EmNN*, Trento 2015

NUCLEON RESONANCE ELECTROCOUPLINGS FROM LIGHT FRONT QUARK MODELS AT Q2 UP TO 12 GEV2

IgorT. Obukhovsky

Institute of Nuclear Physics
Moscow State University

ECT* Trento 12.10.2015

Authors

Moscow (MSU): I.T. Obukhovsky
Tuebingen: V.E. Lyubovitskij
Th. Gutsche
A. Faessler

- **Introduction**
- **The Roper resonance.** $N+\gamma^* \to N^*$ **transition at low** $Q^2 \lesssim$**1-2 GeV**^{2}.
- **High** Q^2. **Nucleon ground state at the light front.**
- **High** Q^2. **Quark configurations and Melosh transformations.**
- **Summary.**

1. Introduction

There are many theoretical approaches to hadron structure:

- Lattice QCD
- DSE and BS equations
- Light-front QCD
- AdS/QCD
- Relativistic and nonrelativistic quark models

Our approach is to fit parameters of light-front quark configurations to the elastic nucleon form factors extracted from recent data on polarized electron scattering and use these to calculate the transition form factors at large Q^2 up to 12 GeV^2.

In the context of projected extensive studies of baryons with $J^P = 1/2^\pm, 3/2^\pm, 5/2^\pm$, etc, there is an interest in calculation of electrocouplings of baryons at large Q^2.

Rough estimates can be made on a basis of light-front quark models. It implies construction of a good basis of quark configurations at the light front possessing the definite value of angular momentum and satisfying Pauli exclusion principle.

The calculation of the Roper resonance electrocouplings will be considered here as example.

Light-front quark wave functions were successfully used by many authors for description nucleon form factors and transition amplitudes

1) before polarized electron data: F.Schlumpf, PRD 47, 4114; S.Capstick and B.D.Keister, PRD 51, 3598; I.G.Aznauryan, PLB 316, 391; F.Cardarelli *et al*, **PLB 371, 7**

2) as well as after these (with taken into account new high-quality data): S.Capstick *et al*, **J.Phys.Conf. 69, 012016; I.G.Aznauryan and V.D.Burkert, PRC 85, 055202; G.Ramalho and K.Tsushima, PRD 81,074020; V.E.Lyubovitskij** *et al*, **PRD 89, 054033.**

Now this work is being continued in the context of the considerably extended experimental program on study of $\gamma_v NN^*$ couplings at large Q^2.

In our recent work (PRD 89, 054033) we have generalized our late results on the Roper resonance electroproduction at $Q^2 < 4\,\mathrm{GeV}^2$ (PRD 84, 014004) by going to more high Q^2 up to 12 GeV^2.

It has taken to rewrite our old non-relativistic model in terms of quark configurations at the light front. We reason that our experience in model description of the Roper resonance electroproduction would be useful for modeling the transition amplitudes for other resonances.

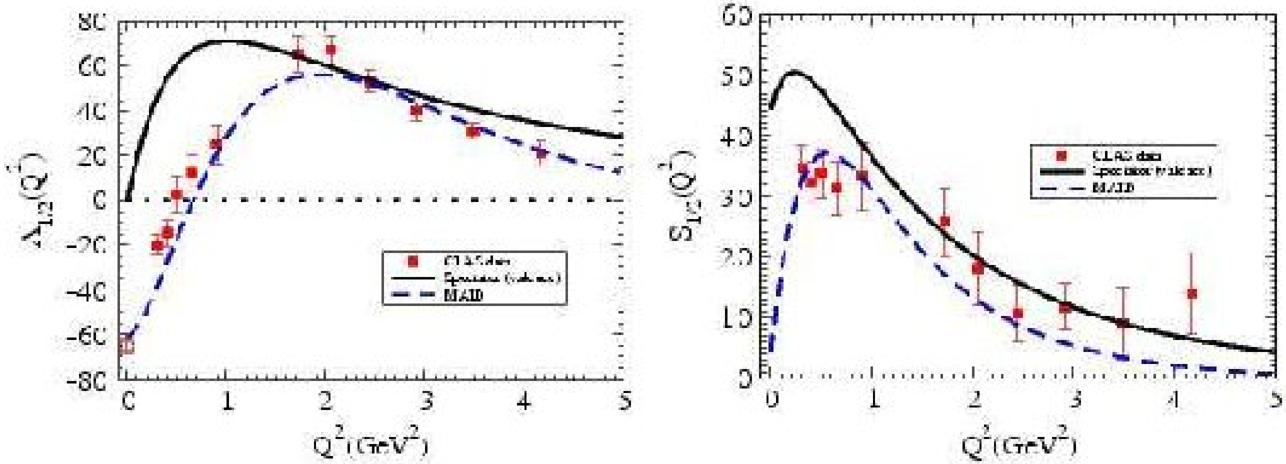
In the case of the Roper resonance the main problem is that its inner structure cannot be adequately described in terms of only constituent quark degrees of freedom, and thus other (more soft) degrees of freedom should be taken into consideration along with the quark core.

It is evident that at high Q^2 the contribution of such soft components to the transition form factors is quickly dying out with rising of the momentum transfer, and only the contribution of the quark core survives.

2. The Roper resonance. $N + \gamma^* \to N^*$ transition at low $Q^2 \lesssim$1-2 GeV^2

Recall, the description of the Roper resonance with the $\boxed{sp^2[3]_X\, L=0}$ configuration, i.e. as the radial two-h.o.-quanta ($2S$) excitation, fails to explain the observed mass $M_R \approx 1440$ MeV and the decay width.

Moreover, in the full interval $0 \le Q^2 \lesssim 4.5$ GeV^2 only the data at $Q^2 \gtrsim$1-2 GeV^2 correlate well with the quark model predictions, while at low Q^2 the transverse amplitude $A_{1/2}$ are in rather poor agreement with the data. For example,



Adapted from G. Ramalho, K.Tsushima, Ph.R. D81, 074020 (2010). Dashed: MAID fit, solid: Gross model.

