From Resonance Extraction to LQCD

and

N* Excitations of Neutron

T.-S. Harry Lee

Argonne National Laboratory

Extraction of Nucleon resonances

Why ?

- N^* are unstable and coupled with meson-baryon continuum to form nucleon resonances

↓

- N^* can only be studied by extracting resonances from the data of meson production reactions:

$\pi N, \gamma N \longrightarrow N^* \longrightarrow \pi N, \eta N, KY, \pi \pi N(\pi\Delta, \rho N, \sigma N)$....

Theoretical formulation of Resonances:

(Gamow, Peierls, Dalitz, Moorhouse, Bohm....)

Resonances are the eigenstates of the Hamiltonian of the underlying fundamental theory with outgoing-wave boundary condition :

$$H\,|\psi_R> = E_R|\psi_R>; \qquad \psi_R(r) \xrightarrow[r \to \infty]{} \frac{e^{ikr}}{r}$$

$$M_R - i\,M_I = [m^2_M + k^2]^{1/2} + [m^2_B + k^2]^{1/2}$$

→

Scattering amplitude:

$$T(E \to E_R) \longrightarrow \frac{\Gamma_R}{E - E_R}$$

Pole on complex-E-plane

Procedures of Resonance Extraction

1. Determine partial-wave amplitudes (PWA) from the available data

2. Extract resonances by analytic continuation of PWA to complex-E plane

Determination of PWA

Theorem :

PWA can be determined up to an overall phase from data of all observables from complete experiment

Ideal situation: Perform complete experiments

A complete measurement of γ N -> π N, KΛ :

$d\sigma/d\Omega$, T, P, Σ (un-polarized γ)

$O_{x'}$, $O_{z'}$ (linear-polarized γ)

$C_{x'}$, $C_{z'}$ (circular-polarized γ)

at all angles at each energy

Reality :

1. Data are incomplete

2. Even data are complete, many solutions are possible in determining PWA

Intrinsic difficulty:

bi-linear relations: $d\sigma/d\Omega = |f^R(\Theta) + i\ f^I(\Theta)|^2$

Example:

Study CLAS data of 8 observables of $\gamma p \rightarrow K \Lambda$

Treat E_{L+}, E_{L-}, M_{L+}, M_{L-} as parameters to fit the data by Monte-Carlo sampling

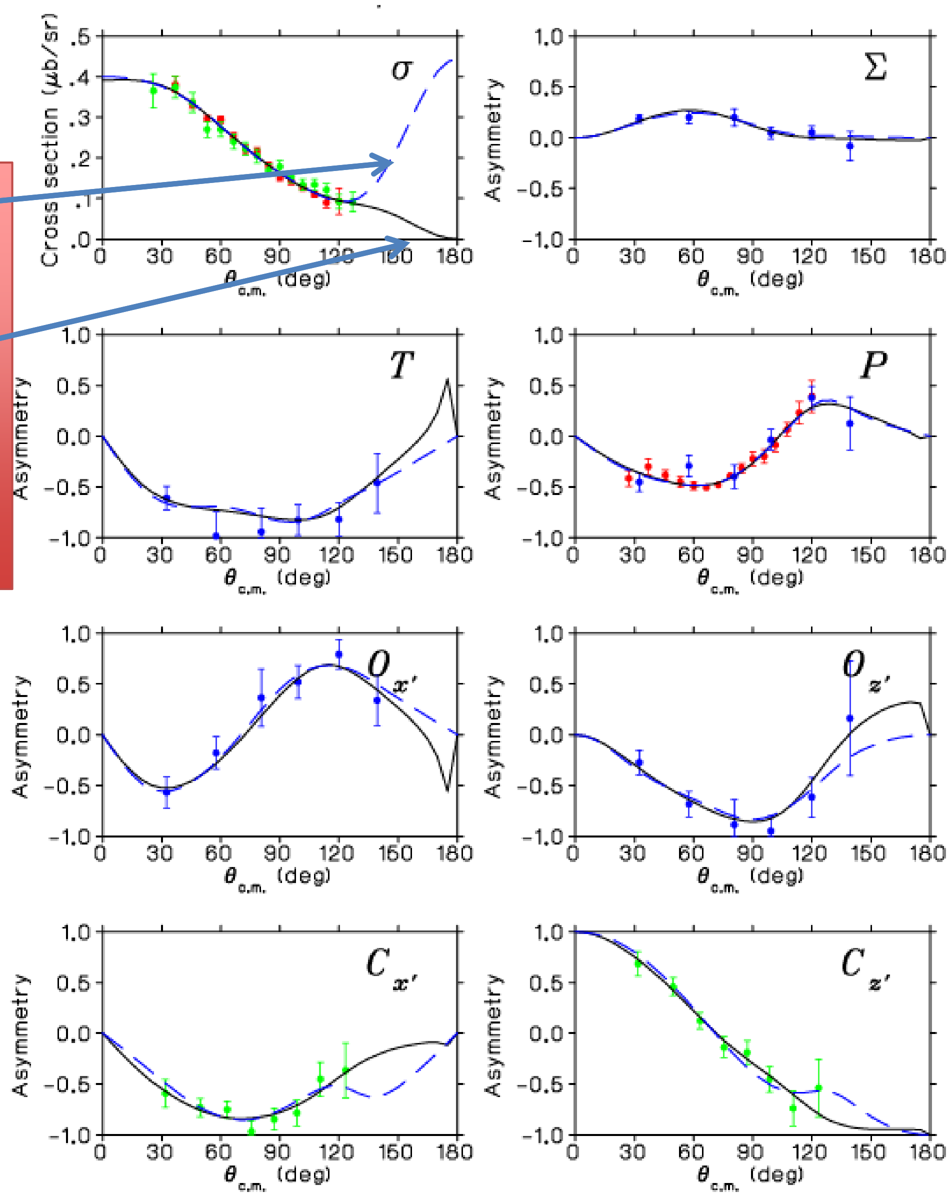
Sandorfi, Hoblit, Kamano, Lee, J. of Phys. G38, 053001 (2011)

Monte-Carlo Fits (Sandorfi, Hobit, Kamano, Lee, J. of Phys. (2011)

χ^2 /data= 0.62

X^2 /data = 0.59

Many solutions
in determining PWA

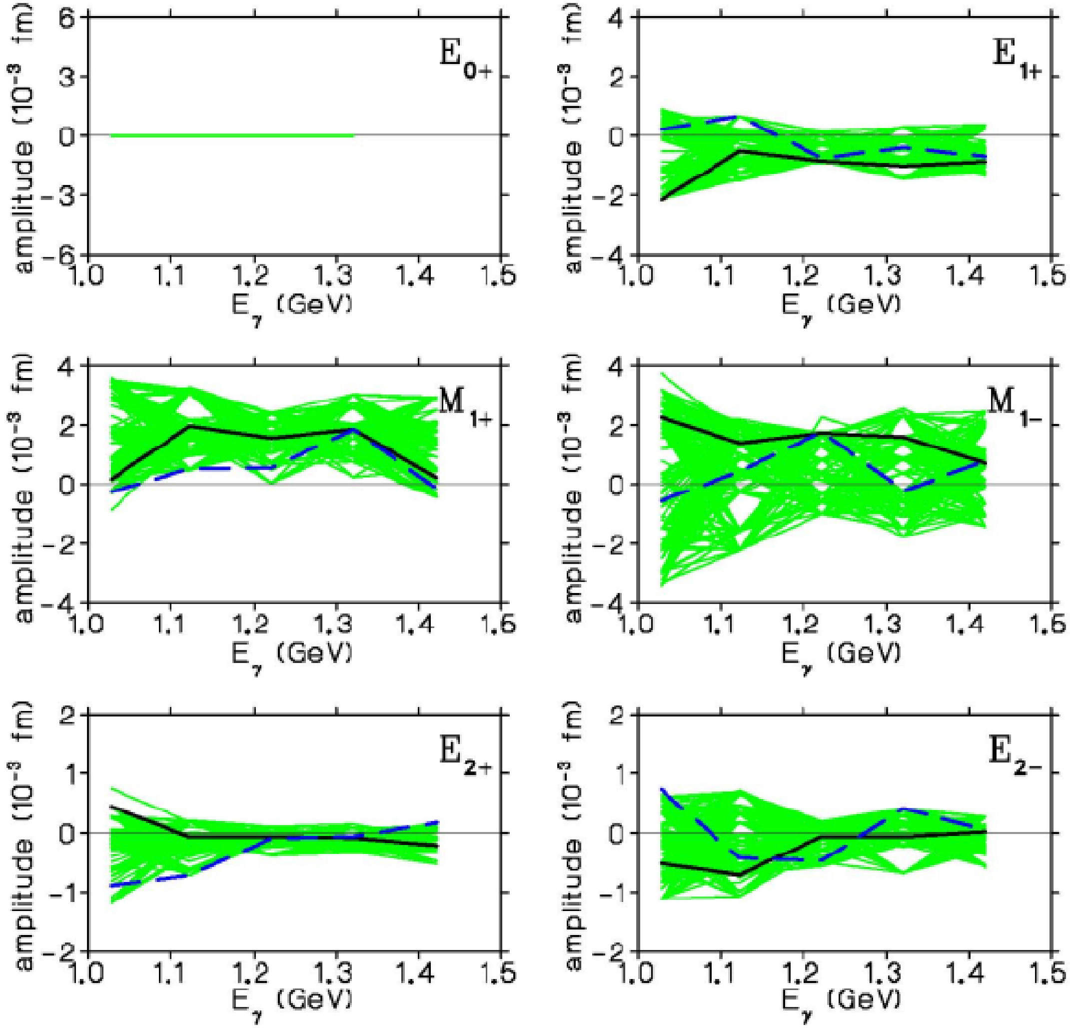


CLAS data of
γ p -> K Λ

Impose constraint

Set $E_{0+} = 0$

PWA with L > 3 are fixed by Born terms

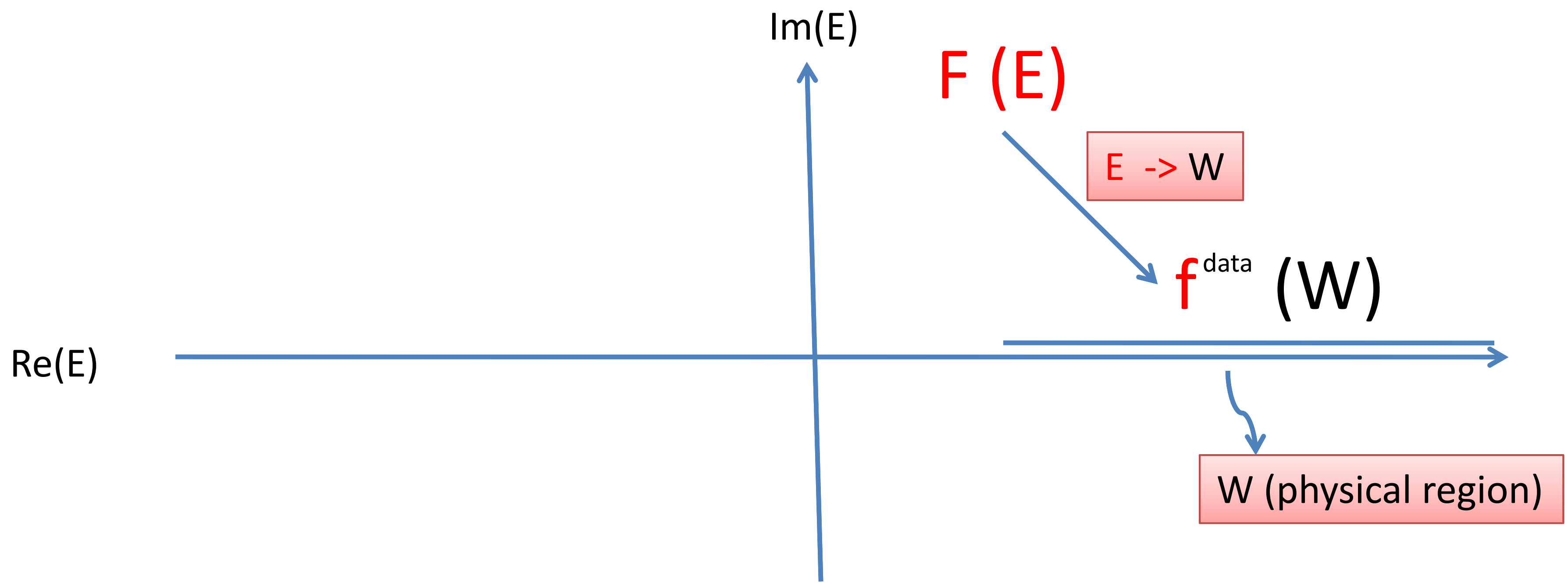


Need theoretical constraints in determining PWA from data

Next Step:

- Use analytic functions F(E) to fit the determined PWA $f^{data}(W)$ in the E = W physical region.

- Extract resonance poles and residues from F (E)



Important condition:

The extracted resonance parameters should be independent of the parameterization of F(E)

To check this condition, study $\pi\pi \to \pi\pi$, $K\bar{K}$ reactions with three models for F(E)

Wu, Lee (2014)

PWA of a dynamical model of $\pi\pi \to \pi\pi, K\bar{K}$

$$T_{i,j}(E) = h_i(k_i)\tau(E)h_j(k_j) + \frac{g_i(k_i)g_j(k_j)}{E - m_0 - \Sigma(E)}$$

$\tau(E)$, $\Sigma(E)$: determined by $h_i(k_i)$, $g_i(k_i)$

I, j = $\pi\pi$, K K

Parameterizations of H'

Model A

$$g(k) = 1/(1+(ck)^2)$$
$$h(k) = 1/(1+(dk)^2)^2$$

Model B

$$g(k) = 1/(1+(ck)^2)^2$$
$$h(k) = 1/(1+(dk)^2)^4$$

Model C

$$g(k) = \exp(-(ck)^2)$$
$$h(k) = \exp(-(dk)^2)$$

Adjust parameters c, d, and m_0

→

PWA from models A, B and C agree within 1%

→

Compare the extracted poles and residues of resonances

PWA from models A, B, and C

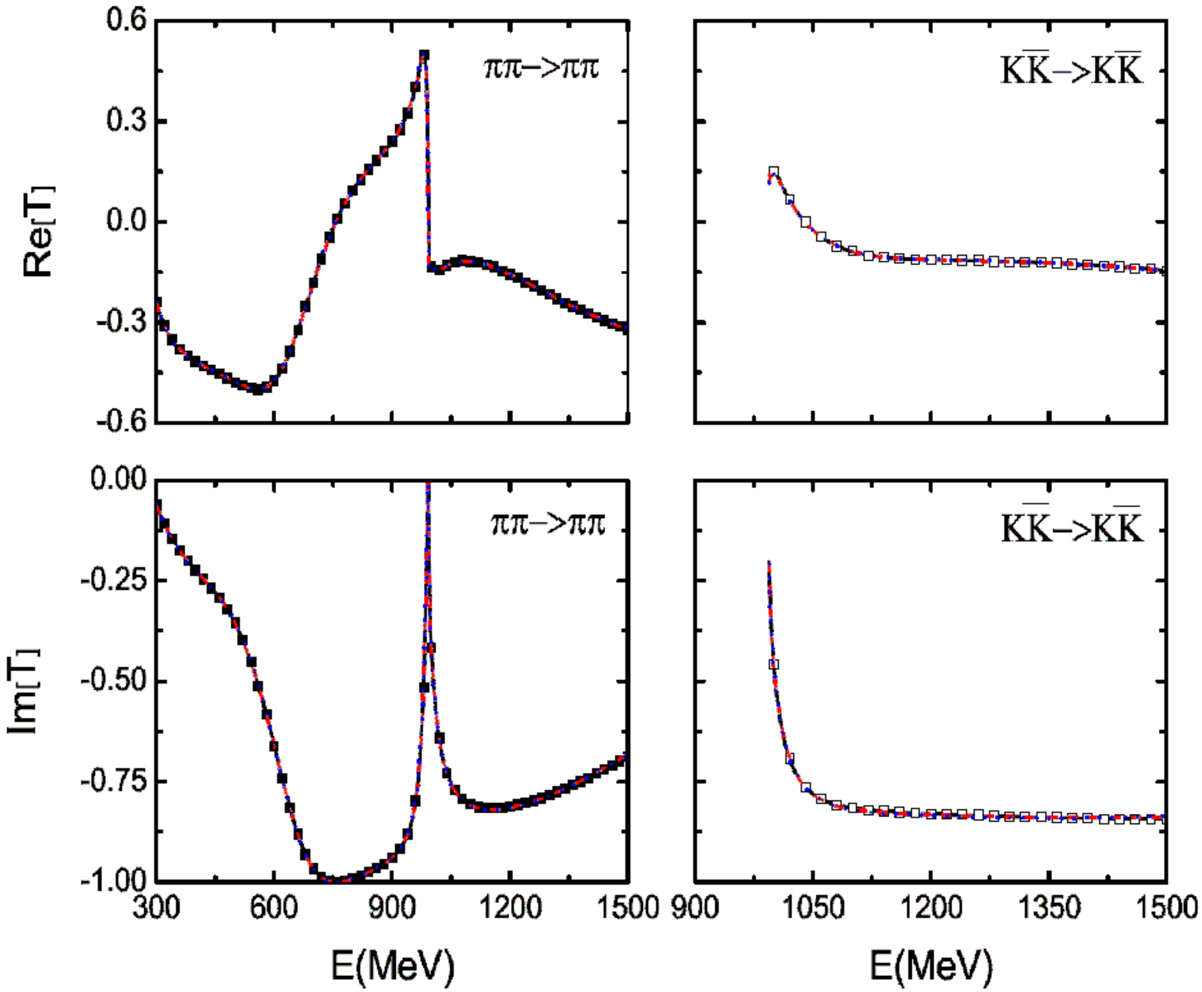


TABLE II: The pole positions and residue of Models I-A, I-B, I-C.

