Nucleon Resonance Spectrum and Form Factors from Superconformal Quantum Mechanics in Holographic QCD

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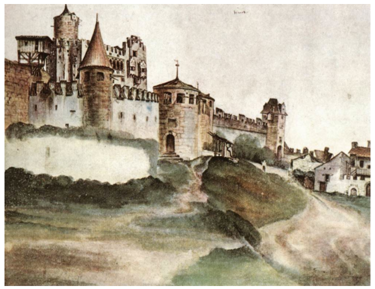
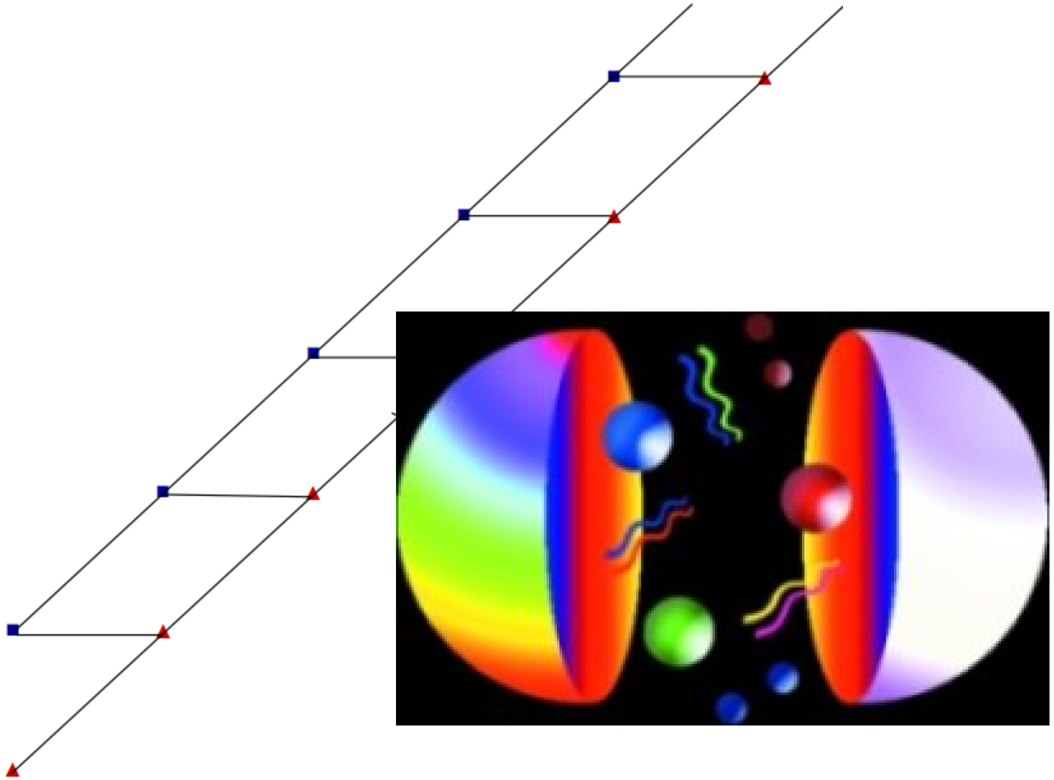
Nucleon Resonances:

From Photoproduction

to High Photon Virtualities

ECT*, Trento, 12 - 16 October 2015





In collaboration with Stan Brodsky and Hans G. Dosch

Quest for a semiclassical approximation to describe bound states in QCD

(Convenient starting point in QCD)

I. Semiclassical approximation to QCD in the light-front: Reduction of QCD LF Hamiltonian leads to a relativistic LF wave equation, where complexities from strong interactions are incorporated in effective potential U

II. Construction of LF potential U: Since the LF semiclassical approach leads to a one-dim QFT, it is natural to extend conformal and superconformal QM to the light front since it gives important insights into the confinement mechanism, the emergence of a mass scale and baryon-meson SUSY

III. Correspondence between equations of motion for arbitrary spin in AdS space and relativistic LF bound-state equations in physical space-time: Embedding of LF wave equations in AdS leads to extension of LF potential U to arbitrary spin from conformal symmetry breaking in the AdS$_5$ action

Outline of this talk

1 Semiclassical approximation to QCD in the light front

2 Conformal quantum mechanics and light-front dynamics: Mesons

3 Embedding integer-spin wave equations in AdS space

4 Superconformal quantum mechanics and light-front dynamics: Baryons

5 Superconformal baryon-meson symmetry

6 Light-front holographic cluster decomposition and form factors

(1) Semiclassical approximation to QCD in the light front

- Start with $SU(3)_C$ QCD Lagrangian

$$\mathcal{L}_{\rm QCD} = \overline{\psi} \left(i\gamma^\mu D_\mu - m \right) \psi - \tfrac{1}{4} G^a_{\mu\nu} G^{a\,\mu\nu}$$

- Express the hadron four-momentum generator $P = (P^+, P^-, \mathbf{P}_\perp)$ in terms of dynamical fields $\psi_+ = \Lambda_\pm \psi$ and $\mathbf{A}_\perp$ $\left(\Lambda_\pm = \gamma^0 \gamma^\pm\right)$ quantized in null plane $x^+ = x^0 + x^3 = 0$

$$\begin{aligned} P^- &= \tfrac{1}{2} \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \, \gamma^+ \frac{\left(i \nabla_\perp\right)^2 + m^2}{i\partial^+} \psi_+ + \text{interactions} \\ P^+ &= \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i\partial^+ \psi_+ \\ \mathbf{P}_\perp &= \tfrac{1}{2} \int dx^- d^2\mathbf{x}_\perp \overline{\psi}_+ \gamma^+ i \nabla_\perp \psi_+ \end{aligned}$$

- LF invariant Hamiltonian $P^2 = P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2$

$$P^2 |\psi(P)\rangle = M^2 |\psi(P)\rangle$$

where $|\psi(P)\rangle$ is expanded in multi-particle Fock states $|n\rangle$: $|\psi\rangle = \sum_n \psi_n |n\rangle$