This is not surprising, since in the quark model the spin-isospin structure of the Roper is identical to the nucleon one, and thus the coordinate parts of N and R should be orthogonal.

The transverse helicity amplitude $A_{1/2} \sim \langle R, +\frac{1}{2}|j^{\mu}\epsilon_{\mu}^{(+)}|N, -\frac{1}{2}\rangle$ is given by the spin-flip transition. If the quark spin part of both N and R is factorized from the coordinate part in the full w.f., the orthogonality of the coordinate parts of R and N $\boxed{\langle R|N\rangle_X = 0}$ brings the value of $A_{1/2}$ to zero at the 'real photon' point $Q^2 = 0$.

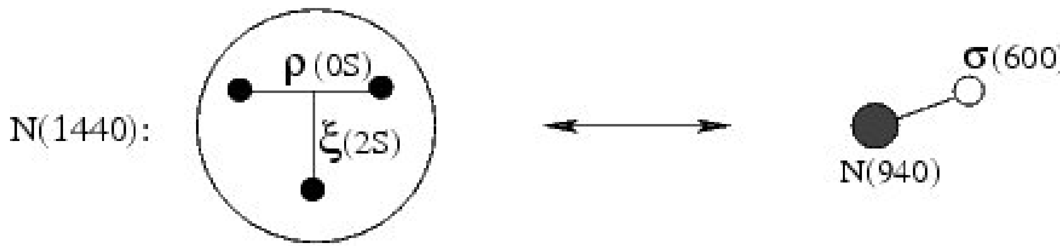
It requires to take into account non-local contributions to the quark current operator. Another problem in the quark approach to the transition form factors is how to evaluate the contribution of the meson cloud.

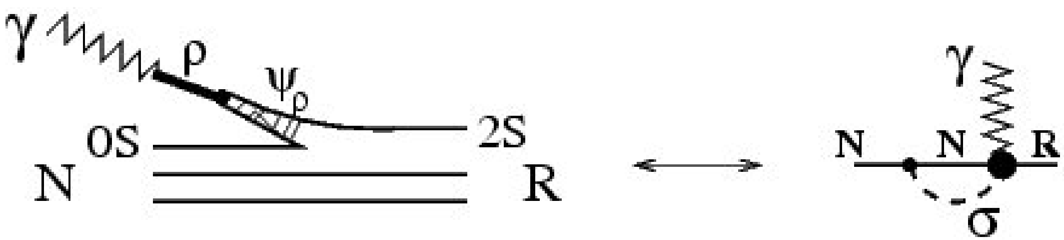
Both problems can be resolved in the framework of an extended version of the quark approach by taken into account the vacuum $q\bar{q}$ pairs and the meson-nucleon loops, but the most important for us would be to show that contributions of such extended components to the transition form factors are quickly dying out at large Q^2.

Possible solutions of the low-Q^2 problems

1) Generalization of CQM:
CQM+$^3P_0$+VMD

2) N+σ molecule:
The resonance pole 1365-i95 MeV
is rather close to the N+σ threshold

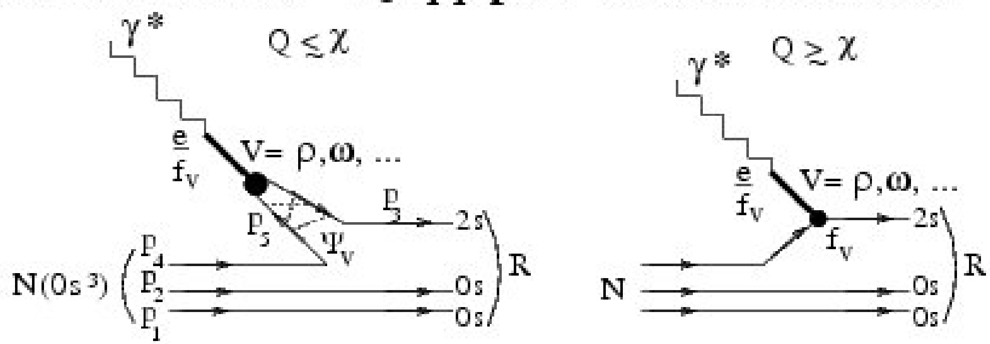




We considered the Roper resonance $R = N_{1/2^+}(1440)$ as a mixed state of the radially excited quark configuration $3q^* = sp^2[3]_X(L = 0)$ and the "hadron molecule" (a loosely bound state of nucleon and σ meson) $(N\sigma)_{mol} = |N + \sigma\rangle$

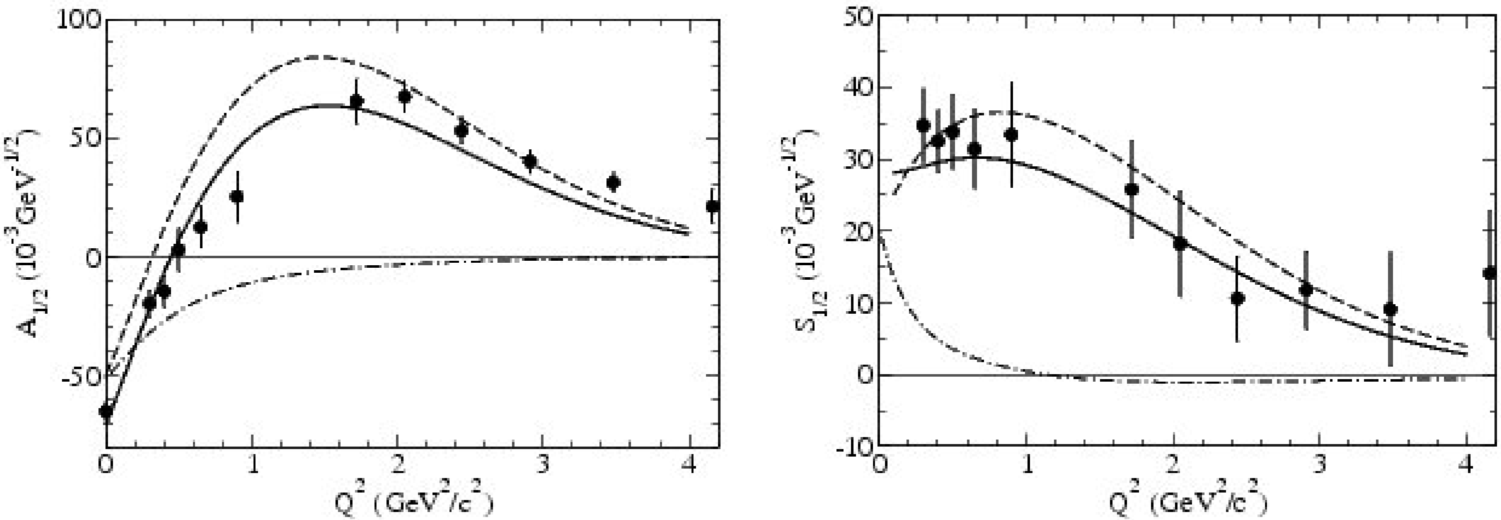
$$R = \cos\theta\,|3q^*\rangle + \sin\theta\,|N + \sigma\rangle$$