Model	Pole Position(MeV)	Residue of $\pi\pi$	Residue of $K\bar{K}$
II sheet-1		$\times 10^{-4}$	
I-A	$639.3 - i158.9$	$5.295 - i2.153$	–
I-B	$637.8 - i159.9$	$5.368 - i2.285$	–
I-C	$634.5 - i156.2$	$5.076 - i2.556$	–
II sheet-2		$\times 10^{-5}$	$\times 10^{-5}$
I-A	$1000.30 - i8.89$	$-3.514 - i3.088$	$1.822 + i33.81$
I-B	$1000.14 - i8.88$	$-3.493 - i3.111$	$2.140 + i34.62$
I-C	$1000.04 - i8.83$	$-3.467 - i3.162$	$2.955 + i35.39$

Poles

Residues

Models A, B, and C agree well

Finding:

If PWA have no error and are fitted perfectly

The extracted resonance parameters
are independent of the parameterization of F(E)

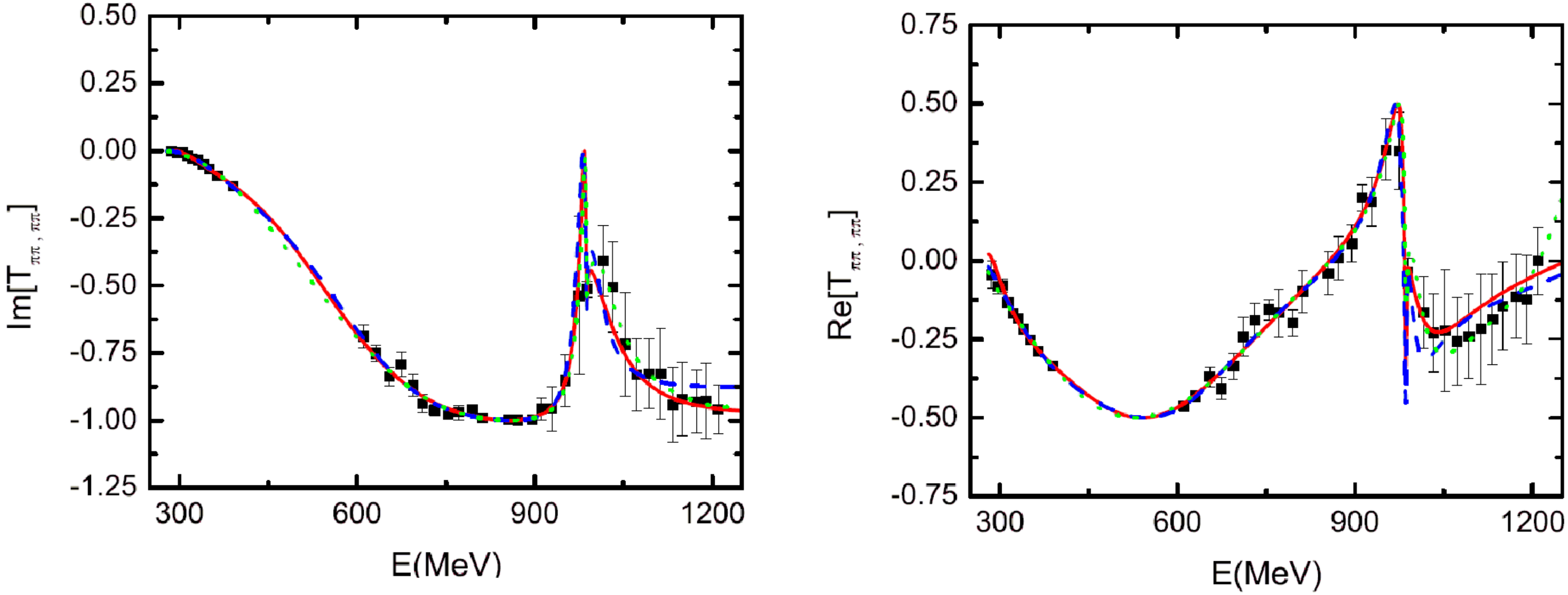
Reality :

- Determined ππ -> ππ PWA have errors

- $K\bar{K}$ -> $K\bar{K}$ PWA are not available

Finding :
The extracted resonance parameters depend on the parameterization of amplitudes

Fits of the current ππ -> ππ data



Models A, B, and C get equally good fits

TABLE VI: The pole positions and residue of Models II-A, II-B, II-C.

Model	χ^2	Pole Position(MeV)	Residue of $\pi\pi$	Residue of $K\bar{K}$
II sheet-1			$\times 10^{-4}$	
II-A	40	$523.7 - i264.6$	$10.78 - i9.323$	–
II-B	36	$597.0 - i217.1$	$6.157 - i3.573$	–
II-C	43	$672.3 - i292.0$	$5.753 + i2.102$	–
II sheet-2			$\times 10^{-5}$	$\times 10^{-4}$
II-A		$992.7 - i9.73$	$-6.356 - i3.709$	$-10.83 + i0.3889$
II-B		$986.6 - i15.25$	$-6.284 - i1.020$	$4.588 + i7.788$
II-C		$998.5 - i11.21$	$-8.870 - i0.9770$	$-15.51 + i2.208$

Poles

Residues

Do not agree well

Reliable amplitude determinations
and resonance extractions (poles, residues)
must Include theoretical constraints

ANL-Osaka approach:
Implement meson-exchange mechanisms within
Hamiltonian Formulation of reactions

Outcome of ANL-Osaka analysis:

a. PWA of πN, γN -> πN, ηN, KY, $\pi \pi N(\pi\Delta, \rho N, \sigma N)$

b. Poles and N-N* form factors of N*up to W=2 GeV

Will be reported by Hiroyuki Kamano

Outcome of ANL-Osaka analysis:

c. A determined Hamiltonian:

1. Provide interpretations of N*:

Example:

Q^2-dependence of meson cloud effects

within Constituent Quark model

Dyson-Schwinger model

………

2.New direction : generate data to test LQCD

ANL-Osaka Hamiltonian

Finite-Volume Hamiltonian Method of Adelaide

Predict spectrum in a finite volume

Test spectrum from LQCD

Adelaide's Finite-Volume Hamiltonian Method

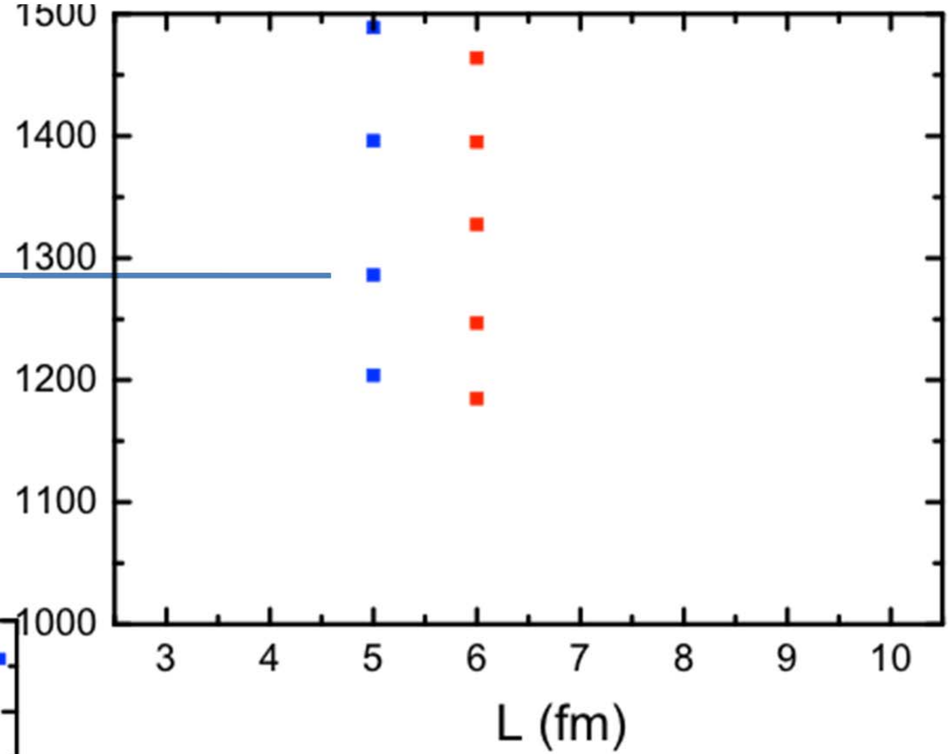
J. M. M. Hall, A. C.-P. Hsu, D. B. Leinweber, A. W. Thomas and R. D. Young, Phys. Rev.D 87, 094510 (2013)

- a method to relate the Experimental data to LQCD

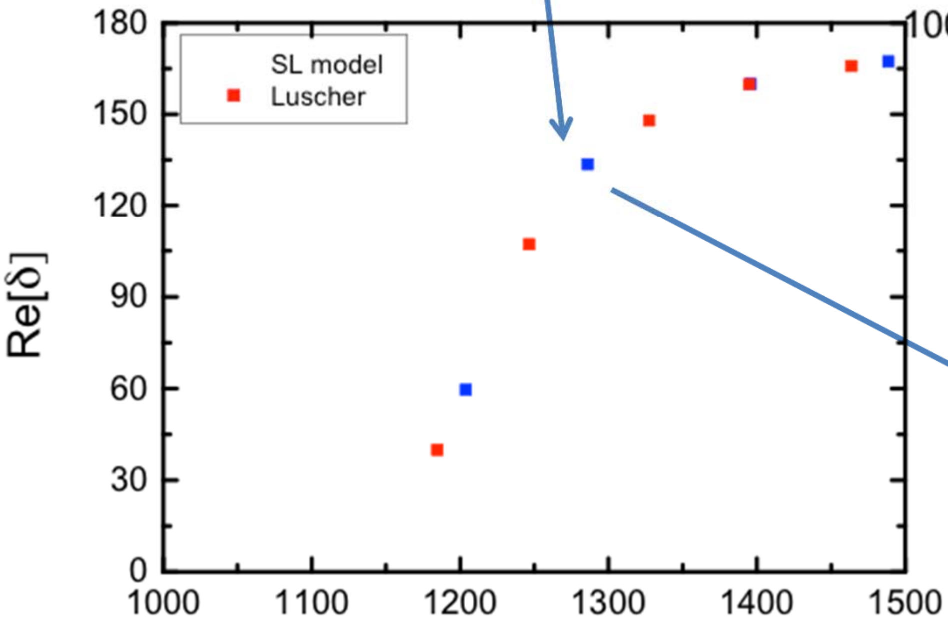
- In one-channel case, it is equivalent to the approach based on Lüscher's formula

Current approach

Lüscher relation



Spectrum from LQCD with L=5,6 fm



Compare with data

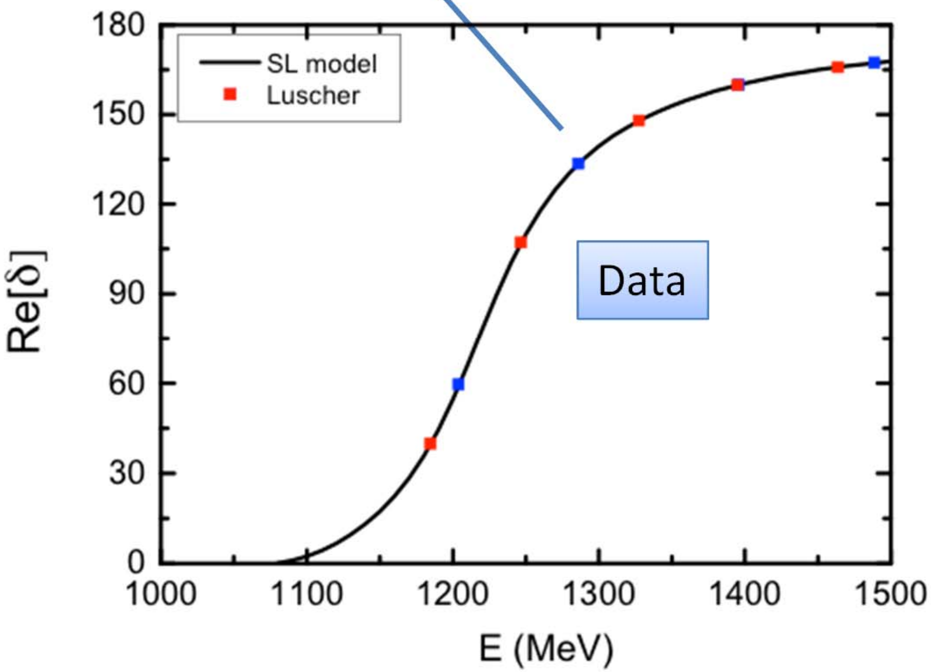
Test LQCD with L=5,6 fm

Finite-Volume Hamiltonian approach

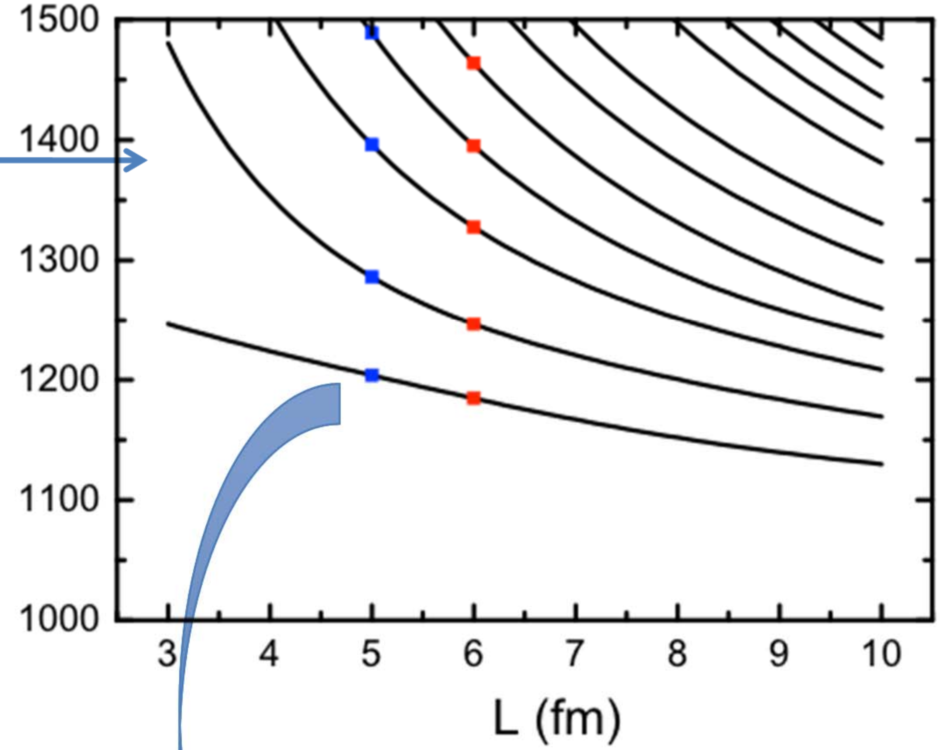
$\mathrm{Det}[H_0+H_I-EI]=0$

In finite volume

$H = H_0+H_I$



Data



Spectrum in finite volume

Test LQCD with L=5 fm

Necessary step to test N* from LQCD :