Effective QCD LF Bound-state Equation

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

- Factor out the longitudinal $X(x)$ and orbital kinematical dependence from LFWF ψ

$$\psi(x,\zeta,\varphi) = e^{iL\varphi} X(x) \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Ultra relativistic limit $m_q \to 0$ longitudinal modes $X(x)$ decouple and LF Hamiltonian equation $P_\mu P^\mu|\psi\rangle = M^2|\psi\rangle$ is a LF wave equation for ϕ

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right)\phi(\zeta) = M^2\phi(\zeta)$$

- Invariant transverse variable in impact space

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2$$

conjugate to invariant mass $\mathcal{M}^2 = \mathbf{k}_\perp^2/x(1-x)$

- Critical value $L = 0$ corresponds to lowest possible stable solution: ground state of the LF Hamiltonian
- Relativistic and frame-independent LF Schrödinger equation: U is instantaneous in LF time and comprises all interactions, including those with higher Fock states.

(2) Conformal quantum mechanics and light-front dynamics

[S. J. Brodsky, GdT and H.G. Dosch, PLB **729**, 3 (2014)]

- Incorporate in 1-dim effective QFT the conformal symmetry of 4-dim QCD Lagrangian in the limit of massless quarks: Conformal QM [V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A **34**, 569 (1976)]

- Conformal Hamiltonian:

$$H = \tfrac{1}{2}\left(p^2 + \frac{g}{x^2}\right)$$

  g dimensionless: Casimir operator of the representation

- Schrödinger picture: $p = -i\partial_x$

$$H = \tfrac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right)$$

- QM evolution

$$H|\psi(t)\rangle = i\frac{d}{dt}|\psi(t)\rangle$$

  H is one of the generators of the conformal group *Conf*(R^1). The two additional generators are:

- Dilatation: $D = -\tfrac{1}{4}\left(px + xp\right)$

- Special conformal transformations: $K = \tfrac{1}{2}x^2$

- H, D and K close the conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

- dAFF construct a new generator G as a superposition of the 3 generators of *Conf*(R^1)

$$G = uH + vD + wK$$

and introduce new time variable τ

$$d\tau = \frac{dt}{u + vt + wt^2}$$

- Find usual quantum mechanical evolution for time τ

$$G|\psi(\tau)\rangle = i\frac{d}{d\tau}|\psi(\tau)\rangle \qquad H|\psi(t)\rangle = i\frac{d}{dt}|\psi(t)\rangle$$

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2.$$

- Operator G is compact for $4uw - v^2 > 0$, but action remains conformal invariant !

- Emergence of scale: Since the generators of *Conf*$(R^1) \sim SO(2,1)$ have different dimensions a scale appears in the new Hamiltonian G, which according to dAFF may play a fundamental role

Connection to light-front dynamics

- Compare the dAFF Hamiltonian G

$$G = \frac{1}{2}u\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right) + \frac{i}{4}v\left(x\,\frac{d}{dx} + \frac{d}{dx}\,x\right) + \frac{1}{2}wx^2.$$

with the LF Hamiltonian H_{LF}

$$H_{LF} = -\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)$$

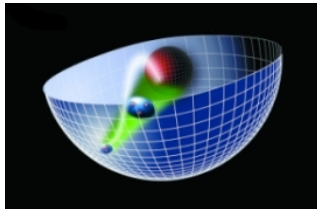
and identify dAFF variable x with LF invariant variable ζ

- Choose $u = 2, \;\; v = 0$
- Casimir operator from LF kinematical constraints: $g = L^2 - \frac{1}{4}$
- $w = 2\lambda^2$ fixes the LF potential to harmonic oscillator in the LF plane $\lambda^2\,\zeta^2$

$$U \sim \lambda^2\zeta^2$$

(3) Embedding integer spin wave equations in AdS space

[GdT, H.G. Dosch and S. J. Brodsky, PRD **87**, 075004 (2013)]



- Integer spin-J in AdS conveniently described by tensor field $\Phi_{N_1 \cdots N_J}$ with effective action

$$S_{\mathit{eff}} = \int d^d x \, dz \, \sqrt{|g|} \, e^{\varphi(z)} \, g^{N_1 N_1'} \cdots g^{N_J N_J'} \Big(g^{M M'} D_M \Phi^*_{N_1 \ldots N_J} \, D_{M'} \Phi_{N_1' \ldots N_J'} - \mu^2_{\mathit{eff}}(z) \, \Phi^*_{N_1 \ldots N_J} \, \Phi_{N_1' \ldots N_J'} \Big)$$

D_M is the covariant derivative which includes affine connection and dilaton $\varphi(z)$ effectively breaks maximal symmetry of AdS$_{d+1}$

$$ds^2 = \frac{R^2}{z^2} \left(dx_\mu dx^\mu - dz^2 \right)$$

- Effective mass $\mu_{\mathit{eff}}(z)$ is determined by precise mapping to light-front physics
- Non-trivial geometry of pure AdS encodes the kinematics and additional deformations of AdS encode the dynamics, including confinement

- Physical hadron has plane-wave and polarization indices along $3{+}1$ physical coordinates and a profile wavefunction $\Phi(z)$ along holographic variable z

$$\Phi_P(x,z)_{\mu_1\cdots\mu_J} = e^{iP\cdot x}\Phi(z)_{\mu_1\cdots\mu_J}, \qquad \Phi_{z\mu_2\cdots\mu_J} = \cdots = \Phi_{\mu_1\mu_2\cdots z} = 0$$

with four-momentum P_μ and invariant hadronic mass $P_\mu P^\mu {=} M^2$

- Variation of the action gives AdS wave equation for spin-J field $\Phi(z)_{\nu_1\cdots\nu_J} = \Phi_J(z)\epsilon_{\nu_1\cdots\nu_J}(P)$

$$\boxed{\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi}(z)}{z^{d-1-2J}}\partial_z\right) + \left(\frac{\mu R}{z}\right)^2\right]\Phi_J = M^2\Phi_J}$$

with

$$(\mu\, R)^2 = (\mu_{\mathit{eff}}(z)R)^2 - Jz\,\varphi'(z) + J(d-J+1)$$

<u>and</u> the kinematical constraints to eliminate the lower spin states $J-1,\, J-2, \cdots$

$$\eta^{\mu\nu}P_\mu\,\epsilon_{\nu\nu_2\cdots\nu_J} = 0, \quad \eta^{\mu\nu}\,\epsilon_{\mu\nu\nu_3\cdots\nu_J} = 0$$