and used a nonzero quark radius of the vector ($V = \rho, ...$) meson b_V. Then, at low Q^2, a non-local γqq vertex could be derived on the basis of the VMD model and of the $^3P_0$ $q\bar{q}$ pair creation model:



Note that this implies the standard values of the adjusted parameters of the quark model (e.g. the size parameter of the nucleon quark core $b_N = 0.48\,fm$) and only the ratios of size parameters b_V/b_N =0.9 and b_R/b_N =0.944 were fitted to the data.

Then we found that at the value of $\cos\theta$ =0.7 - 0.8 this model correlates well with the recent CLAS data. (I.T.Obukhovsky *et al*, PRD 84, 014004)

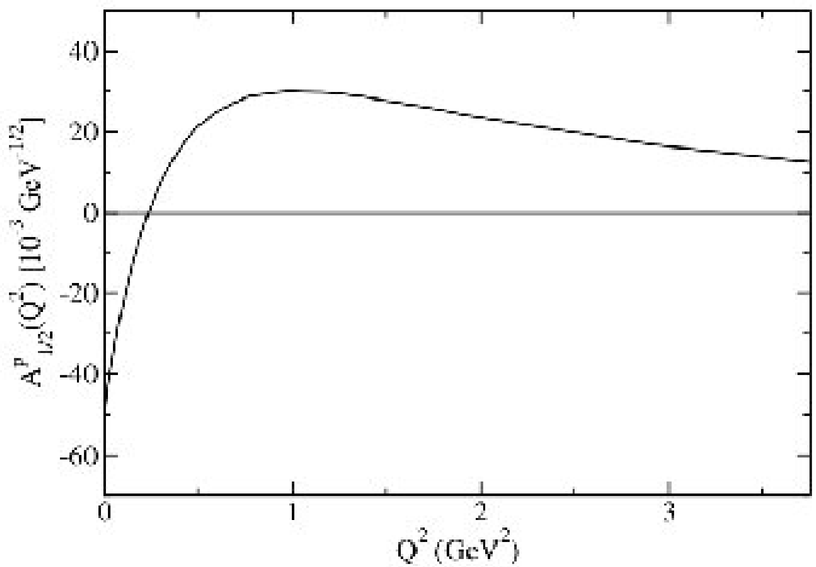


- - - - - **bar quark configuration** $sp^2[3]_X$

- · - · - · **hadron molecule** $N\sigma$

——— $cos\theta 3q^* + sin\theta N\sigma,\ \ cos\theta = 0.8$

It should be noted that in the case of the light-front quark wave functions the spin-flip amplitude $A_{1/2}$ does not vanish at $Q^2 \to 0$, since the Melosh rotation of quark spins in initial and final baryons prevents factorization of the spin part of w.f..



LF: S.Capstick, J.Phys. Conf. 69, 012016 (2007)
(on a large basis of the light-front quark configurations, N=6)

3. High Q^2 . Nucleon ground state at the light front.

At high $Q^2 \gtrsim$ 3-4 Gev^2 the contribution of soft components of the baryon (the meson cloud, "molecular" admixtures, etc.) to transition form factors falls off by comparison with the "quark core" contribution. Hence, only the quark contribution should be considered.

However, the form factors defined by a simple Gaussian form of the quark wave function with the size parameter b=0.5-0.6 fm is quickly dying out at $Q^2 \gtrsim$ 3-4 GeV^2.

A possible alternative to the Gaussian wave function are:
- a superposition of many Gaussians (as in the previous slide);
- a pole-like w.f.,
- a model with the running quark mass (as in the work of I.G.Aznauryan and V.D.Burkert, PRC 85, 055202), following the QCD predictions; etc.

We have chosen a pole-like form of the w.f.

Pole-like form of the nucleon ground state wave function Φ_{0S}

$$\Phi_{0S}(\xi,\eta,k_\perp,K_\perp) = \frac{\mathcal{N}_{0S}}{(1+\mathcal{M}_0^2/\beta^2)^\gamma}$$

$$\mathcal{M}_0^2 = \frac{M^2+k_\perp^2}{\eta\xi(1-\xi)} + \frac{\eta M^2+K_\perp^2}{\eta(1-\eta)}$$

was firstly fitted to the elastic nucleon form factors by Schlumpf (PRD47, 4114) with γ =3.5 and $\beta \approx$2M.

Here k, K – the relative moments in pairs "1-2" and "(12)-3" respectively, the light-front variables are $x_1 = \xi\eta$, $x_2 = (1-\xi)\eta$, $x_3 = 1-\eta$, and $\mathcal{M}_0$ is the mass of the free $3q$ system (M is the constituent quark mass).