Extend Adelaide's Finite-Volume Method to multi-channel

Example:

$P_{11}(1440)$: πN, ππN(πΔ, ρN,σN)

J. Wu, T.-S. H. Lee, A. W. Thomas, R.D. Young, Phy. Rev. C90, 055209 (2014)

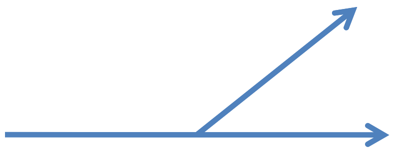
Testing case : ππ, KK scattering

Model Hamiltonian with ππ, KK, σ

$H = H_0 + H_I$

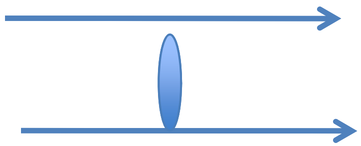
$H_I = G + V$

G: σ



ππ, KK

V: ππ, KK



ππ, KK

Finite-Volume with size L

k : $k_i = n_i \left[\frac{2\pi}{L}\right]^{3/2}$

G : $g_a(k_i) = G_a(k_i) \left[\frac{2\pi}{L}\right]^{3/2}$

V : $v_{ab}(k_i,k_j) \left[\frac{2\pi}{L}\right]^{3}$

$\rightarrow$

$$
[\, H_0 \,] = \begin{pmatrix}
m_0 & 0 & 0 & 0 & 0 & \ldots \\
0 & 2E_a(k_1) & 0 & 0 & 0 & \ldots \\
0 & 0 & 2E_b(k_1) & 0 & 0 & \ldots \\
0 & 0 & 0 & 2E_a(k_2) & 0 & \ldots \\
0 & 0 & 0 & 0 & 2E_b(k_2) & \ldots \\
. & . & . & . & . & .... \\
. & . & . & . & . &
\end{pmatrix}
$$

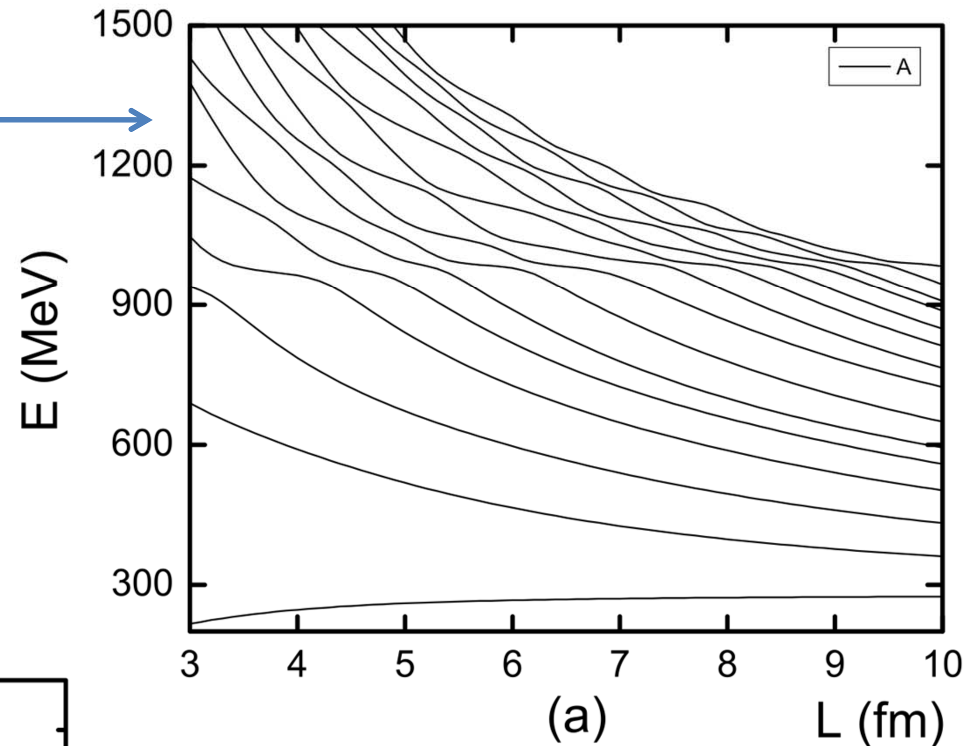
a,b =ππ, KK

$$
[\, H_I \,] = \begin{pmatrix}
0 & g_a(k_1) & G_b(k_1) & g_a(k_2) & g_b(k_2) & \ldots \\
g_a(k_1) & v_{aa}(k_{1,}k_1) & v_{ab}(k_{1,}k_1) & v_{aa}(k_{1,}k_2) & v_{ab}(k_{1,}k_2) & \ldots \\
g_b(k_1) & v_{ba}(k_{1,}k_1) & v_{bb}(k_{1,}k_1) & v_{ba}(k_{1,}k_2) & v_{bb}(k_{1,}k_2) & \ldots \\
g_a(k_2) & v_{aa}(k_{2,}k_1) & v_{ab}(k_{2,}k_1) & v_{aa}(k_2k_2) & v_{ab}(k_2k_2) & \ldots \\
g_b(k_2) & v_{ba}(k_{2,}k_1) & v_{bb}(k_{2,}k_1) & v_{ba}(k_2k_2) & v_{bb}(k_{2,}k_2) & \ldots \\
.. & .. & .. & .. & .. & ..
\end{pmatrix}
$$

$Det[H_0+H_I-EI]=0$

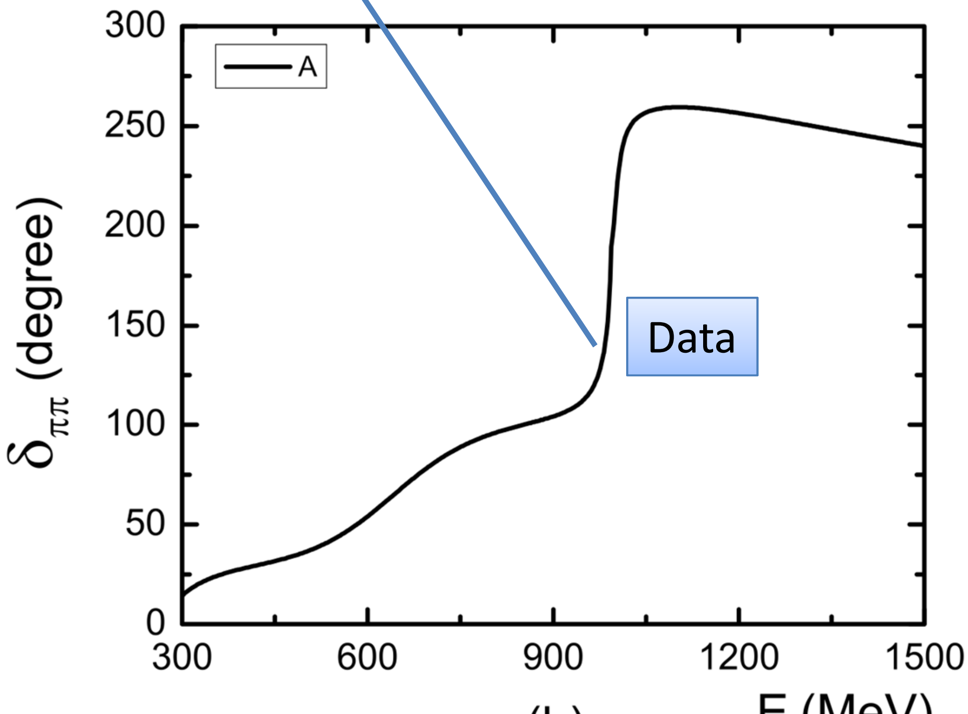
In finite volume

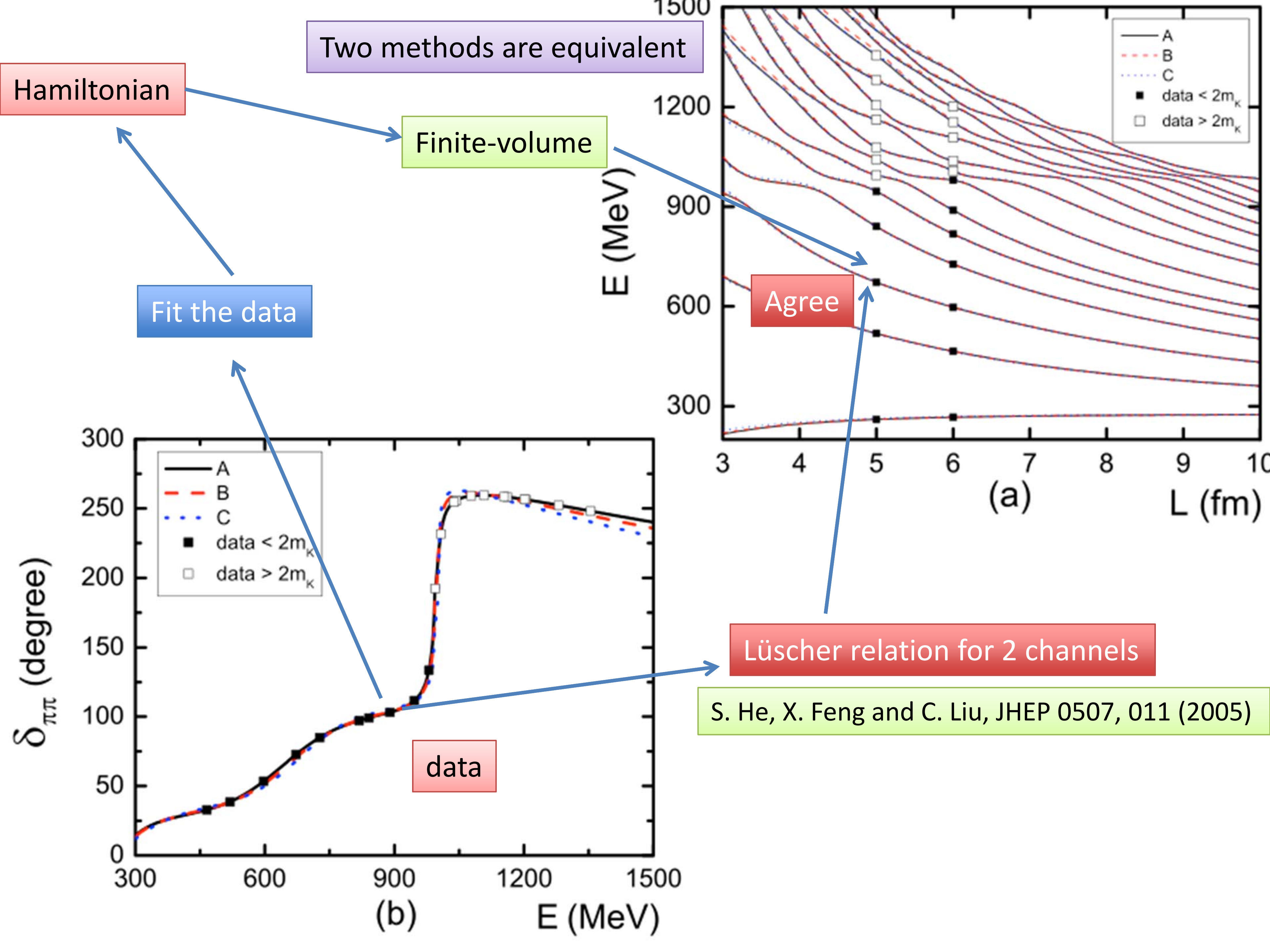
$H = H_0+H_I$



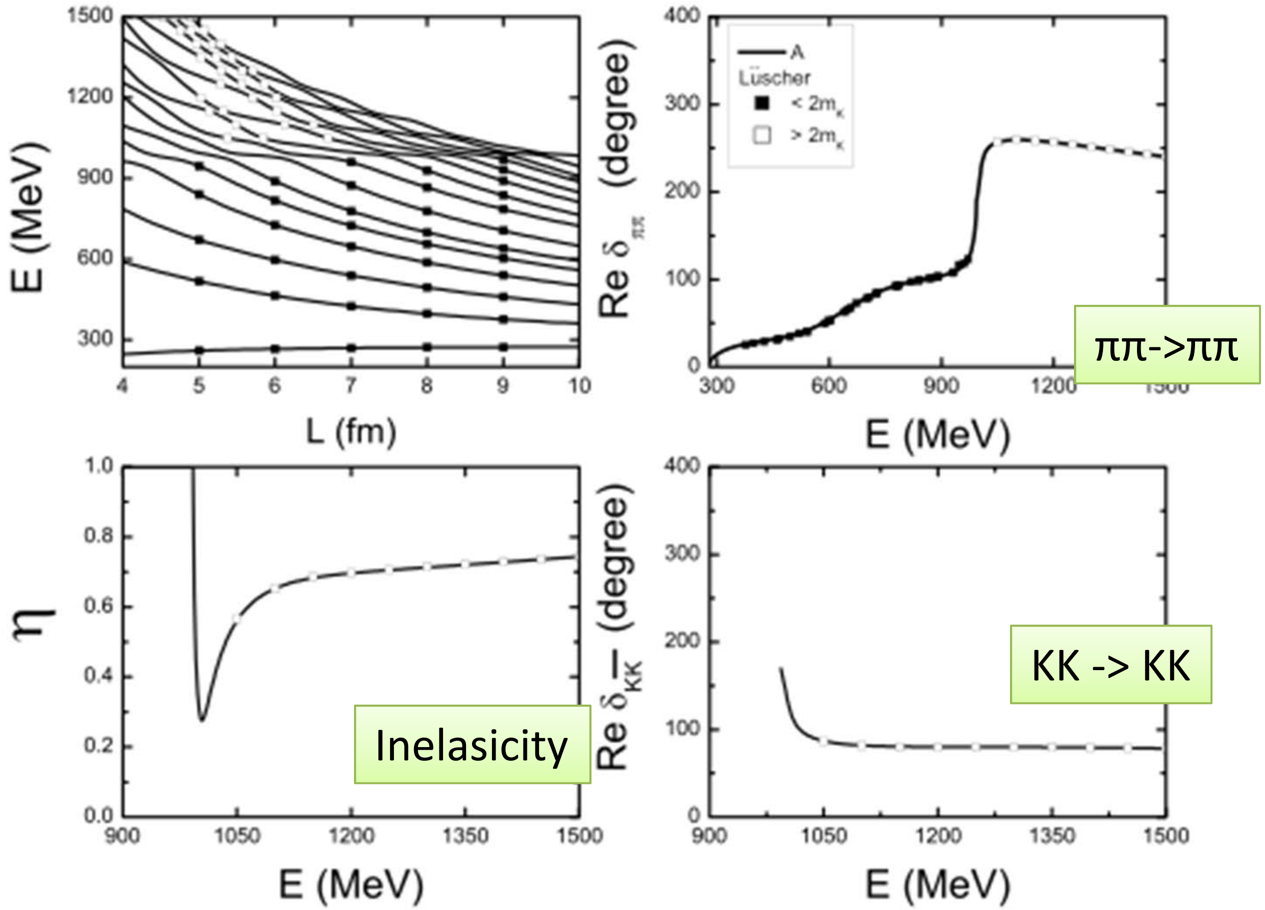
Spectrum in finite-volume

L : size




Two methods are equivalent
Hamiltonian
Finite-volume
Fit the data
Agree
Lüscher relation for 2 channels
S. He, X. Feng and C. Liu, JHEP 0507, 011 (2005)
data
E (MeV)
L (fm)
(a)
A
B
C
data < 2m_K
data > 2m_K
δ_ππ (degree)
E (MeV)
(b)
A
B
C
data < 2m_K
data > 2m_K