- Kinematical constrains in the LF imply that μ must be a constant

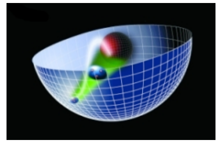
[See also: T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, Phys. Rev. D **85**, 076003 (2012)]

Light-front mapping

[GdT and S. J. Brodsky, PRL **102**, 081601 (2009)]

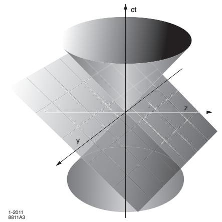
- Upon substitution $\Phi_J(z) \sim z^{(d-1)/2-J} e^{-\varphi(z)/2} \, \phi_J(z)$ and $z \to \zeta$ in AdS WE

$$\left[-\frac{z^{d-1-2J}}{e^{\varphi(z)}}\partial_z\left(\frac{e^{\varphi(z)}}{z^{d-1-2J}}\partial_z\right)+\left(\frac{\mu R}{z}\right)^2\right]\Phi_J(z)=M^2\Phi_J(z)$$



we find LFWE $(d = 4)$

$$\left(-\frac{d^2}{d\zeta^2}-\frac{1-4L^2}{4\zeta^2}+U(\zeta)\right)\phi_J(\zeta)=M^2\phi_J(\zeta)$$



with

$$U(\zeta)=\tfrac{1}{2}\varphi''(\zeta)+\frac{1}{4}\varphi'(\zeta)^2+\frac{2J-3}{2z}\varphi'(\zeta)$$

and $(\mu R)^2 = -(2-J)^2 + L^2$

- Unmodified AdS equations correspond to the kinetic energy terms for the partons
- Effective confining potential $U(\zeta)$ corresponds to the IR modification of AdS space
- AdS Breitenlohner-Freedman bound $(\mu R)^2 \geq -4$ equivalent to LF QM stability condition $L^2 \geq 0$

Meson spectrum

- Dilaton profile in the dual gravity model determined from one-dim QFTh (dAFF)

$$\varphi(z) = \lambda z^2, \qquad \lambda^2 = \tfrac{1}{2} w$$

- Effective potential: $U = \lambda^2 \zeta^2 + 2\lambda(J-1)$

- LFWE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z) = 1$

$$\phi_{n,L}(\zeta) = |\lambda|^{(1+L)/2} \sqrt{\frac{2n!}{(n+L)!}}\, \zeta^{1/2+L} e^{-|\lambda|\zeta^2/2} L_n^L(|\lambda|\zeta^2)$$

- Eigenvalues for $\lambda > 0$

$$\mathcal{M}^2_{n,J,L} = 4\lambda\left(n + \frac{J+L}{2}\right)$$

- $\lambda < 0$ incompatible with LF constituent interpretation

Three relevant points . . .

- A linear potential V_{eff} in the *instant form* implies a quadratic potential U_{eff} in the *front form* at large distances $\longrightarrow$ Regge trajectories

$$U_{\text{eff}} = V_{\text{eff}}^2 + 2\sqrt{p^2 + m_q^2}\, V_{\text{eff}} + 2\, V_{\text{eff}}\sqrt{p^2 + m_q^2}$$

[A. P. Trawiński, S. D. Glazek, S. J. Brodsky, GdT, H. G. Dosch, PRD **90**, 074017 (2014)]

- Results are easily extended to light quarks

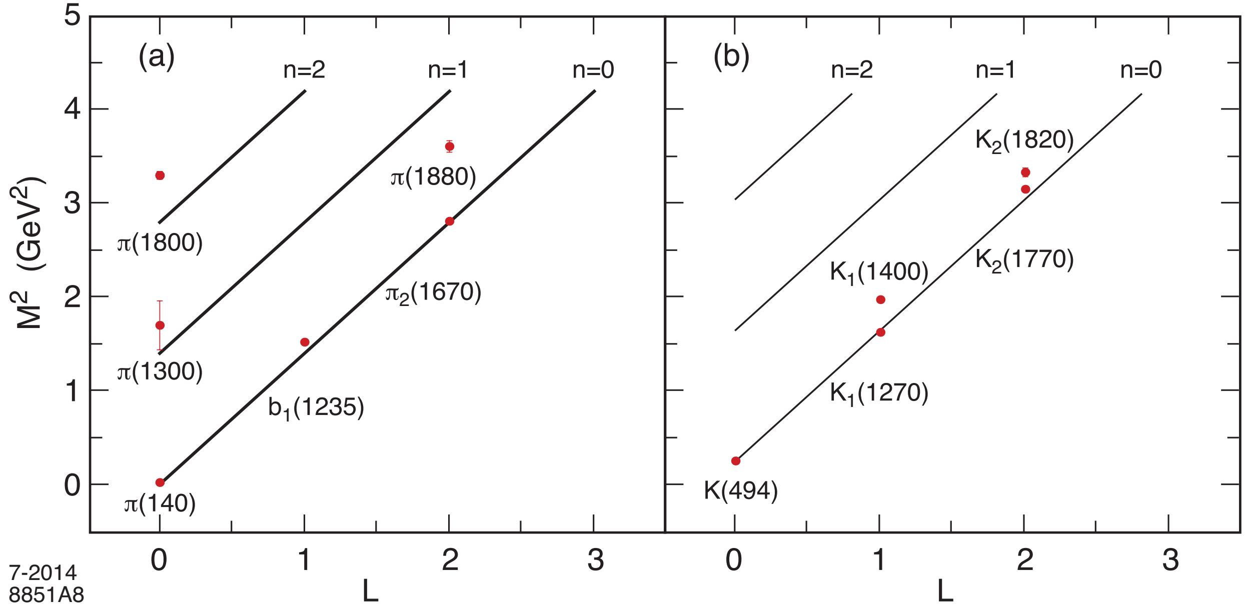
[S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, Phys. Rept. **584**, 1 (2015)