Such form is as yet unjustified, but it is pertinent to note that the pole-like form of the $\bar{q}q$ w.f. of the pion

$$f_\pi \varphi_\pi(x, k_\perp^2) = \frac{9}{4\pi^2} \frac{1}{\left(1 + \frac{k_\perp^2}{4M^2 x(1-x)}\right)^\kappa}, \quad \kappa = 1$$

C.D. Roberts, arXiv:1509.02925 (there are also eval. κ =1 - 2) was reconstructed starting from the projection of the pion's Bethe-Salpeter wave function onto the light front (L.Chang *et al*, PRL110, 132001).

The pole-like w.f. $\Phi_{0S}(\xi, \eta, k_\perp, K_\perp)$ looks like a generalization of this expression to the $3q$ system, where each diquark subsystem has the same color charge as the antiquark, i.e. the color structure of each $q - (qq)$ pair is a counterpart of the $q - \bar{q}$ color wave function of the meson.

Starting from the "+" component of quark current on the light front

$$I^{(i)+} = e^{(i)}\left(I f_1 + i\,\hat{n}_z \cdot [\sigma^{(i)} \times q_\perp] f_2\right),$$

(without quark form factors, but with the anomalous quark magnetic moment $\varkappa_q$, i.e. $f_1 = 1$, $f_2 = \varkappa_q$),
we have fitted the free parameters of the pole-like w.f. Φ_{0S} to the modern data on nucleon form factors (we only preserved the characteristic values of the basic parameters, find by Schlumpf: $\gamma \approx$3.5 and $\beta \approx$2M) and with the constant value of the constituent (dressed) quark mass M.

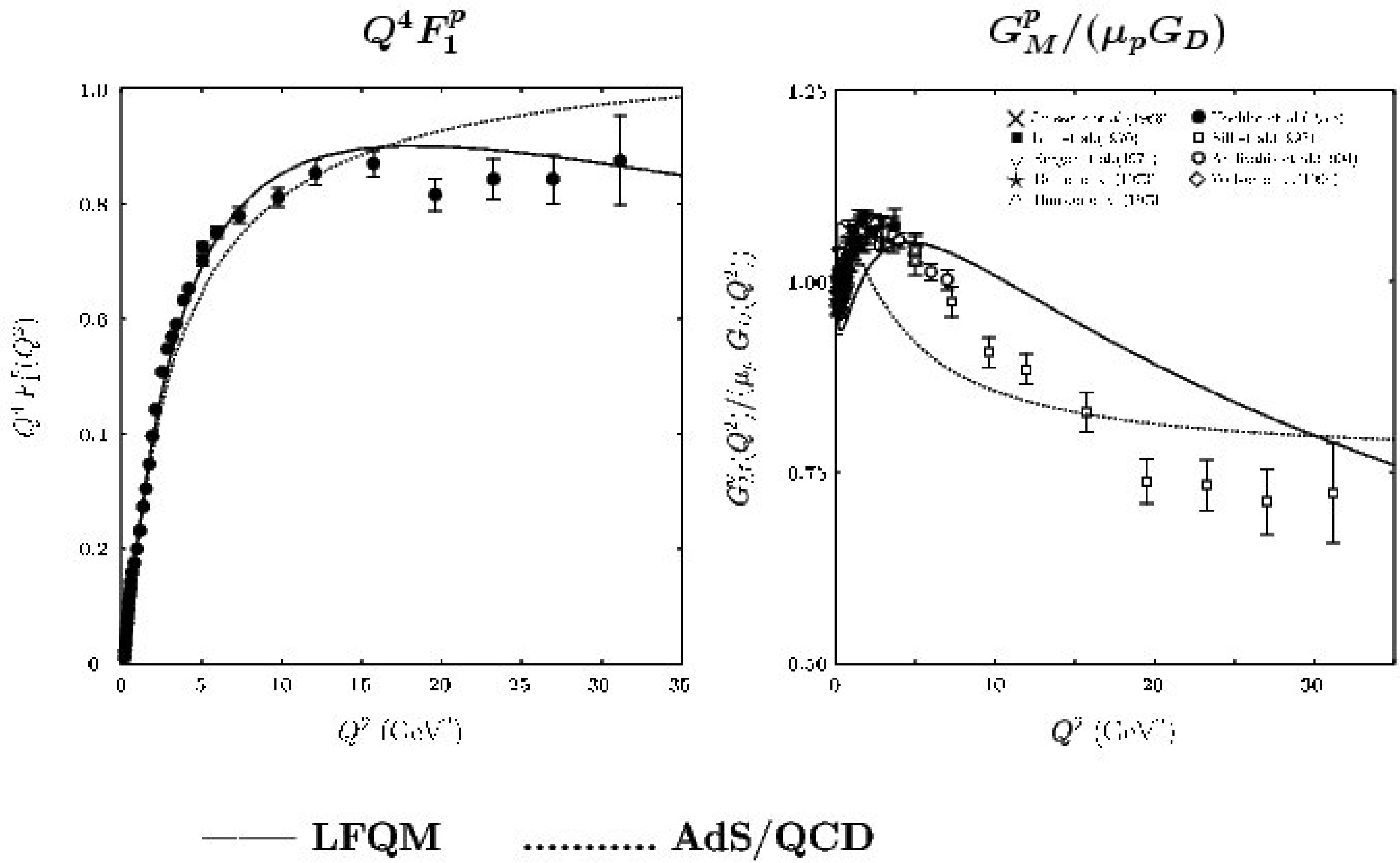
We used the data on elastic nucleon form factors within a full measured range 0$\leq Q^2 \leq$32 GeV^2 including the electron polarization data on the ratio G_E/G_M at $Q^2 \lesssim$6-8 GeV^2.