Full results



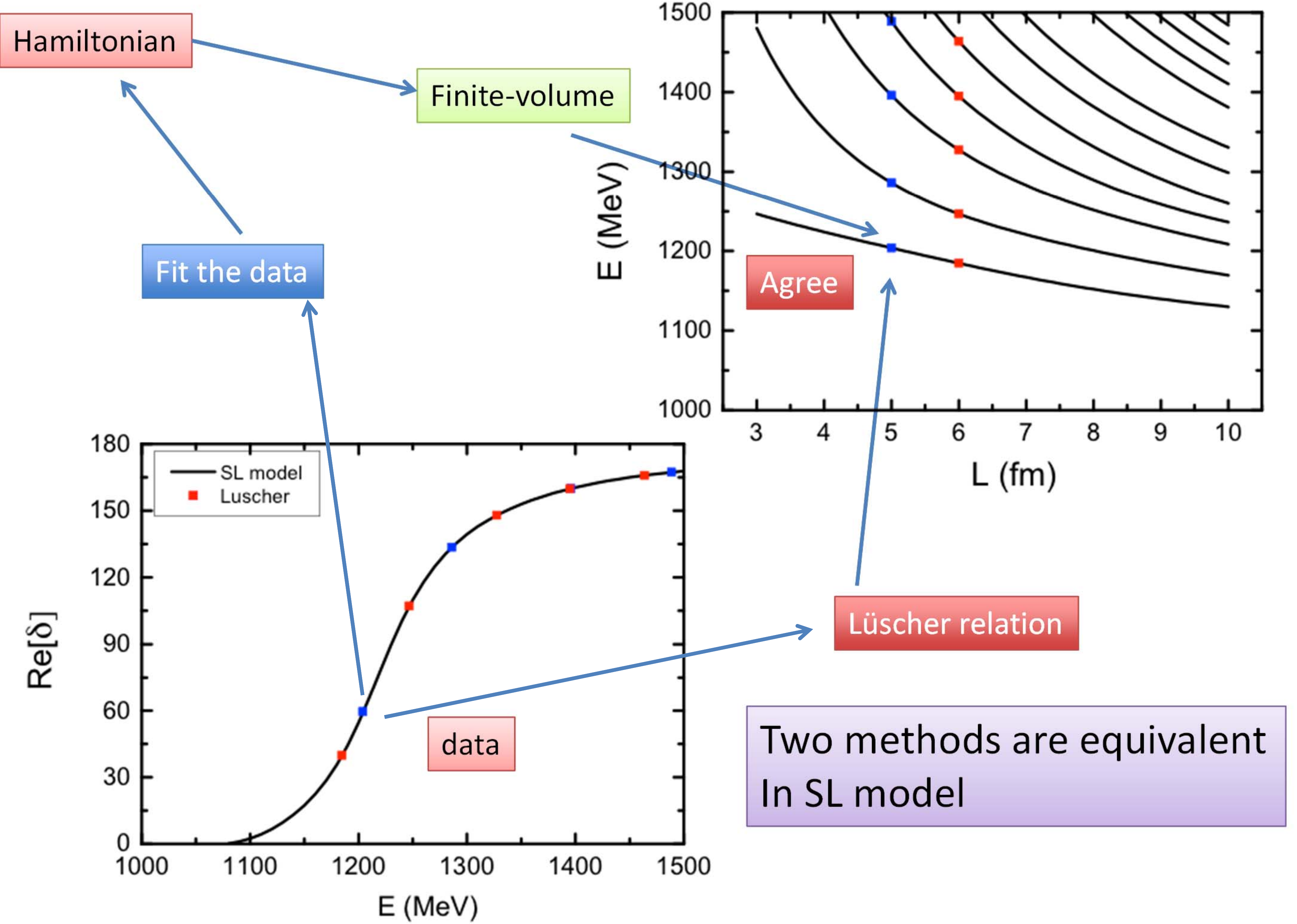
Two methods are equivalent for two-channel case

- Finite-volume Hamiltonian method is equivalent to Lüscher's method for one-channel and two-channel cases

- Finite-volume Hamiltonian method can readily be used for multi-channel cases

Can be applied to ANL-Osaka Hamiltonian for predicting spectrum to test LQCD

First step (Wu, Lee, 2015):
One-channel limit of ANL-Osaka Model: SL Model


Hamiltonian
Finite-volume
Fit the data
Agree
Lüscher relation
data
Two methods are equivalent
In SL model
E (MeV)
L (fm)
Re[δ]
E (MeV)
SL model
Luscher
1500
1400
1300
1200
1100
1000
3
4
5
6
7
8
9
10
180
150
120
90
60
30
0
1000
1100
1200
1300
1400
1500

Extraction of N* excitation of neutron

No neutron target

→

Perform analysis of

d(γ, π)NN
d(e,e'π)NN

Developments:

1. Apply SL Model

K. Hafidi, T.-S. H. Lee, Phys. Rev. C (2001)
Jiajun Wu, T. Sato, T.-S. H. Lee, Phys. Rev. C 91, 035203 (2014)

2. Apply ANL-Osaka Model

In progress with preliminary results

Model Hamiltonian with N, π, Δ, and γ

$$H = H_0 + H'_1 + H'_2$$

$$H'_2 = V_{NN,NN} + V_{N\Delta,NN} + V_{N\Delta,N\Delta}$$

$$H'_1 = v_{\pi N,\pi N} + h_{\Delta,\pi N}$$

$$+ v_{\pi N,\gamma N} + h_{\Delta,\gamma N}$$

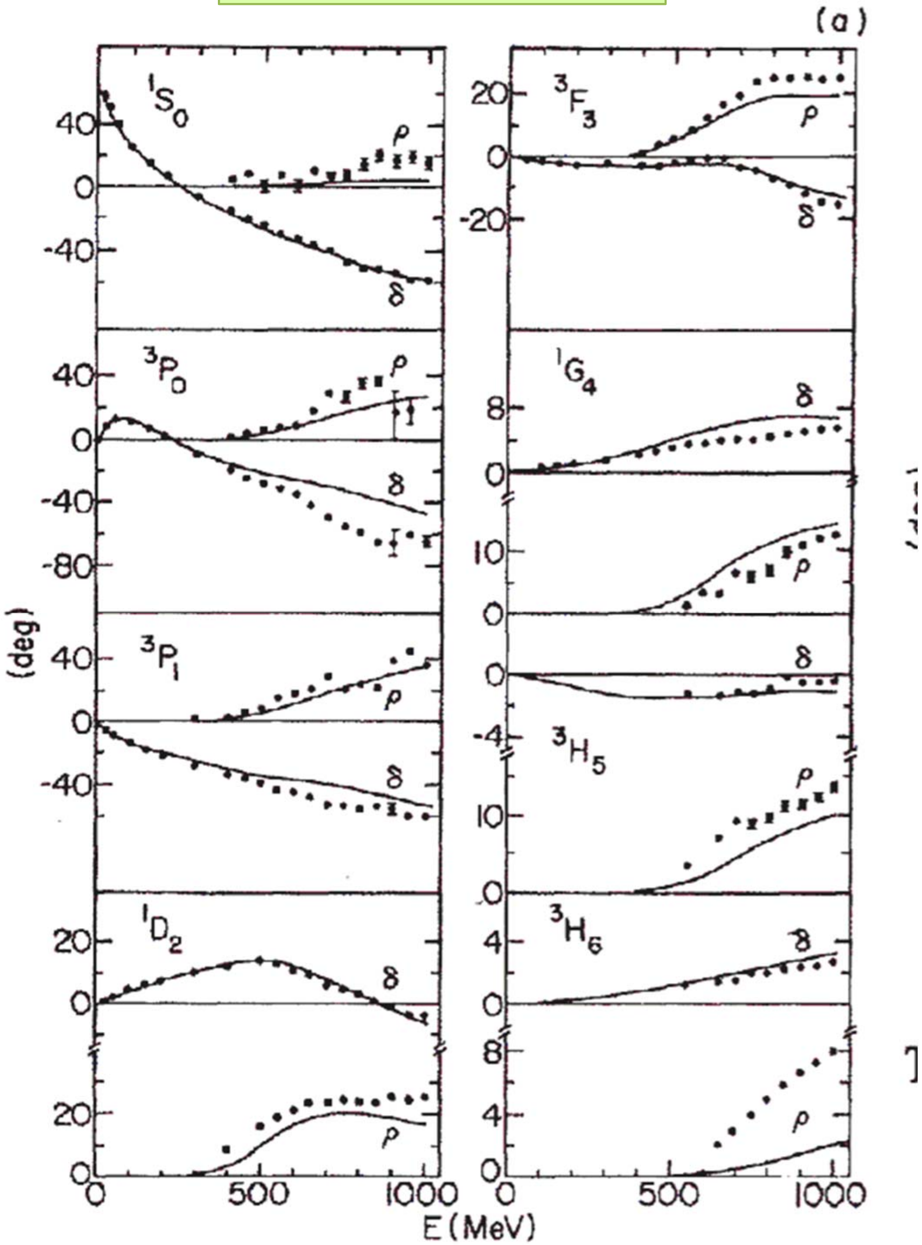
Lee, Matsuyama (1985-1992)

Determined by
πN -> πN
NN-> NN, πNN

Determined by
γN-> πN
N(e,e'π) N

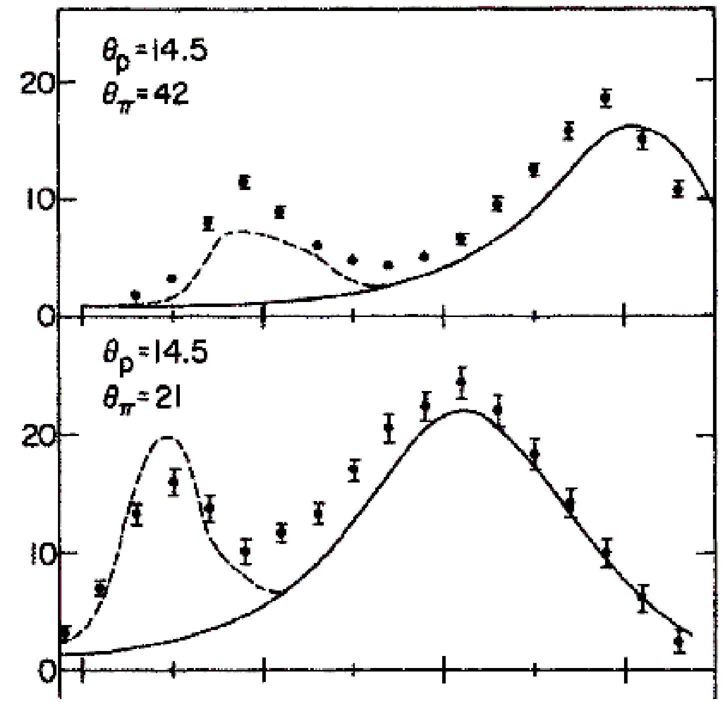
Sato, Lee, 1996-2001

NN phase shifts

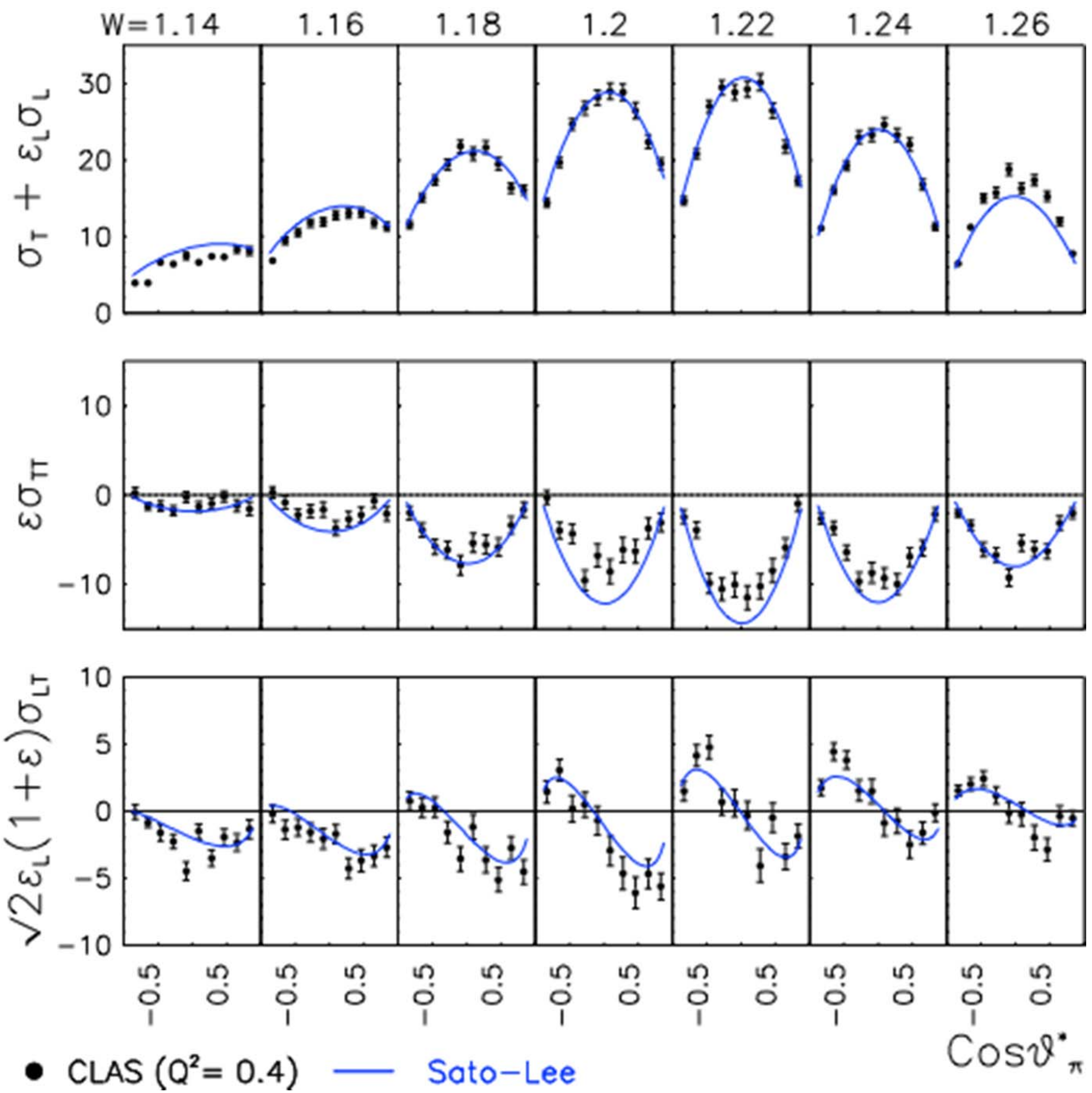


Lee, Matsuyama (1985-1992)