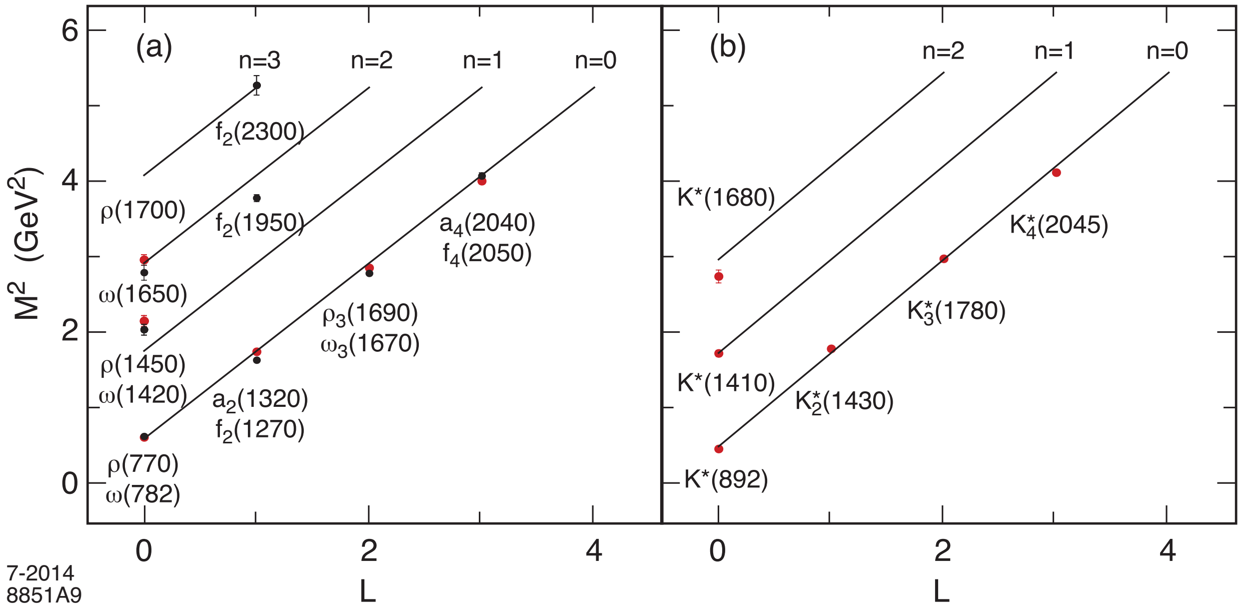
$$\Delta M^2_{m_q, m_{\overline{q}}} = \frac{\int_0^1 dx\, e^{-\frac{1}{\lambda}\left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)} \left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)}{\int_0^1 dx\, e^{-\frac{1}{\lambda}\left(\frac{m_q^2}{x} + \frac{m_{\overline{q}}^2}{1-x}\right)}}$$

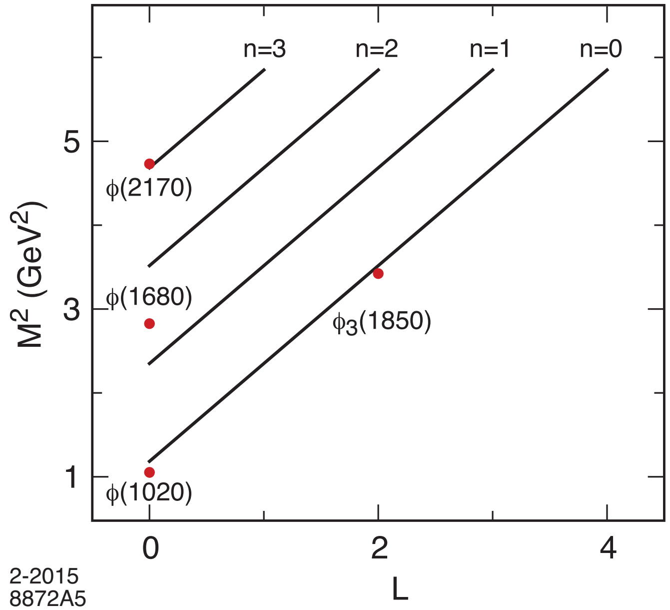
- For n partons invariant LF variable ζ is [S. J. Brodsky and GdT, PRL **96**, 201601 (2006)]

$$\zeta = \sqrt{\frac{x}{1-x}}\Big|\sum_{j=1}^{n-1} x_j \mathbf{b}_{\perp j}\Big|$$

where x_j and x are longitudinal momentum fractions of quark j in the cluster and of the active quark







Orbital and radial excitations for $\sqrt{\lambda} = 0.59$ GeV (pseudoscalar) and 0.54 GeV (vector mesons)

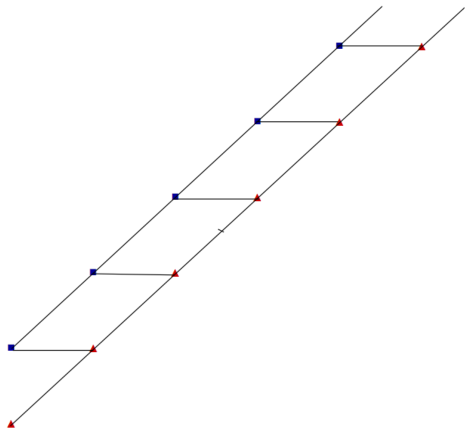
(4) Superconformal quantum mechanics and light-front dynamics

[GdT, H.G. Dosch and S. J. Brodsky, PRD **91**, 045040 (2015)]