**With the values γ =3.51, M =0.251 GeV, $\varkappa_u$ =-0.0028, $\varkappa_u$ =0.0224, β_u =0.579(0.59) GeV, β_d =0.5(0.48) GeV
we have obtained a not so bad description of the data.**

**We also compared the obtained fit with the results of another relativistic approach, in which an effective light-front wave function was derived from the matching of soft-wall AdS/QCD and light-front QCD.
Th. Gutsche, V.E. Lyubovitskij, I. Schmidt, A. Vega, PRD 89, 054033**

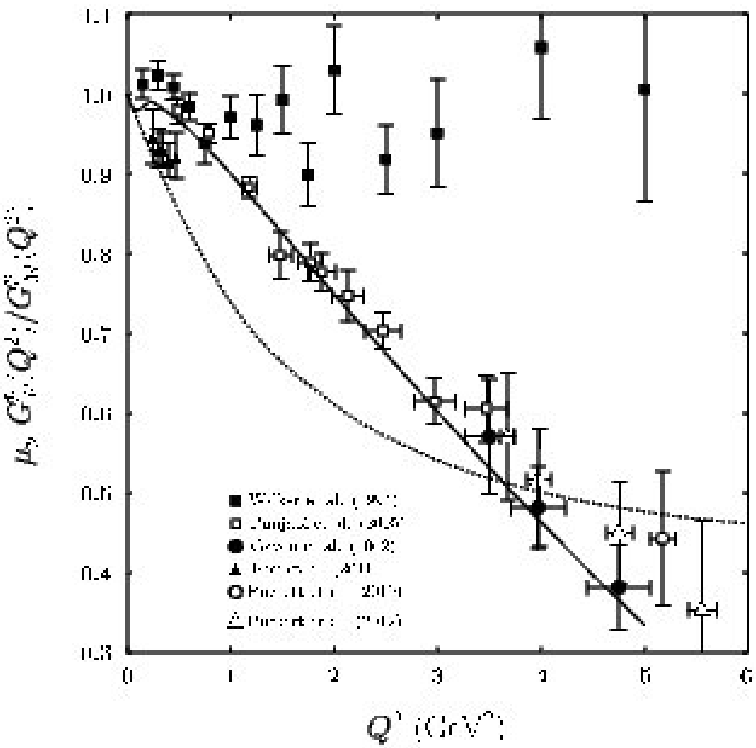
Таблица 1: Electromagnetic properties of nucleons in LF quark models (the results for Var2 are given in parenthesises in the second column)

Quantity	LFQM	AdS/QCD	Data
μ_p (in n.m.)	2.820 (2.820)	2.793	2.793
μ_n (in n.m.)	-1.920 (-1.920)	-1.913	-1.913
μ_u (in n.m.)	3.720 (-1.920)	3.673	1.673
μ_d (in n.m.)	-1.020 (-1.020)	-1.033	-2.033
r_E^p (fm)	0.871 (0.872)	0.789	0.8921 ± 0.0073
$\langle r_E^2 \rangle^n$ (fm^2)	-0.014 (-0.022)	-0.108	-0.1161 ± 0.0022
r_M^p (fm)	0.883 (0.872)	0.757	$0.777 \pm 0.013 \pm 0.010$
r_M^n (fm)	0.898 (0.893)	0.773	$0.862^{+0.009}_{-0.008}$
r_E^u (fm)	0.867 (0.866)	0.754	0.8589 ± 0.0107
r_E^d (fm)	0.855 (0.846)	0.638	0.7507 ± 0.0094
r_M^u (fm)	0.875 (0.832)	0.749	0.7288 ± 0.0151
r_M^d (fm)	0.938 (0.949)	0.815	1.0582 ± 0.0434