pp -> π^+ np

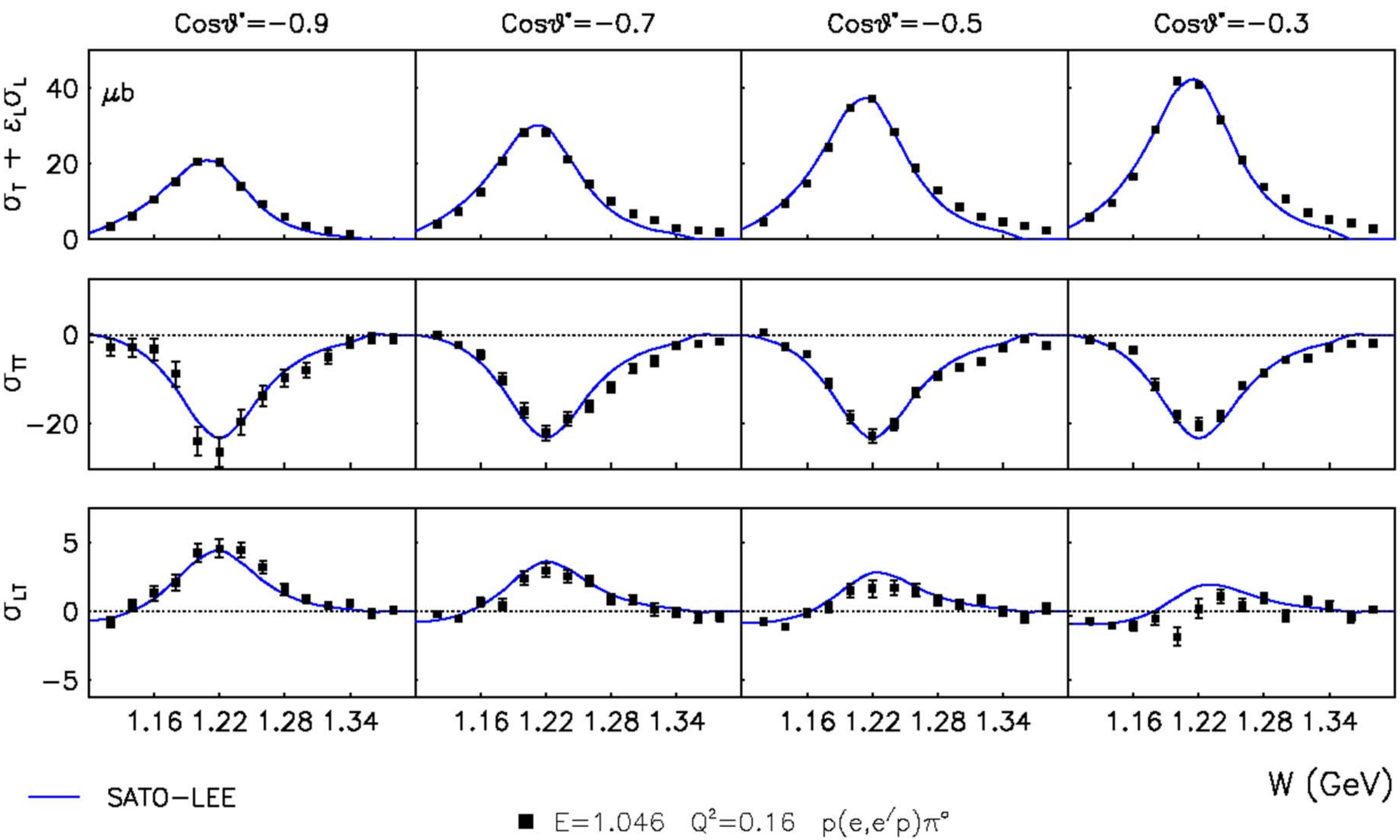


CLAS data of p(e, e' π^0)p (From Joo, 2000)



Pion electroproduction Structure functions

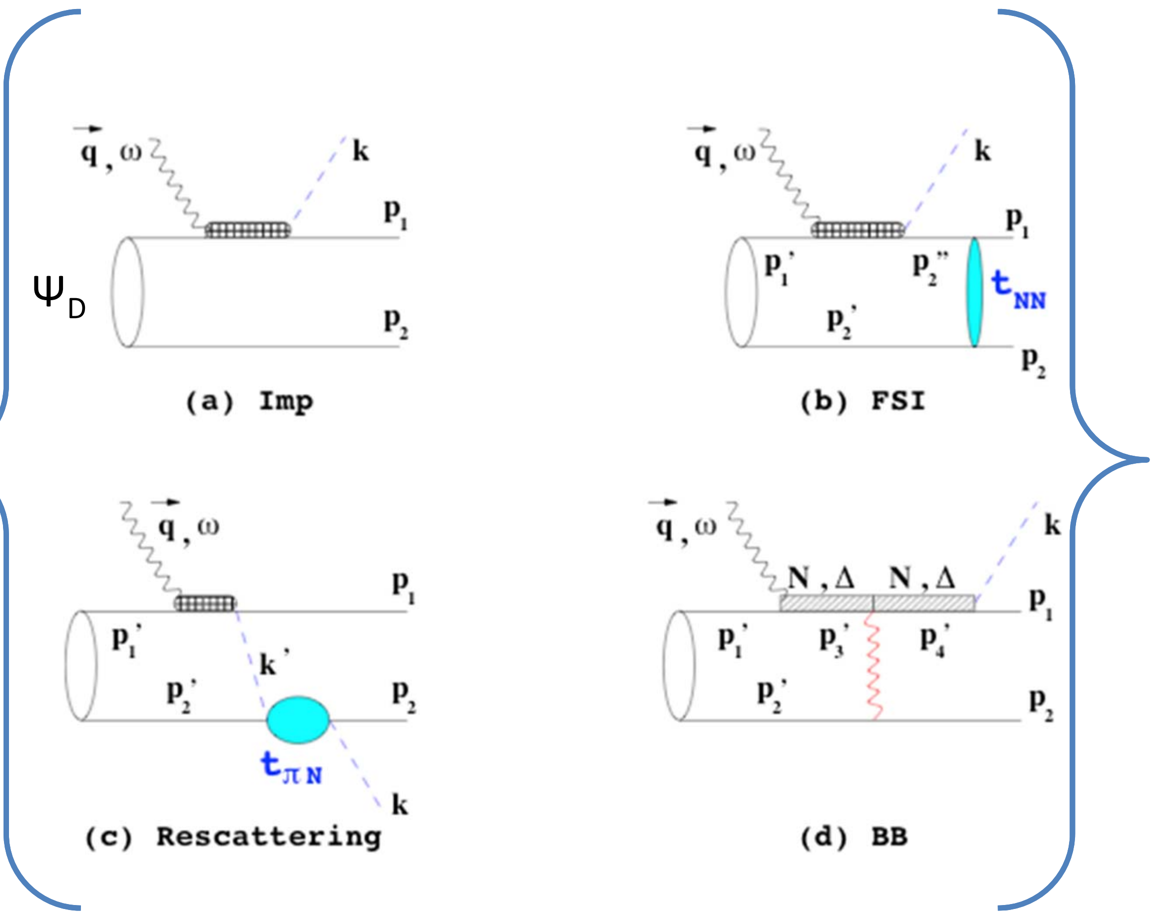
(data CLAS from C. Smith,2004)



Model Hamiltonian with N, π, Δ, and γ

multiple scattering theory

$$<\pi NN|T|\gamma^* \Psi_d>$$



$\sigma \ (\gamma^* d \to \pi NN)$

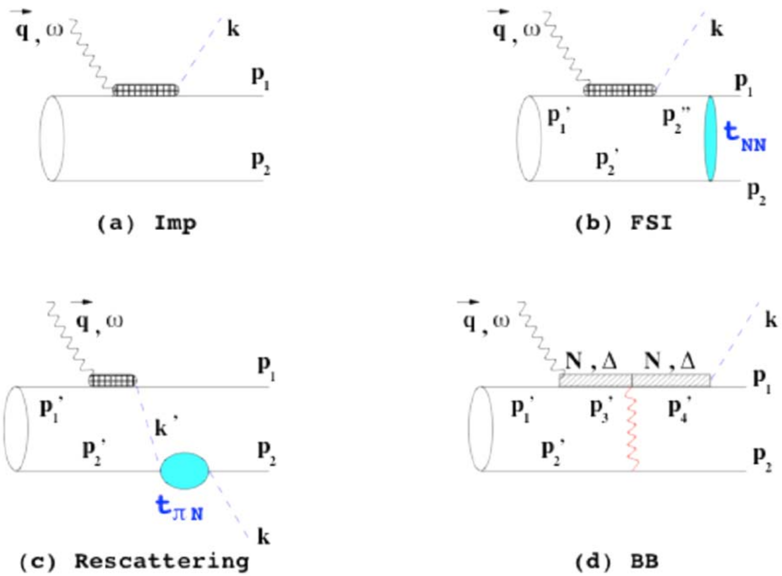
Calculations include :

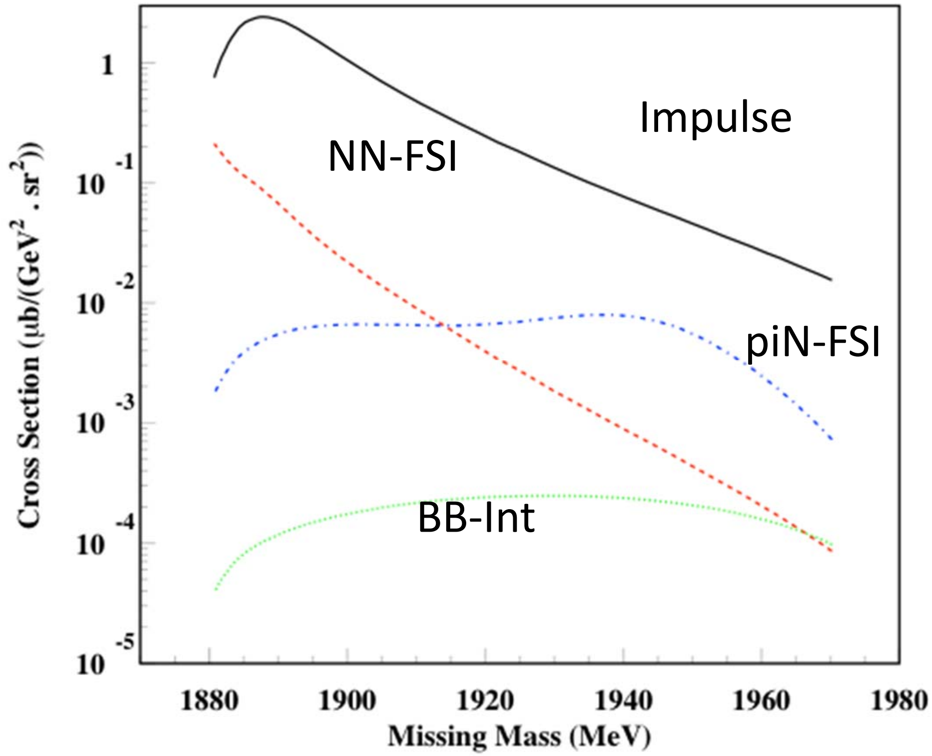
- Fermi motion effects
- Spin rotaion effects