- SUSY QM contains two fermionic generators Q and $Q^\dagger$, and a bosonic generator, the Hamiltonian H

  [E. Witten, NPB **188**, 513 (1981)]

- Closure under the graded algebra $sl(1/1)$:



$$\frac{1}{2}\{Q, Q^\dagger\} = H$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[Q, H] = [Q^\dagger, H] = 0$$

Note: Since $[Q^\dagger, H] = 0$ the states $|E\rangle$ and $Q^\dagger|E\rangle$ have identical eigenvalues E

- A simple realization is

$$Q = \chi\,(ip + W)\,, \qquad Q^\dagger = \chi^\dagger\,(-ip + W)$$

where χ and $\chi^\dagger$ are spinor operators with anticommutation relation

$$\{\chi, \chi^\dagger\} = 1$$

- In a 2×2 Pauli-spin matrix representation: $\chi = \frac{1}{2}\,(\sigma_1 + i\sigma_2)\,,\;\; \chi^\dagger = \frac{1}{2}\,(\sigma_1 - i\sigma_2)$

$$[\chi, \chi^\dagger] = \sigma_3$$

- Following Fubini and Rabinovici consider a 1-dim QFT invariant under conformal and supersymmetric transformations [S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

- Conformal superpotential (f is dimensionless)

$$W(x) = \frac{f}{x}$$

- Thus 1-dim QFT representation of the operators

$$Q = \chi\left(\frac{d}{dx} + \frac{f}{x}\right), \qquad Q^\dagger = \chi^\dagger\left(-\frac{d}{dx} + \frac{f}{x}\right)$$

- Conformal Hamiltonian $H = \frac{1}{2}\{Q, Q^\dagger\}$ in matrix form

$$H = \frac{1}{2}\begin{pmatrix} -\frac{d^2}{dx^2} + \frac{f(f-1)}{x^2} & 0 \\ 0 & -\frac{d^2}{dx^2} + \frac{f(f+1)}{x^2} \end{pmatrix}$$

- Conformal graded-Lie algebra has in addition to Hamiltonian H and supercharges Q and $Q^\dagger$, a new operator S related to generator of conformal transformations $K \sim \{S, S^\dagger\}$

$$S = \chi\, x, \qquad S^\dagger = \chi^\dagger x$$

- Find enlarged algebra (Superconformal algebra of Haag, Lopuszanski and Sohnius (1974))

$$\begin{aligned} \tfrac{1}{2}\{Q, Q^\dagger\} &= H, \qquad \tfrac{1}{2}\{S, S^\dagger\} = K \\ \tfrac{1}{2}\{Q, S^\dagger\} &= \frac{f}{2} + \frac{\sigma_3}{4} + iD \\ \tfrac{1}{2}\{Q^\dagger, S\} &= \frac{f}{2} + \frac{\sigma_3}{4} - iD \end{aligned}$$

where the operators

$$\begin{aligned} H &= \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 - \sigma_3 f}{x^2}\right) \\ D &= \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right) \\ K &= \tfrac{1}{2}x^2 \end{aligned}$$

satisfy the conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

- Following F&R define a supercharge R, a linear combination of the generators Q and S

$$R = \sqrt{u}\, Q + \sqrt{w}\, S$$

and consider the new generator $G = \frac{1}{2}\{R, R^\dagger\}$ which also closes under the graded algebra $sl(1/1)$

$$\frac{1}{2}\{R, R^\dagger\} = G \qquad \frac{1}{2}\{Q, Q^\dagger\} = H$$
$$\{R, R\} = \{R^\dagger, R^\dagger\} = 0 \qquad \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[R, H] = [R^\dagger, H] = 0 \qquad [Q, H] = [Q^\dagger, H] = 0$$

- New QM evolution operator

$$G = uH + wK + \frac{1}{2}\sqrt{uw}\,(2f + \sigma_3)$$

is compact for $uw > 0$: Emergence of a scale since Q and S have different units

- Light-front extension of superconformal results follows from

$$x \to \zeta, \quad f \to \nu + \tfrac{1}{2}, \quad \sigma_3 \to \gamma_5, \quad 2G \to H_{LF}$$

- Obtain:

$$H_{LF} = -\frac{d^2}{d\zeta^2} + \frac{\left(\nu + \frac{1}{2}\right)^2}{\zeta^2} - \frac{\nu + \frac{1}{2}}{\zeta^2}\gamma_5 + \lambda^2\zeta^2 + \lambda(2\nu + 1) + \lambda\gamma_5$$

where coefficients u and w are fixed to $u = 2$ and $w = 2\lambda^2$

Nucleon Spectrum

- In 2×2 block-matrix form

$$H_{LF} = \begin{pmatrix} -\frac{d^2}{d\zeta^2} - \frac{1-4\nu^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(\nu+1) & 0 \\ 0 & -\frac{d^2}{d\zeta^2} - \frac{1-4(\nu+1)^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda\nu \end{pmatrix}$$

