 1700	$\frac{3}{2}^+$	0^+	$\frac{3}{2}$	1	1	1687
$\Delta(1620)\ S_{31}$	****	1600 - 1660	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	0	1600
$\Delta(1700)\ D_{33}$	****	1670 - 1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	0	1600
$\Delta(1750)\ P_{31}$	*	1708 - 1780	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1857
$\Delta(1900)\ S_{31}$	**	1840 - 1920	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	1	1963
$\Delta(1905)\ F_{35}$	****	1855 - 1910	$\frac{5}{2}^+$	2^+	$\frac{3}{2}$	1	0	1958
$\Delta(1910)\ P_{31}$	****	1860 - 1920	$\frac{1}{2}^+$	2^+	$\frac{3}{2}$	1	0	1958
$\Delta(1920)\ P_{33}$	***	1900 - 1970	$\frac{3}{2}^+$	2^+	$\frac{3}{2}$	1	0	1958
$\Delta(1930)\ D_{35}$	***	1900 - 2000	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	2064
$\Delta(1940)\ D_{33}$	**	1940 - 2060	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	1	1963
$\Delta(1950)\ F_{37}$	****	1915 - 1950	$\frac{7}{2}^+$	2^+	$\frac{3}{2}$	1	0	1958

Nucleon Wave Function

- The SI interaction allows scalar and axial-vector diquarks components in nucleon WF with probability:

State	Scalar component	Axial-vector component
N	53%	47%
N(1440)	51%	49%
Δ(1232)	0	100%

- Important also in the calculation of several other observables: e.m. form factors, open-flavor decays, magnetic moments, …

DE SANCTIS ET AL., ARXIV:1410.0590

Future developments

- Rel. Interacting qD Model extended to heavy baryons
- Baryon magnetic moments in qD model
- Improved nucleon elastic and transition (helicity amplitudes) e.m. form factors
- Open-flavor decays in a qD model

Conclusions

- Three quark QM vs qD Model
- A relativistic Interacting qD Model
  Ferretti, Vassallo and Santopinto, PRC83, 065204 (2011)
- Nonstrange baryon spectrum
- Extension to strange baryons
  Santopinto and Ferretti, PRC92, 025202 (2015)
- A relativistic Interacting qD Model with a spin-isospin transition interaction
  De Sanctis et al., arxiv:1410.0590
- Improved nonstrange spectrum and scalar-axial-vector diquark mixing effects

Thank you for you attention!

Extra slides

SI transition interaction

- Operator:

$$M_{\mathrm{tr}}(r) = V_0 \; e^{-\frac{1}{2}\nu^2 r^2} (\vec{s}_2 \cdot \vec{S})(\vec{t}_2 \cdot \vec{T})$$

- Matrix elements defined as:

$$\langle \; s_1', m_{s_1}' \; | \, S_\mu^{[1]} \, | s_1, m_{s_1} \rangle \neq 0 \text{ for } s_1' \neq s_1$$

$$\langle 1 \| \; S_1 \; \| 0 \rangle = 1 \;\; \langle 0 \| \; S_1 \; \| 1 \rangle = -1$$

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Point Form Relativistic Dynamics

Point Form is one of the Relativistic Hamiltonian Dynamics
for a fixed number of particles (Dirac)

Construction of a representation of the Poincaré generators
P_μ (tetramomentum), J_k (angular momenta), K_i (boosts)
obeying the Poincaré group commutation relations
in particular

$$[P_k , K_i] = i \, \delta_{kj} \, H$$

Three forms:
Light (LF), Instant (IF), Point (PF)
Differ in the number and type of (interaction) free generators

Point form: P_μ interaction dependent

J_k and K_i free

Composition of angular momentum states as in the non relativistic case

Mass operator $M = M_0 + M_I$

$M_0 = \Sigma_i \sqrt{\vec{p}_i^{\,2} + m^2}$

$\Sigma_i \vec{p}_i = 0$

$\vec{P}_i$ undergo the same Wigner rotation -> M_0 is invariant

The eigenstates of the relativistic qD Model are interpreted as eigenstates of the mass operator M

Moving three-quark states are obtained through (interaction free) Lorentz boosts (velocity states)