  $|p_L, m_s>_d = R_w(\Lambda)\ |p_{c,} m_s>$
- Lorentz transformaion of currents

  $[J]_d\ = \Lambda\ [j]_N\ \Lambda^{-1}$
- Exact loop-integrations of FSI terms

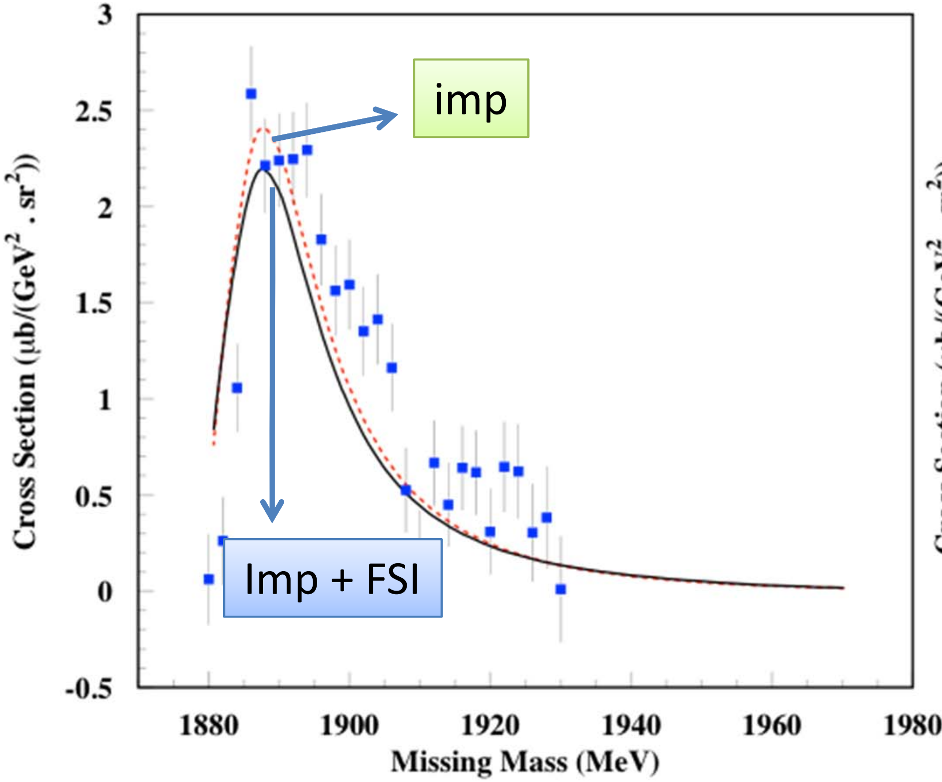
d(e,e’π⁺)nn

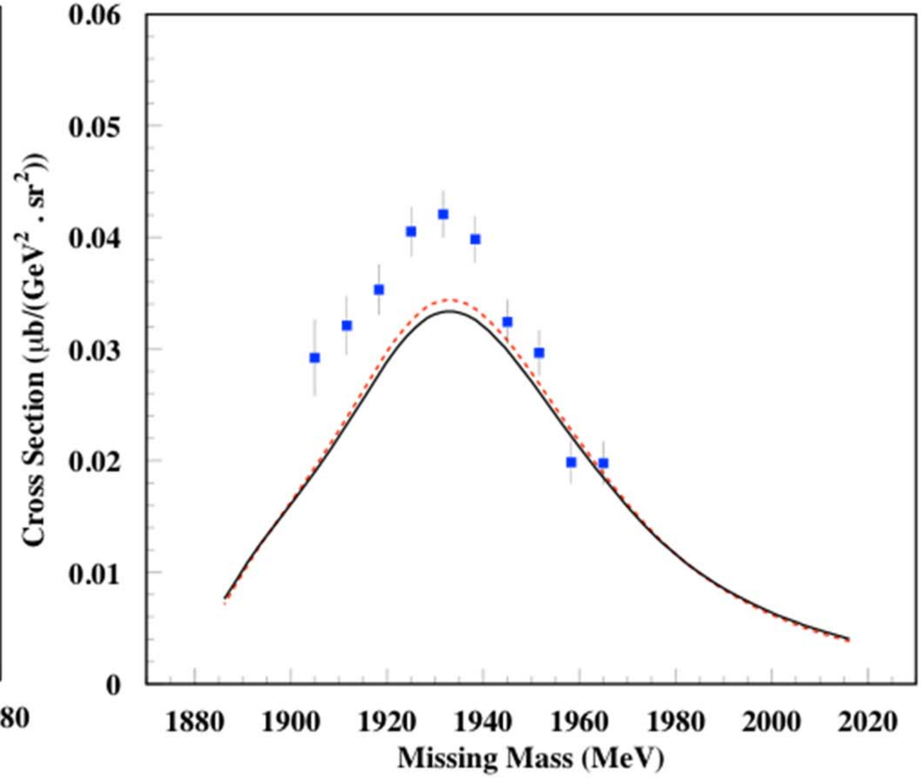




K. Hafidi, T.-S. H. Lee, Phys. Rev. C (2001)

Impluse term dominants
BB Int can be neglected





Saclay data

Q^2=0.08 $(GeV/c)^2$
W=1.16 GeV
E_e=645 MeV

Jlab data

Q^2=0.4$(GeV/c)^2$
W=1.16 GeV
E_e=844 MeV

Apply the SL model to study

$\gamma d \to \pi^- pp$

$\gamma d \to \pi^0 np$

J. Wu, T. Sato, T.-S. H. Lee Phys. Rv. C91, 035203 (2014)

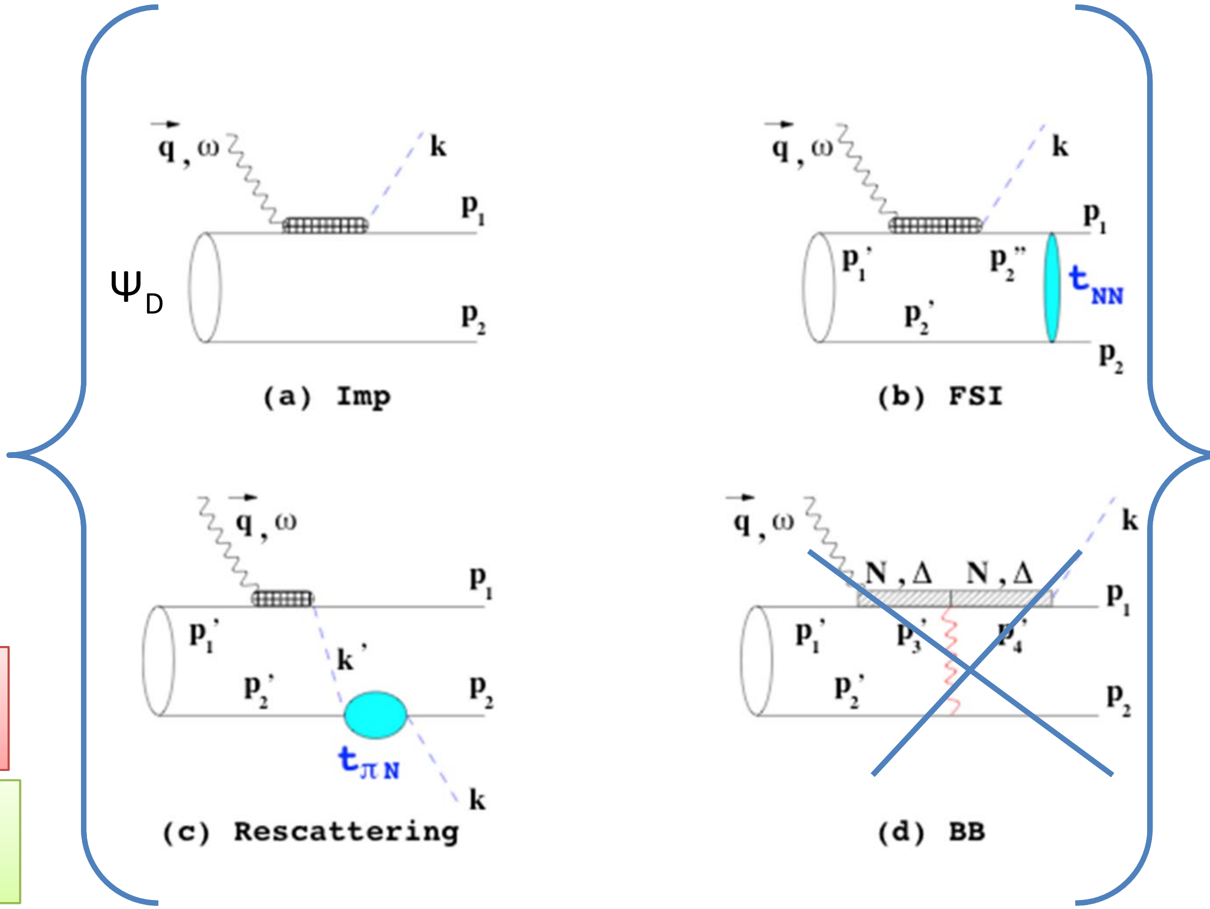
Model Hamiltonian with N, π, Δ, and γ

multiple scattering theory

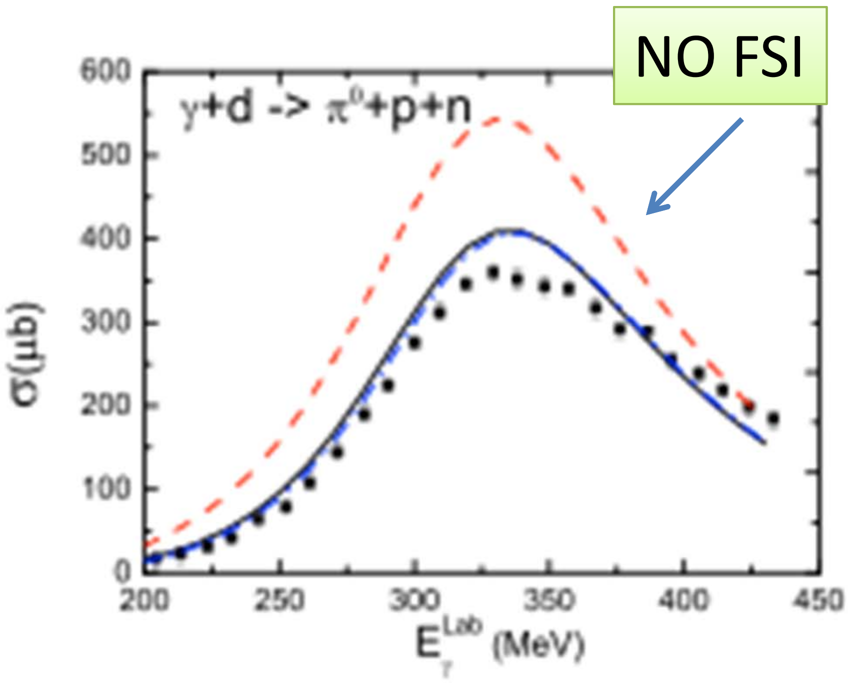
$<\pi NN|T|\gamma\Psi_D>$

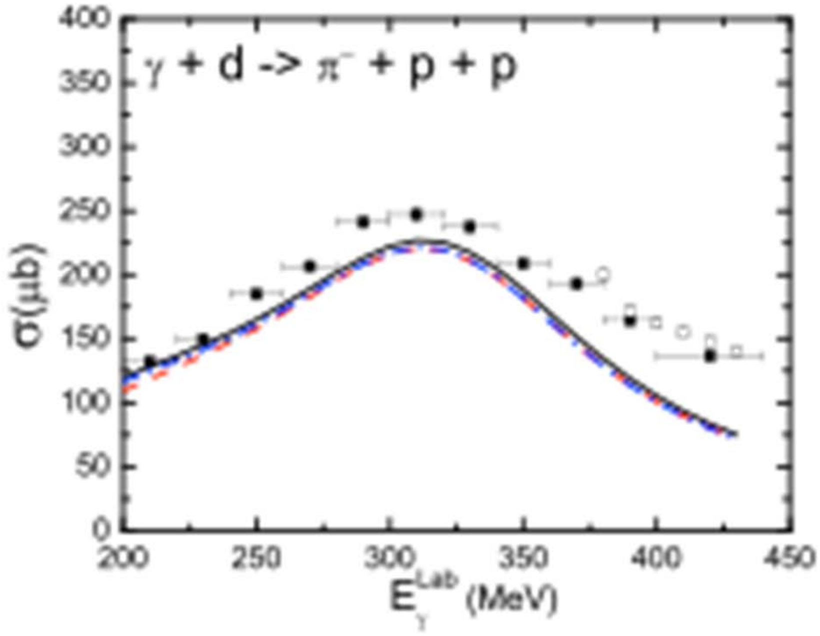
σ (γ d -> π^- pp)

σ (γ d -> π^0 np)

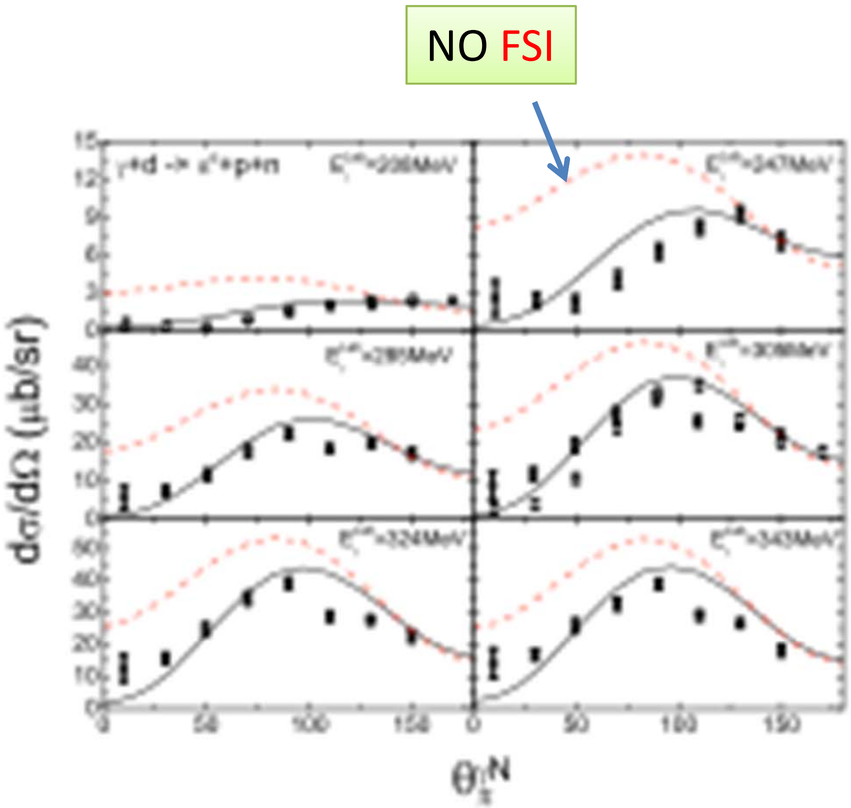


In the Δ(1232) region

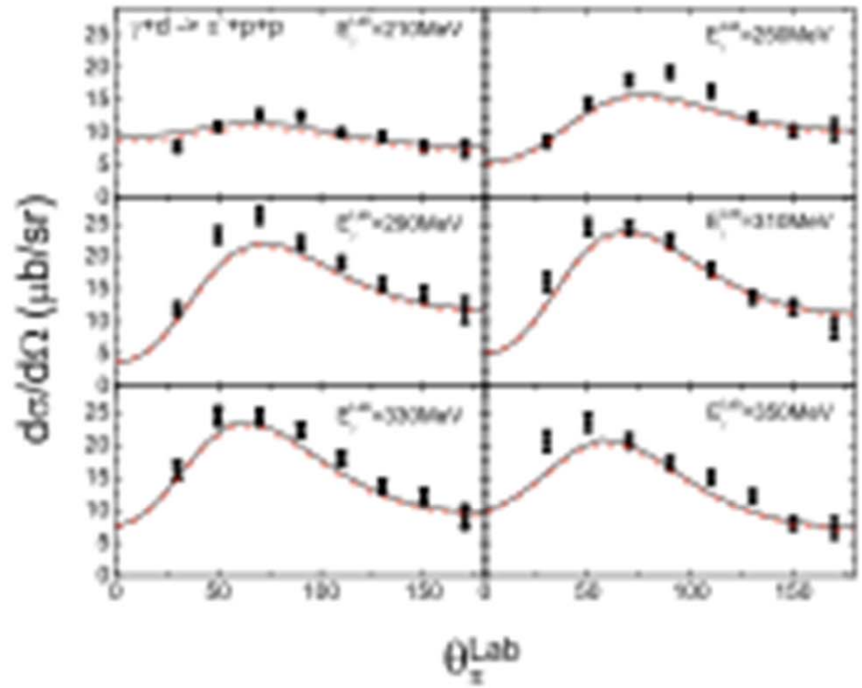




FSI is large for T=0 NN state
FSI is weak for T=1 NN state



$\gamma d \rightarrow \pi^0 np$



$\gamma d \rightarrow \pi^- nn$

FSI is large for T=0 NN state
FSI is weak for T=1 NN state

Apply ANL-Osaka Model

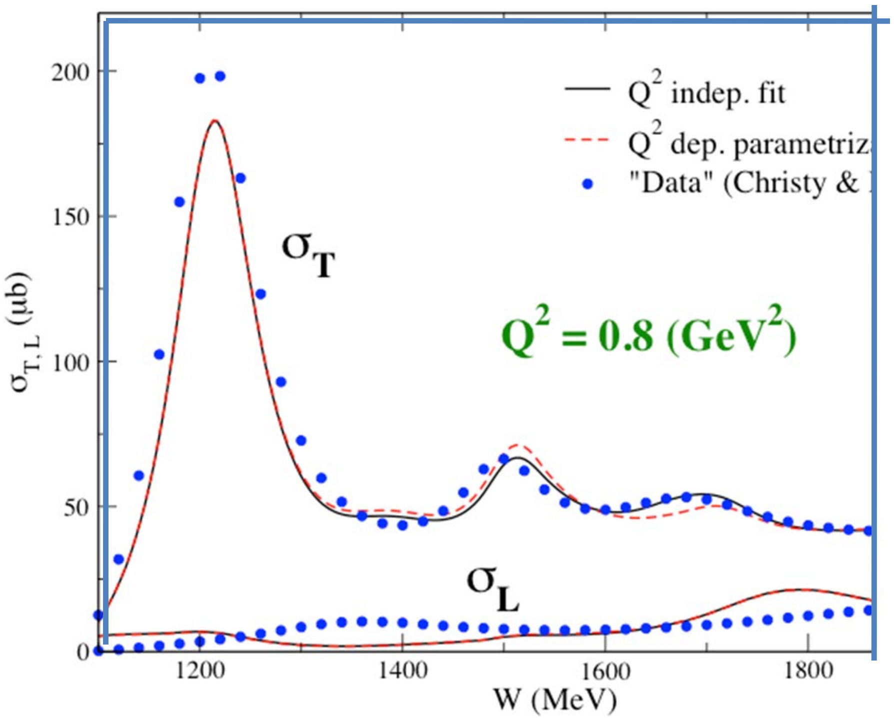
Make Predictions for the analysis of JLab data

- $d(\gamma, \pi^-)nn$, E_γ = 250 – 1600 MeV
- $d(e,e'\pi^-)nn$, E_e = 2.039 GeV, $Q^2 < 2.0$ (GeV/c)^2
  W = 1.236, 1.600 GeV

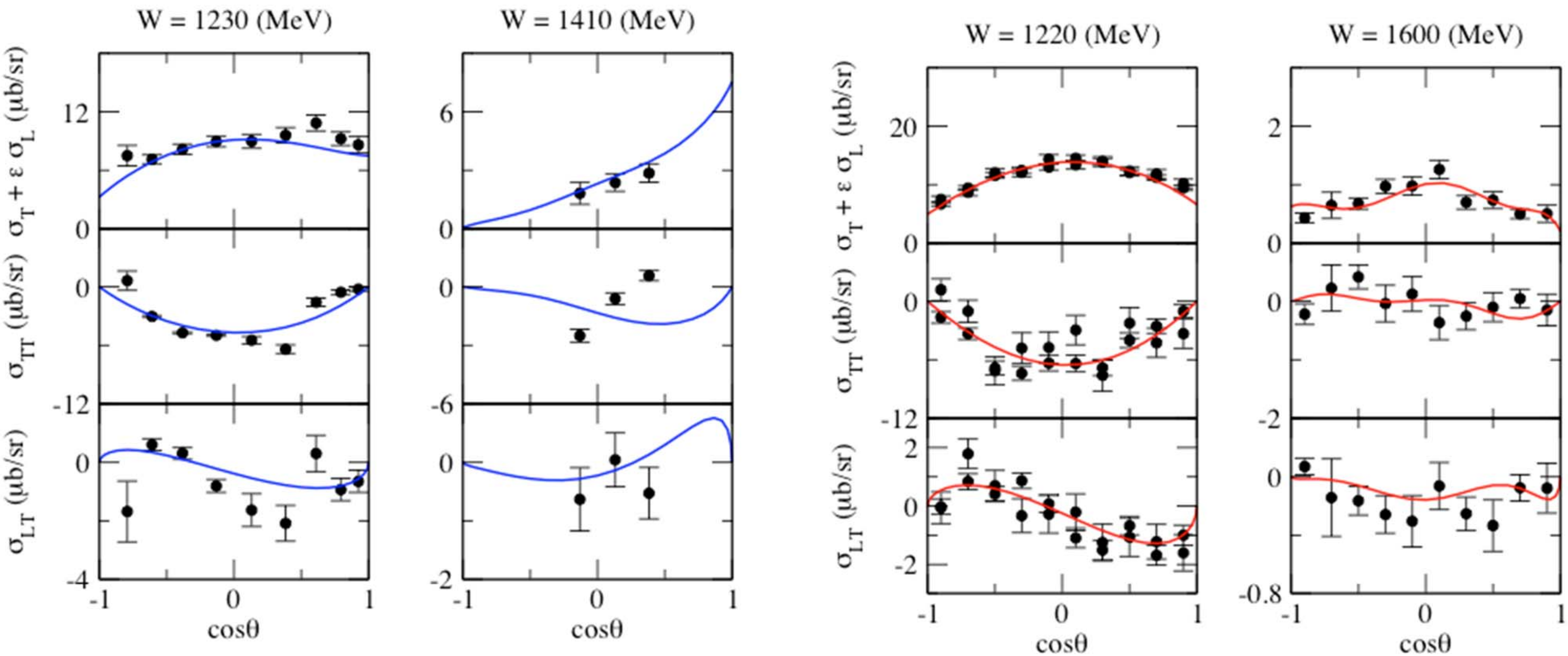
neutron-N* form factors are determined by simultaneous fits to the data of