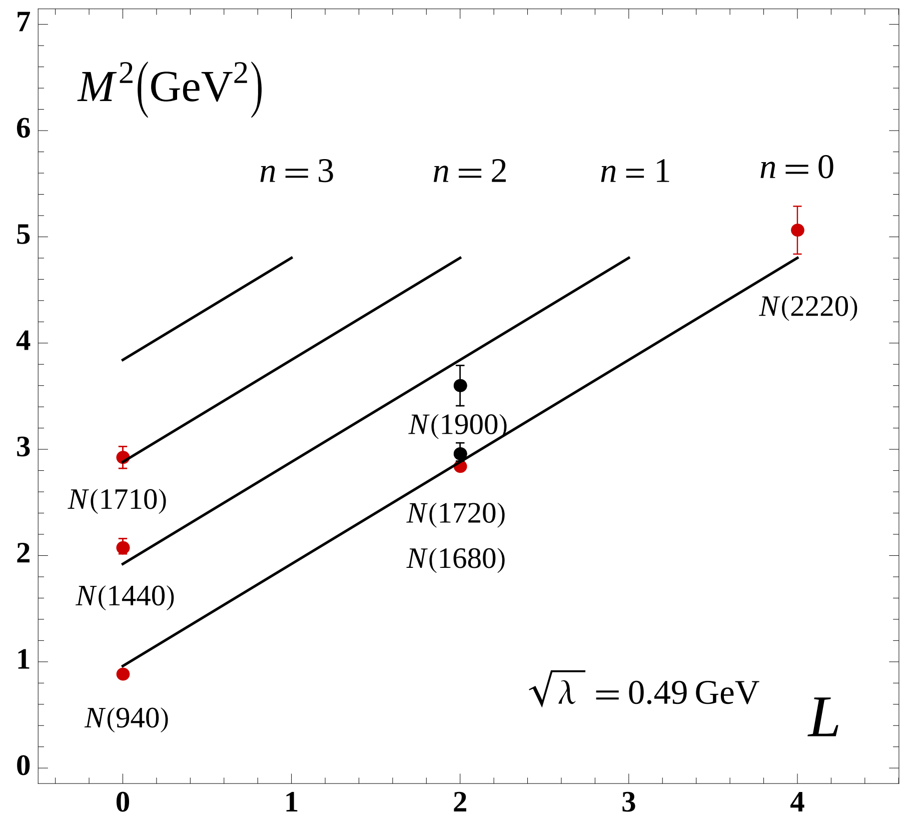
- Eigenfunctions

$$\psi_+(\zeta) \sim \zeta^{\frac{1}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu}(\lambda\zeta^2)$$
$$\psi_-(\zeta) \sim \zeta^{\frac{3}{2}+\nu} e^{-\lambda\zeta^2/2} L_n^{\nu+1}(\lambda\zeta^2)$$

- Eigenvalues

$$M^2 = 4\lambda(n + \nu + 1)$$

- Lowest possible state $n = 0$ and $\nu = 0$
- Orbital excitations $\nu = 0, 1, 2 \cdots = L$
- L is the relative LF angular momentum between the active quark and spectator cluster



(6) Superconformal baryon-meson symmetry

[H.G. Dosch, GdT, and S. J. Brodsky, PRD **91**, 085016 (2015)]

- Previous application: positive and negative chirality components of baryons related by supercharge R

$$R^\dagger|\psi_+\rangle = |\psi_-\rangle$$

with identical eigenvalue M^2 since $[R, G] = [R^\dagger, G] = 0$

- Conventionally supersymmetry relates fermions and bosons

$$R|\text{Baryon}\rangle = |\text{Meson}\rangle \quad \text{or} \quad \text{R}^\dagger\,|\text{Meson}\rangle = |\text{Baryon}\rangle$$

- If $|\phi\rangle_M$ is a meson state with eigenvalue M^2, $G\,|\phi\rangle_M = M^2|\phi\rangle_M$, then there exists also a baryonic state $R^\dagger\,|\phi\rangle_M = |\phi\rangle_B$ with the same eigenvalue M^2:

$$G\,|\phi\rangle_B = G\,R^\dagger|\phi\rangle_M = R^\dagger\,G\,|\phi\rangle_M = M^2|\phi\rangle_B$$

- For a zero eigenvalue M^2 we can have the trivial solution

$$|\phi(M^2 = 0)\rangle_B = 0$$

Special role played by the pion as a unique state of zero energy

Baryon as superpartner of the meson trajectory

$$|\phi\rangle = \begin{pmatrix} \phi_{\text{Meson}} \\ \phi_{\text{Baryon}} \end{pmatrix}$$

- Compare superconformal meson-baryon equations with LFWE for nucleon (leading twist) and pion:

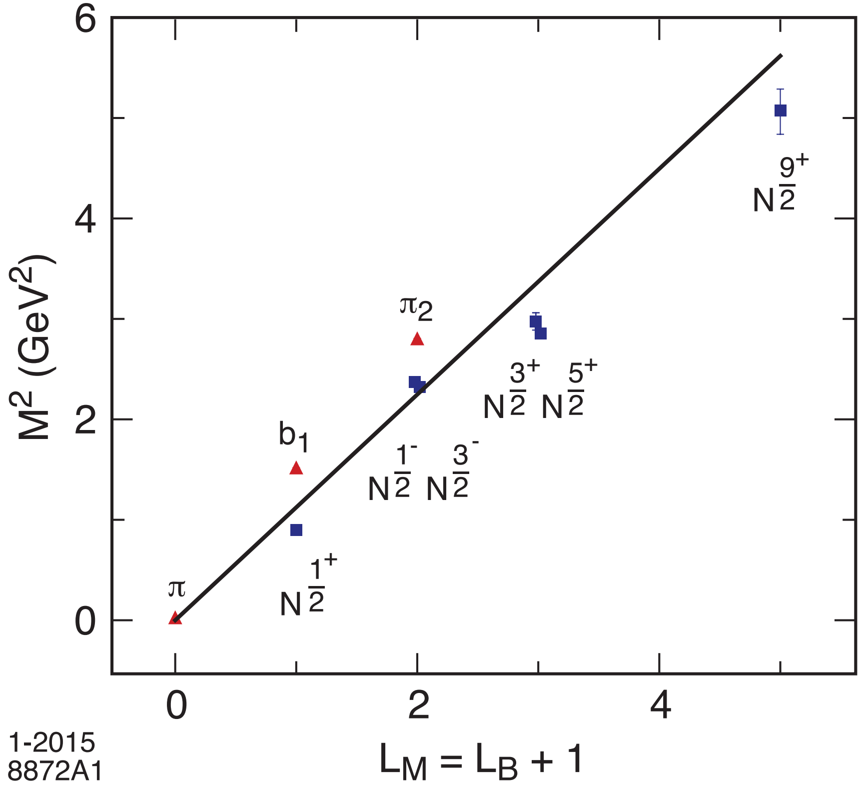
$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f + \lambda + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2}\right)\phi_{\text{Baryon}} = M^2 \phi_{\text{Baryon}}$$