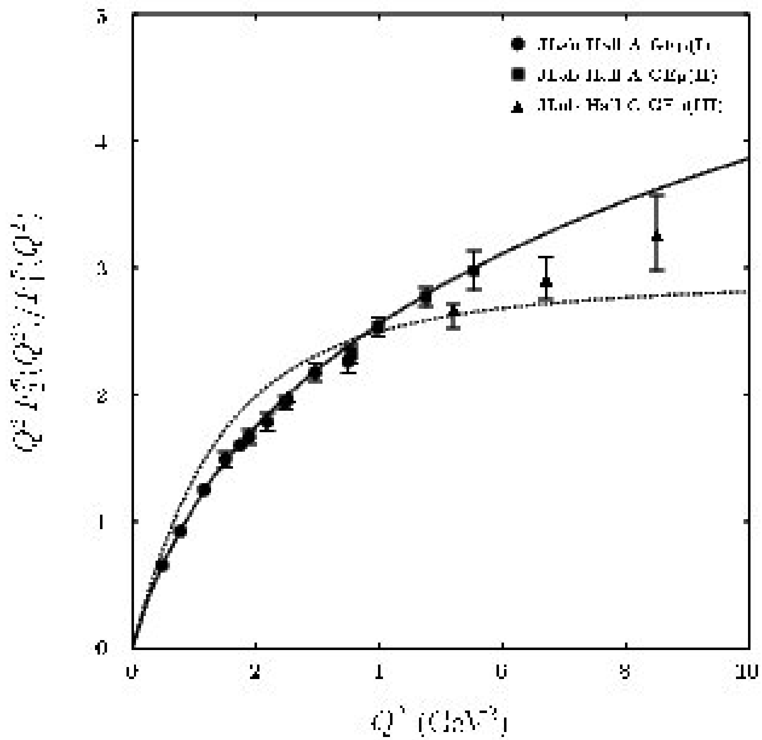


$Q^4 F_1^p$
$G_M^p/(\mu_p G_D)$
Q^2 (GeV^2)
Q^2 (GeV^2)
LFQM
AdS/QCD

$\mu_p G_E^p / G_M^p$



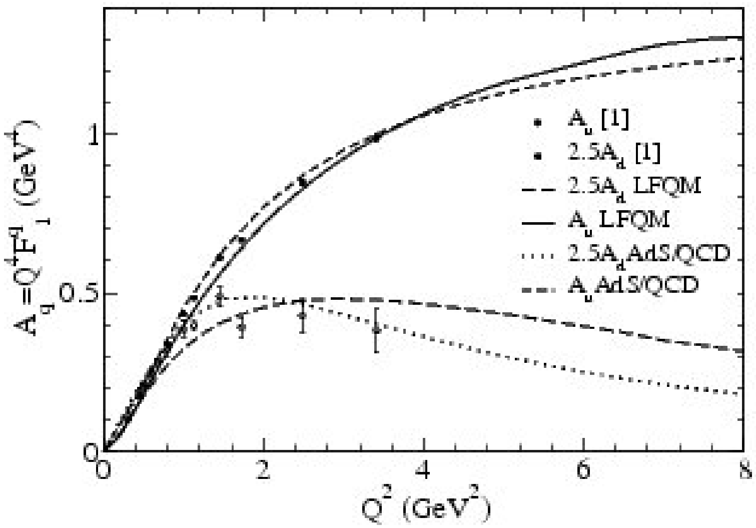
$Q^2 F_2^p / F_1^p$

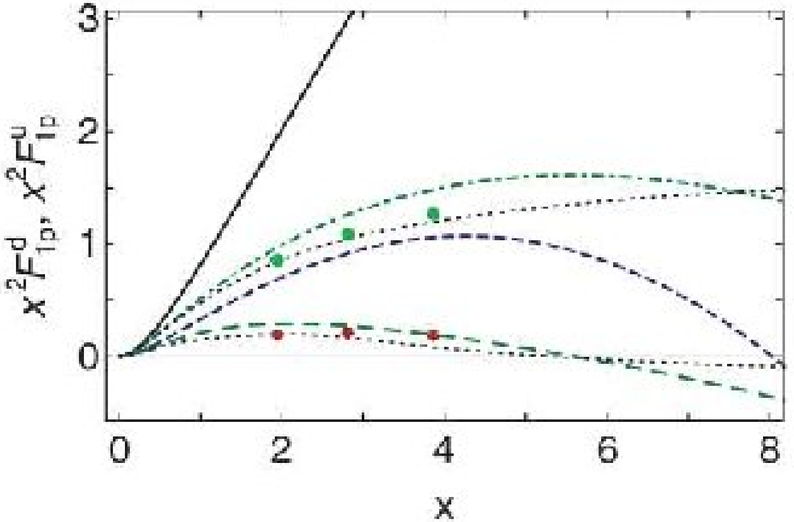


—— **LFQM** .... **AdS/QCD**

Flavor decomposition: $F_i^u = 2F_i^p + F_i^n$, $F_i^d = 2F_i^n + F_i^p$.

Data: G.D. Cates *et al*, PRL 106, 252003





$x = Q^2/m_N^2$

I.T.Obukhovsky *et al* GPG41,095005
LFQM and AdS/QCD

D.J.Wilson *et al*, PRC85, 025206
Bethe-Salpeter (q-$2q$)

4. High Q^2. Quark configurations and Melosh transformations.

Before passing to the Roper at high Q^2 some comment is well-timed:

A good basis of relativistic quark configurations possessing definite values of angular momentum and satisfying the Pauli exclusion principle is needed now for model calculation (evaluation) electrocouplings of baryon resonances with $J^P = 1/2^{\pm}, 3/2^{\pm}, 5/2^{\pm},$ etc.

As our experience shows such basis can be constructed starting from the nonrelativistic shell-model configurations by changing the harmonic oscilator wave functions for the light-front analogous w.f. (Gaussian or pole-like) dependent on the relativistic relative moments k, K and expressed by the light-front invariants $\xi, \eta, \mathcal{M}_0, \mathcal{M}^{ij_0}$.

The problem is only one of coupling spin and orbital moments.
However, if the Bethe-Salpeter or DSE wave function are known, they can be readily projected onto the light front.

But in common case, when there are no such wave functions, one should start from the canonical states $|p_i; s_i\mu_i\rangle_c$ in the abstract Hilbert space, where are formally defined Lorentz boosts $U(\lambda(p'_i \leftarrow p_i)) = exp\{i\vec{K}\cdot\hat{n}\chi\}$, e.g. by the rule

$$|p_i; s_i\mu_i\rangle_c \rightarrow U(\lambda(p'_i \leftarrow p_i))\,|p_i; s_i\mu_i\rangle_c$$

$$= \sum_{\mu'_i} |p'_i; s_i\mu'_i\rangle_c \; D^{s_i}_{\mu'_i\mu_i}(\underbrace{R[\lambda(\mathring{p}_i \leftarrow p'_i)\lambda(p'_i \leftarrow p_i)\lambda(p_i \leftarrow \mathring{p}_i)]}_{Wigner\ rotation})$$

Here $p'_i = \lambda(p'_i \leftarrow p_i)p_i$ is the rotationless Lorentz transformation, $\mathring{p}_i = \{M, 0\}$ is the 4-momentum of particle in its rest frame, $e^\chi = (E + |\vec{P}|)/\mathcal{M}_0$ is the boost parameter ($\hat{n}$ is the direction), and $\vec{K}$ is three generators of boost in the Hilbert space (they are only known for free particles).

The canonical spin state is defined by the rotationless Lorentz boost

$$|p_i; s_i\mu_i\rangle_c = U(\lambda(p_i \leftarrow \overset{\circ}{p}_i))\,|\,\overset{\circ}{p}_i; s_i\mu_i\rangle\ ,$$

while the light-front spin state is defined by another type of Lorentz transformation, which leads to the same momentum p_i, but would be represented as a two-step process:

$l(p_i \leftarrow \overset{\circ}{p}_i) = \lambda(p_i \leftarrow p_\infty)\lambda(p_\infty \leftarrow \overset{\circ}{p}_i)$**, where p_∞ is the quark momentum at the infinite momentum frame.**

$$|p_i; s_i\mu_i\rangle_f = U(l(p_i \leftarrow \overset{\circ}{p}_i))\,|\,\overset{\circ}{p}_i; s_i\mu_i\rangle\ ,$$

The full manifold of l's forms a subgroup of the Lorentz (Poincare) group. It is a kinematic subgroup for the light-front dynamics — an analog of the rotational subgroup for the instant form of dynamics.