a. inclusive d(e,e') X total cross sections

b. $\sigma_T + \varepsilon\sigma_L$ of $p(e,e'\pi^0)p$, $p(e,e'\pi^+)n$



d(e,e')X



$\sigma_T + \varepsilon\sigma_L$: p(e,e'π^+)n , p(e,e'π^0)p

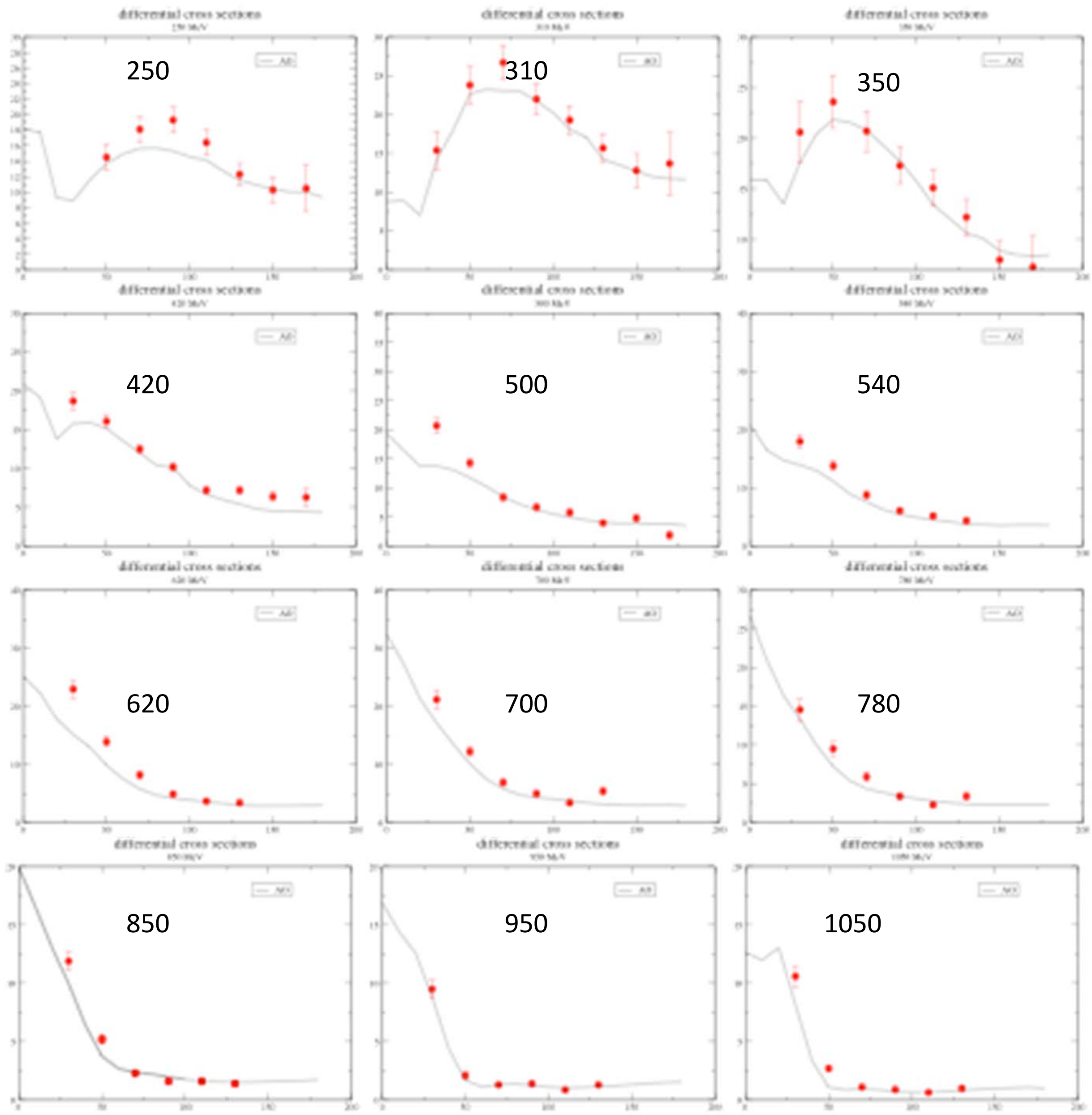
Jlab data (from K. Joo and L.C. Smith)

d (γ,π⁻)pp

Agree with the old data

-

$d\sigma / d\Omega_{\pi}$



Compare the predictions with

Data from JLab-g14-E06-101 (from A. Sandorfi)

1. ϕ-dependence of the spectator proton
2. Double polarization E

Preliminary conclusions:

- Can describe unpolarized $d\sigma/d\Omega$
- Need to tune ANL-Osaka model to fit polarization data P, Σ, E, G…

can improve $\gamma\ n \to \pi^{-} p$ amplitudes which are needed to determine the isospin structure of $\gamma N \to N^*$ transitions

d(e,e′ π^-) pp

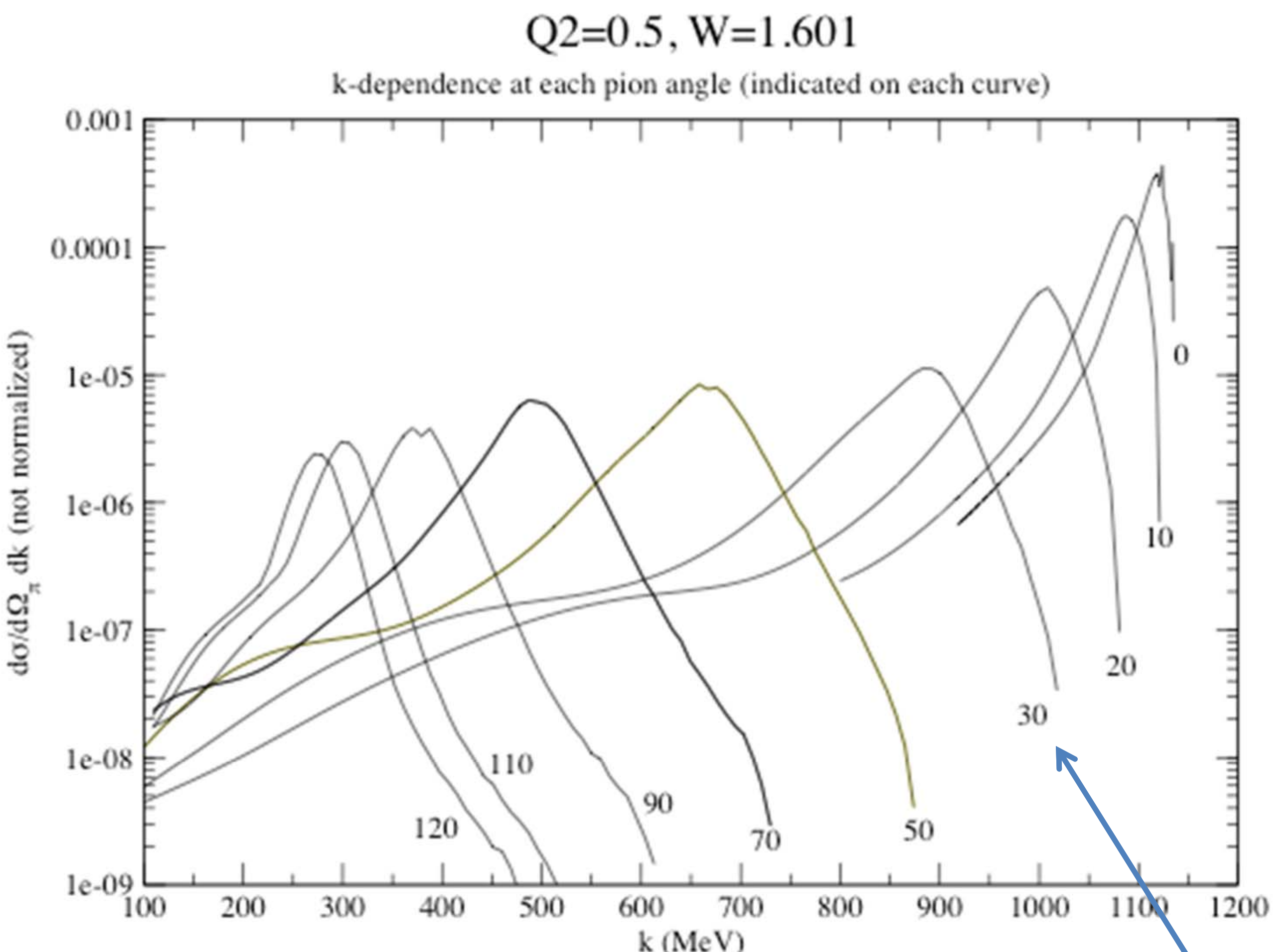
(Ralf Gothe's talk)

Predictions have been made for

E_e = 2.039 GeV, $Q^2 < 2.0$ (GeV/c)^2

W = 1.236, 1.600 GeV

$$\frac{d\sigma_T}{d\Omega_\pi dk}(Q^2, W, k, \theta_\pi, \phi_\pi = 0)$$



k(pion momentum)

θ_Π

$$\frac{d\sigma_{T,L}}{d\Omega_\pi}(Q^2, W, \theta_\pi, \phi_\pi = 0) = \int dk \frac{d\sigma_T}{d\Omega_\pi dk}(Q^2, W, k, \theta_\pi, \phi_\pi = 0)$$

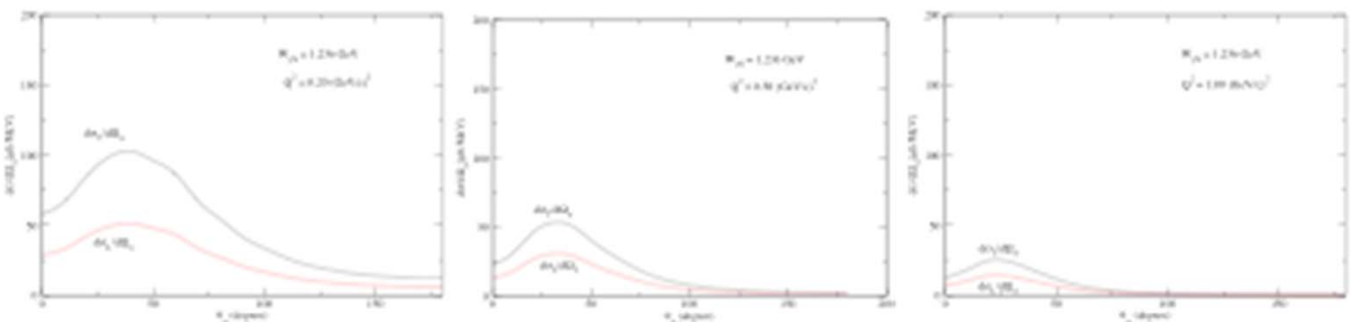


FIG. 1. $d(e, e'\pi^-)pp$ at $W = 1.236$ GeV.

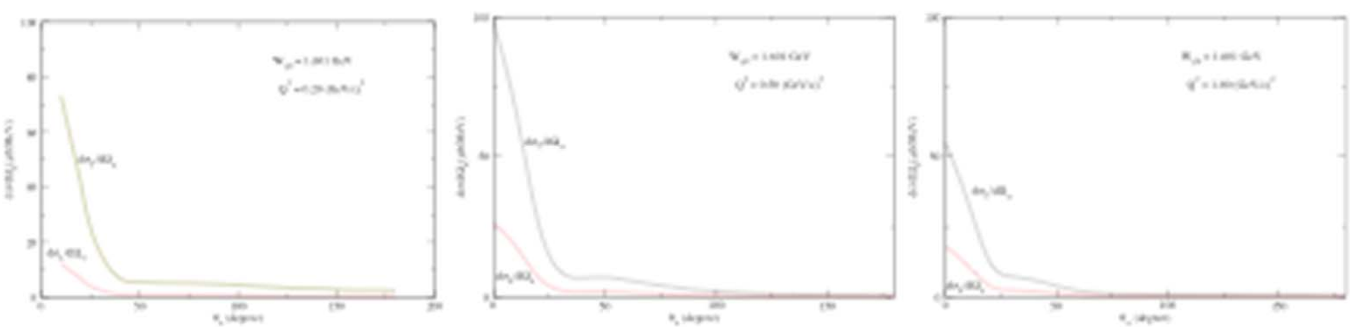
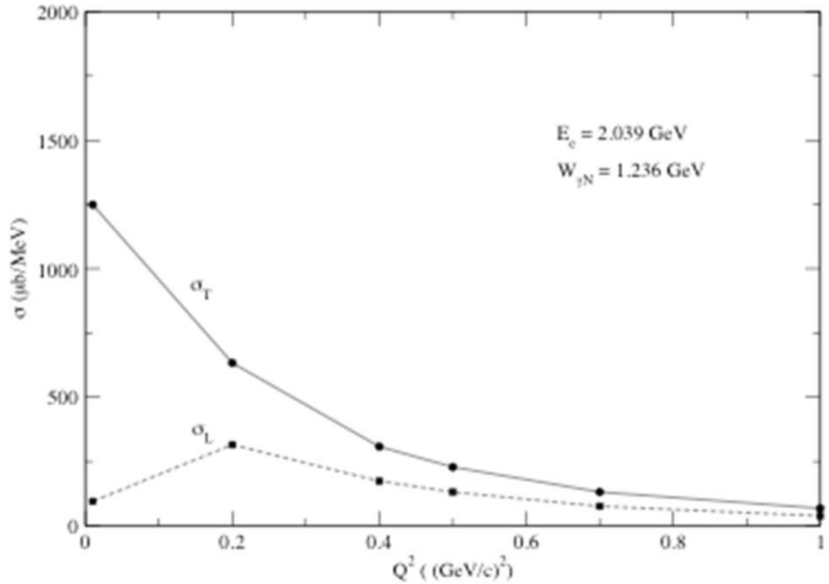
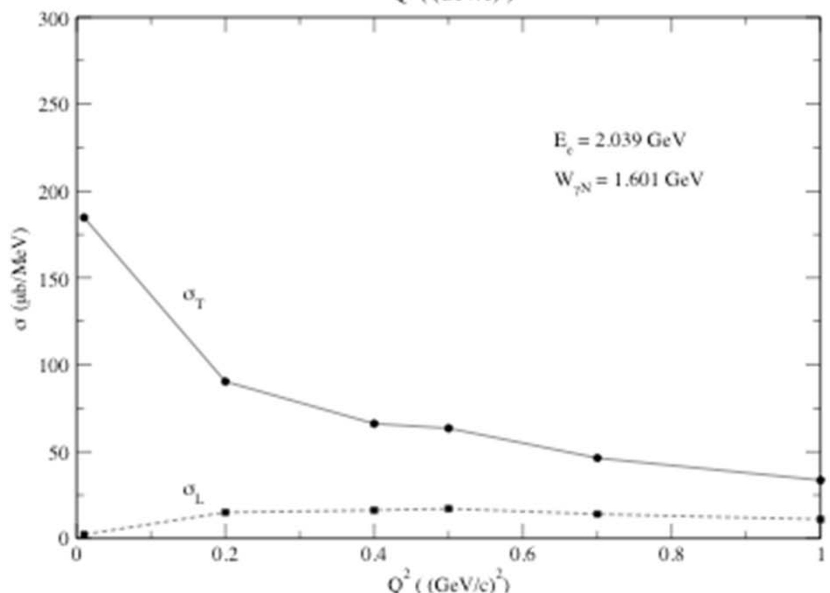