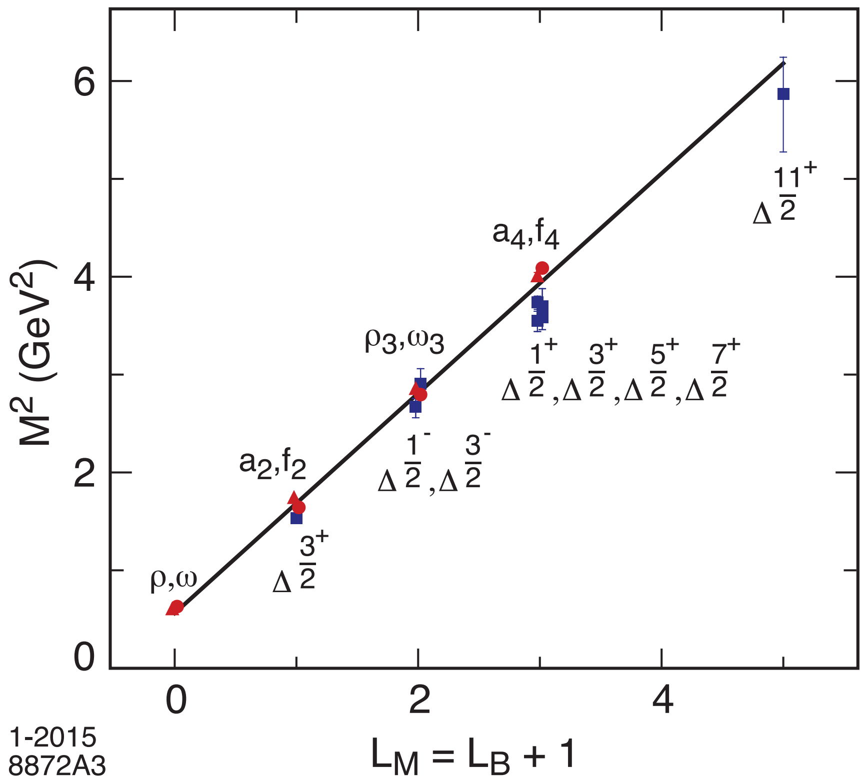
$$\left(-\frac{d^2}{d\zeta^2} + \lambda_B^2 \zeta^2 + 2\lambda_B(L_B + 1) + \frac{4L_B^2 - 1}{4\zeta^2}\right)\psi^+_{L_B} = M^2 \psi^+_{L_B}$$

$$\left(-\frac{d^2}{dx^2} + \lambda^2 x^2 + 2\lambda f - \lambda + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2}\right)\phi_{\text{Meson}} = M^2 \phi_{\text{Meson}}$$

$$\left(-\frac{d^2}{d\zeta^2} + \lambda_M^2 \zeta^2 + 2\lambda_M(L_M - 1) + \frac{4L_M^2 - 1}{4\zeta^2}\right)\phi_{L_M} = M^2 \phi_{L_M}$$

- Find: $\lambda = \lambda_M = \lambda_B, \quad f = L_B + \frac{1}{2} = L_M - \frac{1}{2} \quad \Rightarrow \quad \boxed{L_M = L_B + 1}$