As a result, canonical and front states are related by a specific rotation

$$|p_i; s_i\mu_i\rangle_c = \sum_{\mu_i'} |p_i; s_i\mu_i'\rangle_f \, D^{s_i}_{\mu_i'\mu_i}(\underbrace{R[\lambda(\mathring{p}_i \leftarrow p_i \lambda(p_i \leftarrow p_\infty)\lambda(p_\infty \leftarrow \mathring{p}_i)]}_{Melosh\ rotation})$$

which is known as the Melosh transformation. For the quark ($s_i = 1/2$) one can write:

$$D^{\frac{1}{2}}_{\mu_i'\mu_i}(\theta_M) = \frac{M + p_{iz} + i\hat{n}_z \cdot [\vec{\sigma}_i \times \vec{p}_{i\perp}]}{\sqrt{(M + p_{iz})^2 + p_{i\perp}^2}}$$

The Lorentz transformation $l(p_i' \leftarrow p_i)$ does not rotate the front states

$$|p_i; s_i\mu_i\rangle_f \rightarrow U(l(p_i' \leftarrow p_i))\,|p_i; s_i\mu_i\rangle_f = |p_i'; s_i\mu_i\rangle_f$$

that is very convenient to use in moving reference frames.

But the front states are not convenient to use in construction of rotationally covariant states. Such states can be only constructed in terms of canonical states.

To construct the rotationally covariant basis for 3q configurations $|\overset{\circ}{P}, J(LS)M\rangle$ with definite values of total (JM), orbital (L) and spin (S) angular moments in the proper rest frame ($\overset{\circ}{P}= \{\mathcal{M}_0, 0\}$) one should

- start from the canonical quark states $|p_1\mu_1\rangle_c|p_2\mu_2\rangle_c|p_3\mu_3\rangle_c$,
- introduce the relative moments and
- project this canonical state onto the rotationally covariant $J(LS)M$ states by the methods developed for the relativistic Hilbert space on the basis of the standart angular momentum theory.

Such states should possess the definite permutation symmetry (i.e. the definite Yang scheme $[f^c]_s$ and the Yamanouchi symbol y^c_s) in the canonical spin space (s). For example, in the simplest case of $L = 0$ $(J = S)$ the spin state is

$$|\overset{\circ}{P}, S(S_{12})M\rangle = \sum_{\mu} (\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | S_{12}\mu_1)(S_{12}\mu_{12}\frac{1}{2}\mu_3|SM)|p_1\mu_1\rangle_c|p_2\mu_2\rangle_c|p_3\mu_3\rangle_c$$

where the Clebsch-Gordon coefficients provide the definite value of $[f^c]_s y^c_s$ dependent on the values of S and S_{12}. But on the light front this state will be modified by Melosh transformation

$$|\overset{\circ}{P}, S(S_{12})M\rangle = \sum_{\mu} (\frac{1}{2}\mu_1 \frac{1}{2}\mu_2 | S_{12}\mu_1)(S_{12}\mu_{12}\frac{1}{2}\mu_3|SM) \sum_{\mu'_1} D^{\frac{1}{2}}_{\mu'_1\mu_1}(\theta_1)|p_1\mu_1\rangle_f$$

$$\times \sum_{\mu'_2} D^{\frac{1}{2}}_{\mu'_2\mu_2}(\theta_2)|p_2\mu_2\rangle_f \sum_{\mu'_3} D^{\frac{1}{2}}_{\mu'_3\mu_3}(\theta_3)|p_3\mu_3\rangle_f$$

$\theta_1 \neq \theta_2 \neq \theta_3$, and thus in the front spin space there will be other values of $[f^f]_s y^f_s$ different from the initial value of $[f^c]_s y^c_s$.

Nevertheless, the exclusion Pauli principle is not violated for the full wave function, though the calculation technique (fraction parentage coefficients, etc.) becomes more cumbersome, especially in the case of nonzero orbital momentum L.

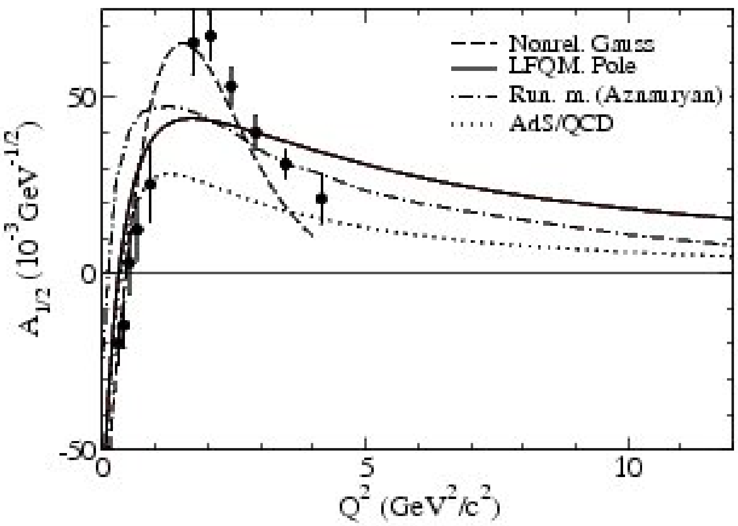
Light-front configuration $sp^2[3]_X L = 0$