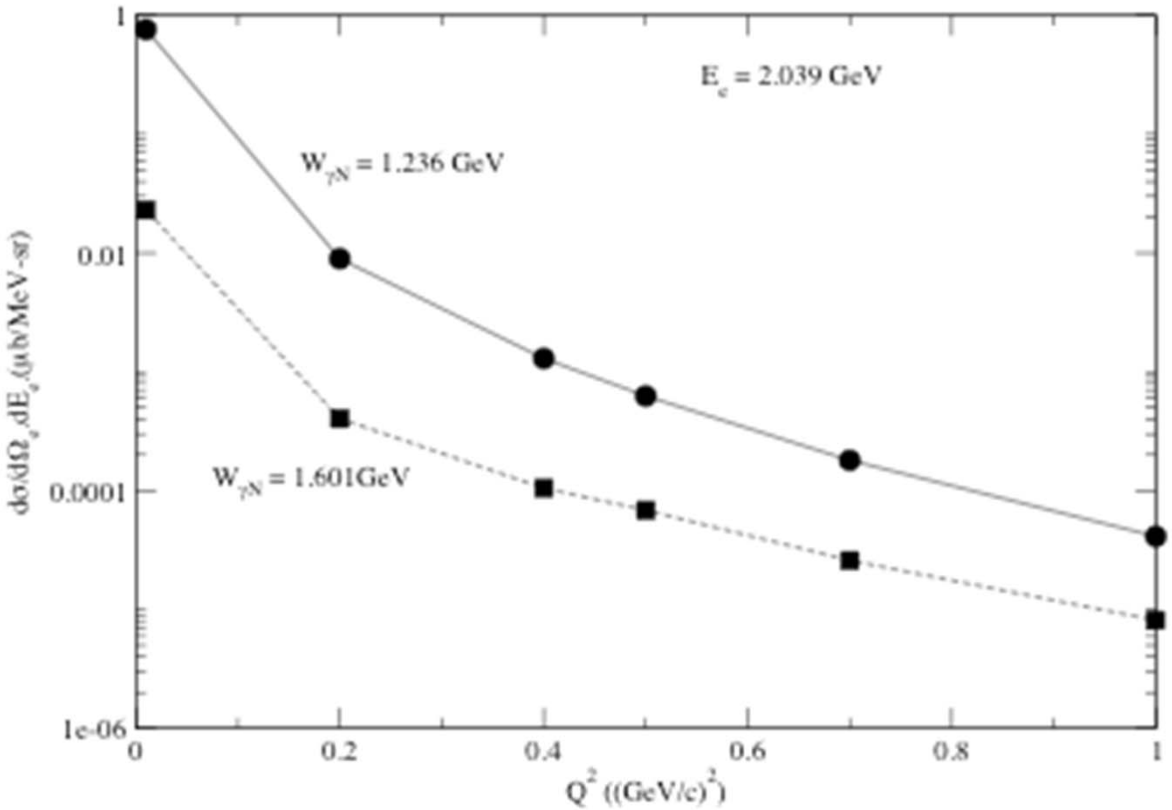
FIG. 2. $d(e, e'\pi^-)pp$ at $W = 1.601$ GeV.

$$\sigma_{T,L}(Q^2, W) = (2\pi) \times \int d\ cos\theta_\pi \frac{d\sigma_{T,L}}{d\Omega}(Q^2, W, k, \theta_\pi, \phi_\pi = 0)$$





$$\frac{d\sigma}{d\Omega_{e'} dE'_e} = \Gamma_v[\sigma_T(Q^2, W) + \epsilon\sigma_L(Q^2, W)]$$

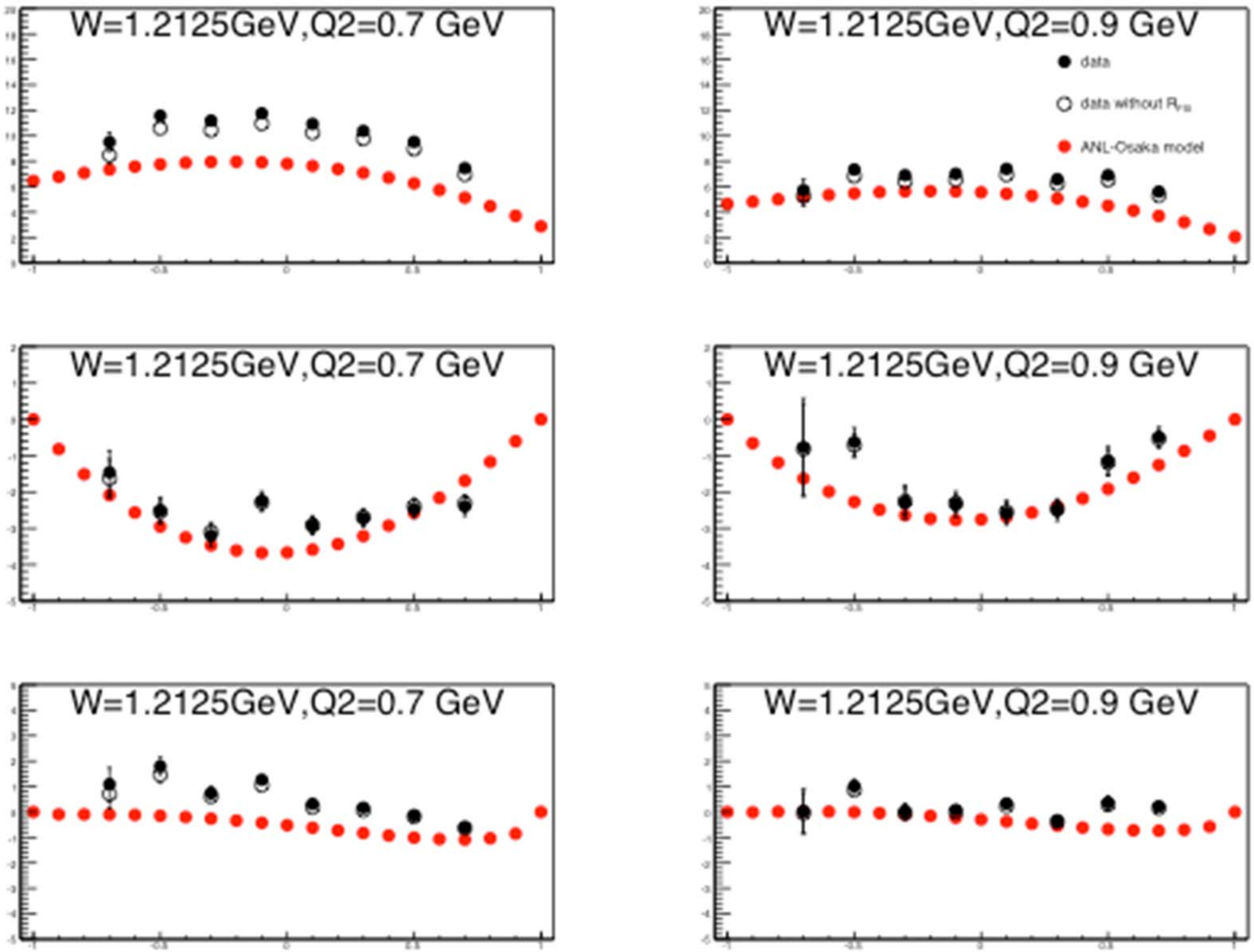


To be tested by the Jlab data
being analyzed by Ye Tian, Ralf Gothe …

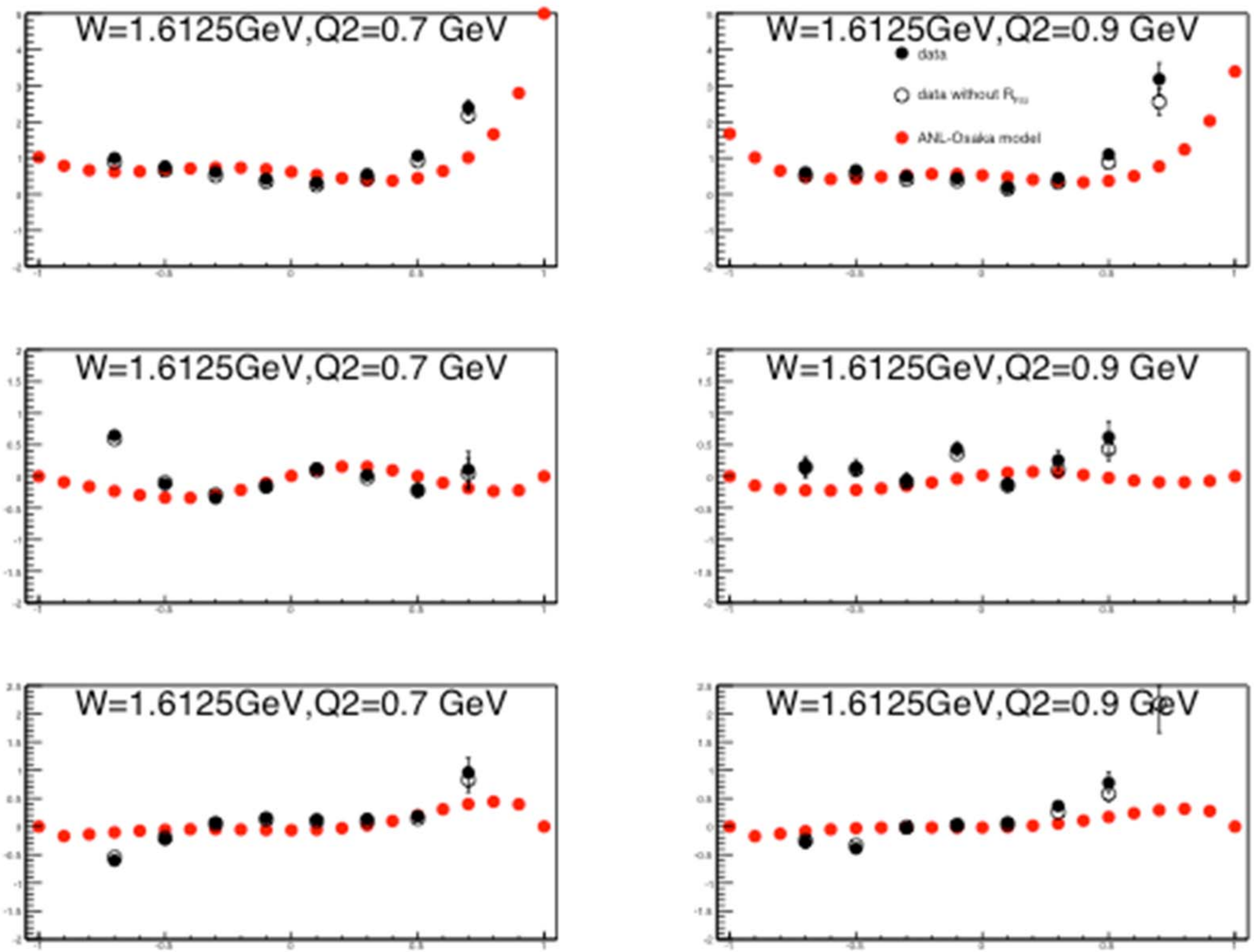
First comparisons (Oct. 7, 2015) :
Structure functions of n(e,e’ π^-) p from

a. Extracted from d(e,e’π^-)pp data (Gothe’s talk)
b. Calculated from ANL-Osaka model

Preliminary data from Ye Tian

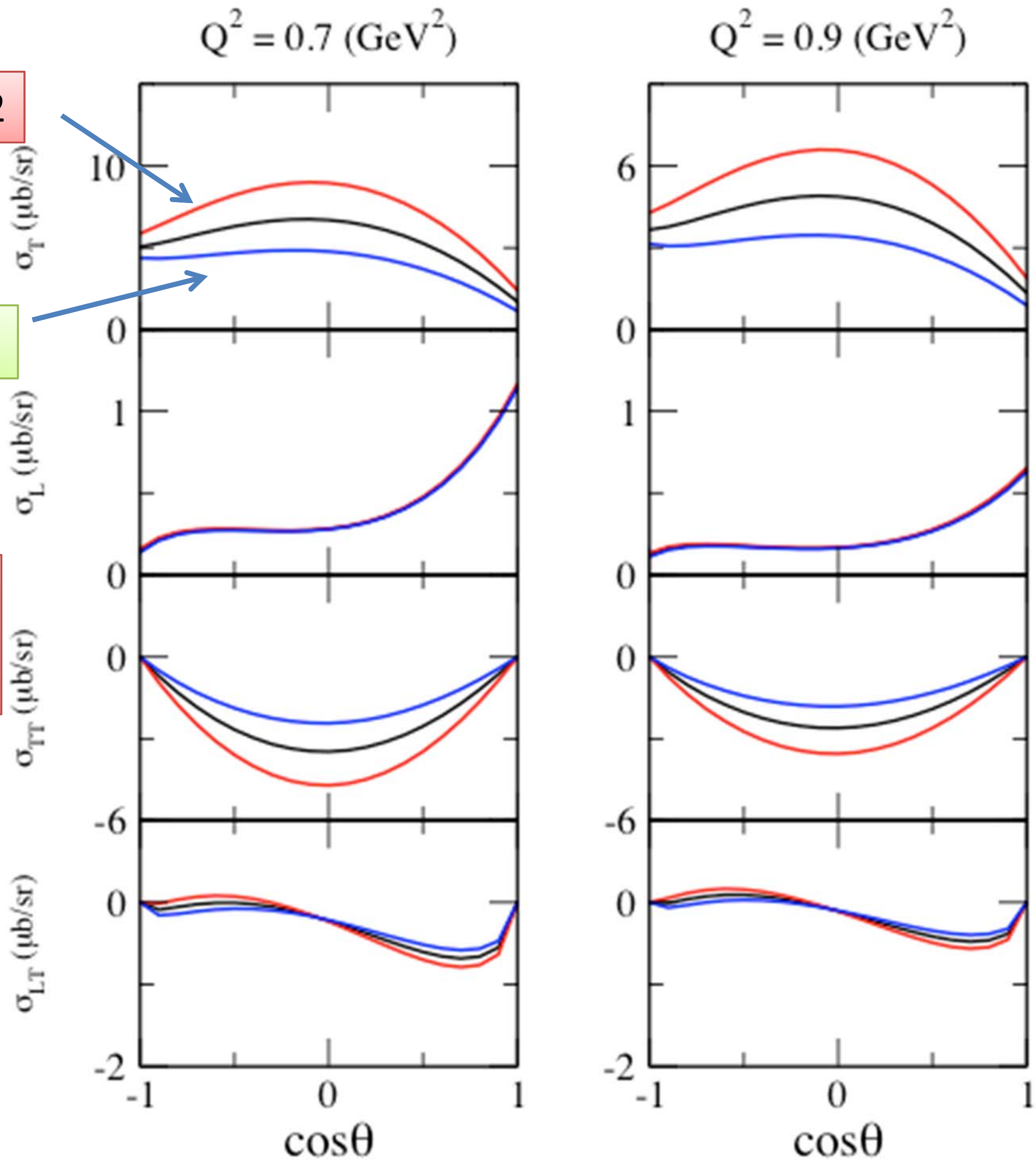


Preliminary data from Ye Tian



$g_{\Delta,\gamma N}$ × 1.2

$g_{\Delta,\gamma N}$ × 0.8



New data can be used to determine $g_{\Delta,\gamma N}$

Much more works are needed to extract quantitative Information on the N* excitations of the neutron

Summary

1. An approach including final state interactions has been developed to extract N* of neutron from meson production data on deuteron target

Interactions between theoretical calculations and data analysis are essential

2. Theoretical constraints must be included in the partial-wave analysis of data and the extractions of nucleon resonances

3. Resonance poles are related to the eigenstates of the underlying fundamental Theory and should be extracted by each analysis group to minimize the errors

4. New development:

ANL-Osaka Hamiltonian

↓

Finite-Volume Hamiltonian Method of Adelaide

↓

Test spectrum from LQCD

Main challenge:
Including ππN channels for N* study