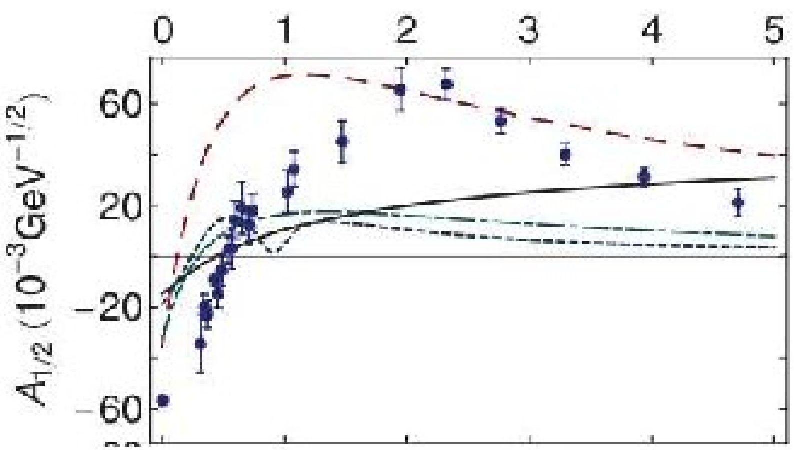
The quark wave function of the Roper resonance corresponds to the lowest radial excitation with $L = 0$. We used the light-front analogue of the radially excited quark configuration for the case a pole-like ground state Φ_{0S}:

$$\Phi_{2S} = \mathcal{N}_{2S}\left(1 - c_R\frac{\mathcal{M}_0^2}{\beta^2}\right)\Phi_{0S}$$

The parameter c_R is defined by the orthogonality condition $\langle\Phi_{2S}|\Phi_{0S}\rangle = 0$

Transverse helicity amplitude $A_{1/2}$ of the Roper resonance electroproduction at high Q^2 in different approaches.





Tuebingen and JLab groups

Adapted from D.J.Wilson *et al*, PRC 85, 025205

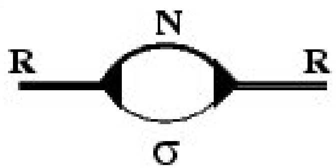
5. Summary.

- The pole-like light-front nucleon wave function provides a good description of the elastic form factors in a large interval $0 \leq Q^2 \leq 30$ GeV^2, where the data demonstrate asymptotic behavior predicted by pQCD, $F_1^p \sim Q^{-4}$ and $F_2^p \sim Q^{-6}$.
- However, the transition form factors for the Roper resonance, calculated with the same basis of wave functions, considerably differ from the data demostrating the more gentle slope at $Q^2 \gtrsim$1-2 GeV^2
- Comparison with the results another approaches have shown that independently of the concrete approach (holograph QCD, DSE, light front with the running mass or with the pole-like wave function) the results demonstrate qualitatively similar behavior, if the adjusted parameters of the model are fitted to the data on nucleon form factors in a full interval $0 \leq Q^2 \leq 30$ GeV^2. But absolut values of electrocouplings for different models could be different in 30% - 60%.

We consider the Roper resonance as a superposition of the radially excited three-quark configuration $|3q^*\rangle = |sp^2[3]_X\, L{=}0\rangle$ and the hadron molecule component $N+\sigma$

$$|R\rangle = \cos\theta|3q^*\rangle + \sin\theta|N+\sigma\rangle$$

The parameter θ is adjusted to optimize the description of the helicity amplitude $A_{1/2}$ only.

The hadron loop 

$\Sigma_{N\sigma}$ gives a negative contribution to the mass of the Roper resonance, and the $RM\sigma$ coupling constant $g_{_{RN\sigma}}$ is defined by the 'compositeness condition'

$$Z_R \equiv 1 - \frac{d}{d\not{p}}\Sigma_{N\sigma}(\not{p})|_{\not{p}=m_{_R}} = 0,$$

i.e. the elementary particle R has a zero weight in the hadron molecule.

We use effective Lagrangians (Dubna group: G. Efimov, M. Ivanov, V. Lyubovitskij) for description of nonlocal RNσ and NNσ interactions, e.g.

$$\mathcal{L}_{str}(x) = g_{RN\sigma}\bar{R}(x)\int d^4y\,\Phi_R(y^2)N(x+\alpha y)\sigma(x-(1-\alpha)y), \quad \alpha = \frac{M_\sigma}{m_N + M_\sigma},$$

and h.o. Gaussians as Fourier transforms of $\Phi_N(y^2)$ and $\Phi_R(y^2)$

$$\tilde{\Phi}_N(k_E^2) = exp(-\frac{k_E^2}{\Lambda^2}) \quad \text{and} \quad \tilde{\Phi}_R(k_E^2) = (1-\lambda\frac{k_E^2}{\Lambda^2})exp(-\frac{k_E^2}{\Lambda^2})$$

with the orthogonality condition $\int\tilde{\Phi}_R(^2k_E)\tilde{\Phi}_N(k_E^2)d^4k_E = 0$.

The electromagnetic interaction term for this nonlocal vertex

$$\mathcal{L}^{(1)}_{em} = g_{_{RN\sigma}} \bar{R}(x) \int dy\, e^{-ieI(x+\alpha y, x, P)} N(x+\alpha y)\sigma(x-(1-\alpha)y) + h.c.$$

is generated when the nonlocal Lagrangian are gauged with a gauge field exponential $e^{-ieI(x+\alpha y, x, P)}$ where

$$I(y, x, P) = \int_x^y dz_\mu A^\mu(z), \quad P \text{ is the path of integration}$$

S.Mandelstam, Ann.Phys. 19, 1 (1963); J.Terning, Ph.Rev. D44, 887 (1991)

The full Lagrangian of electromagnetic interaction

$$\mathcal{L}_{em} = \mathcal{L}_{em}^{(1)} + \mathcal{L}_{em}^{(2)}$$

includes also the standard term

$$\mathcal{L}_{em}^{(2)} = e_{_B}\bar{B}(x)\not{A}(x)B(x), \quad B = N, R$$

obtained by minimal substitution $\partial^\mu B \to (\partial^\mu - e_{_B}A^\mu)B$

Only the total sum of the first order diagrams (including the contact terms $\mathcal{L}_{em}^{(1)}$) satisfies the gauge invariance