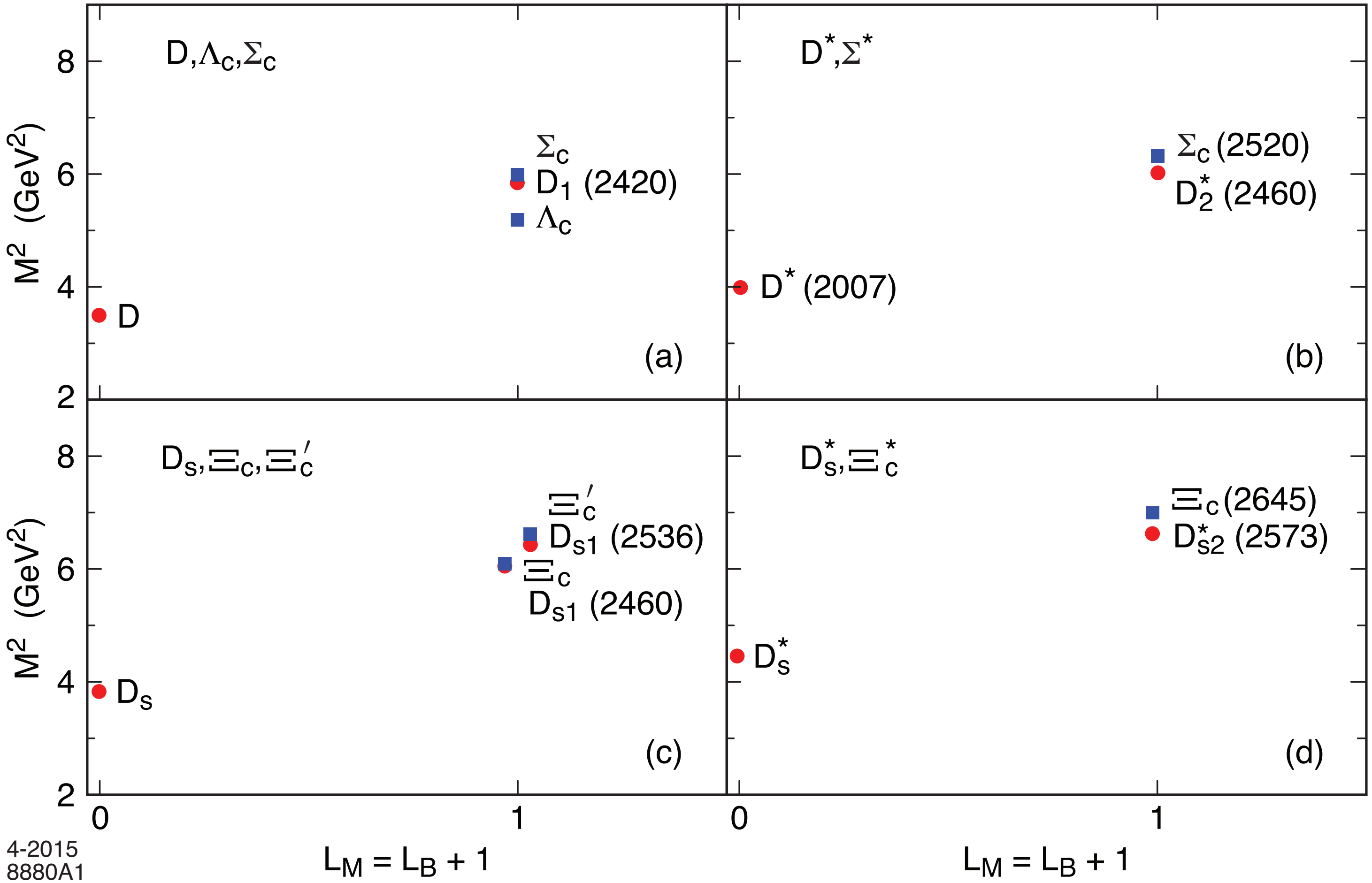


Superconformal meson-nucleon partners: solid line corresponds to $\sqrt{\lambda} = 0.53$ GeV

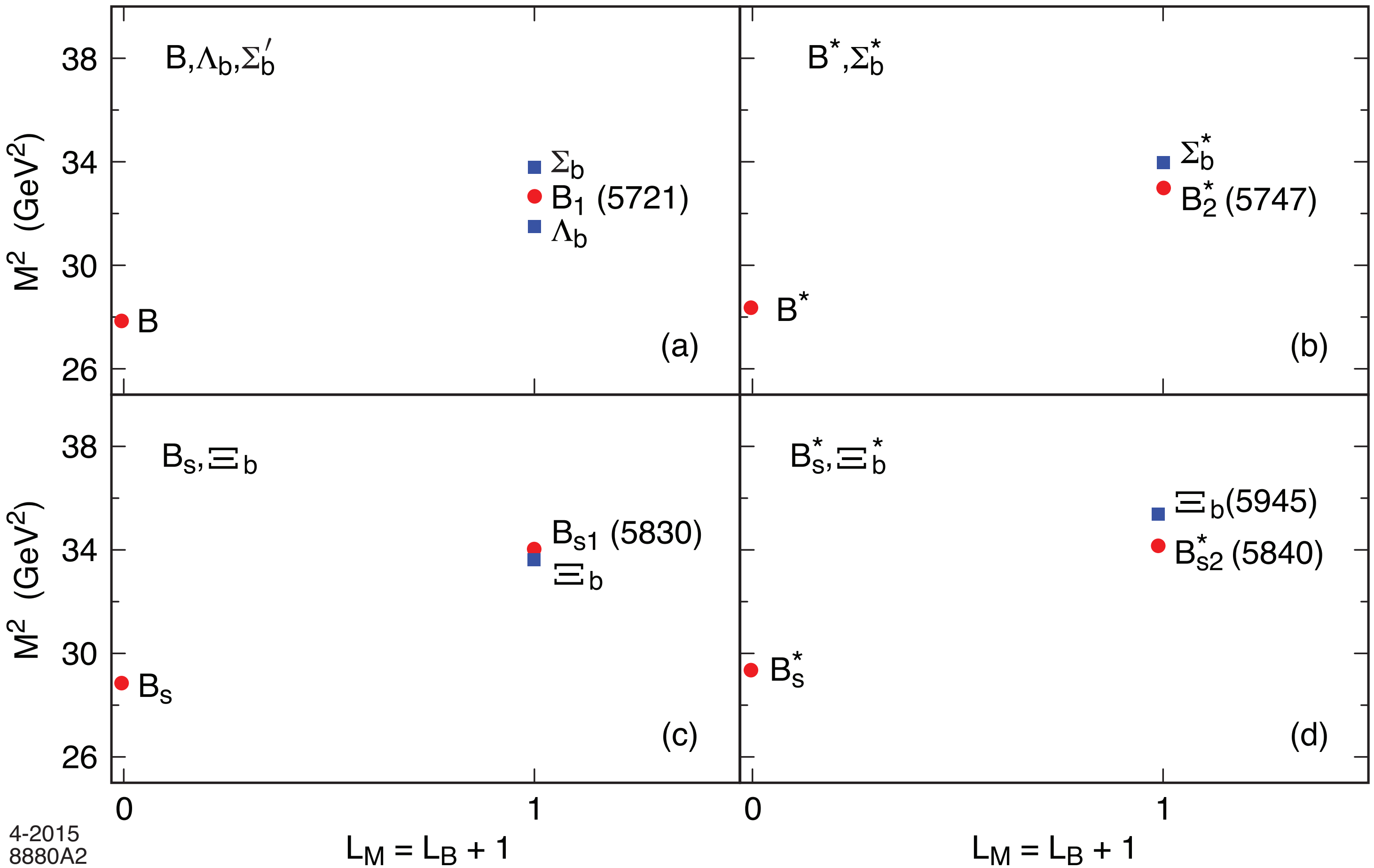
Supersymmetry across the light and heavy-light hadronic spectrum

[H.G. Dosch, GdT, and S. J. Brodsky, Phys. Rev. D **92**, 074010 (2015)]

- Introduction of quark masses breaks conformal symmetry without violating supersymmetry



Supersymmetric relations between mesons and baryons with charm



Supersymmetric relations between mesons and baryons with beauty

Emerging SUSY from color dynamics $\bar{\mathbf{3}} \to \mathbf{3} \times \mathbf{3}$ **Work in progress**

[S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé]

- Superconformal spin-dependent Hamiltonian to describe mesons and baryons (chiral limit)

$$\boxed{G = \{R_\lambda^\dagger, R_\lambda\} + 2\lambda \, \mathbf{I} \, s} \qquad R \sim Q + \lambda S$$

- LFWE for mesons

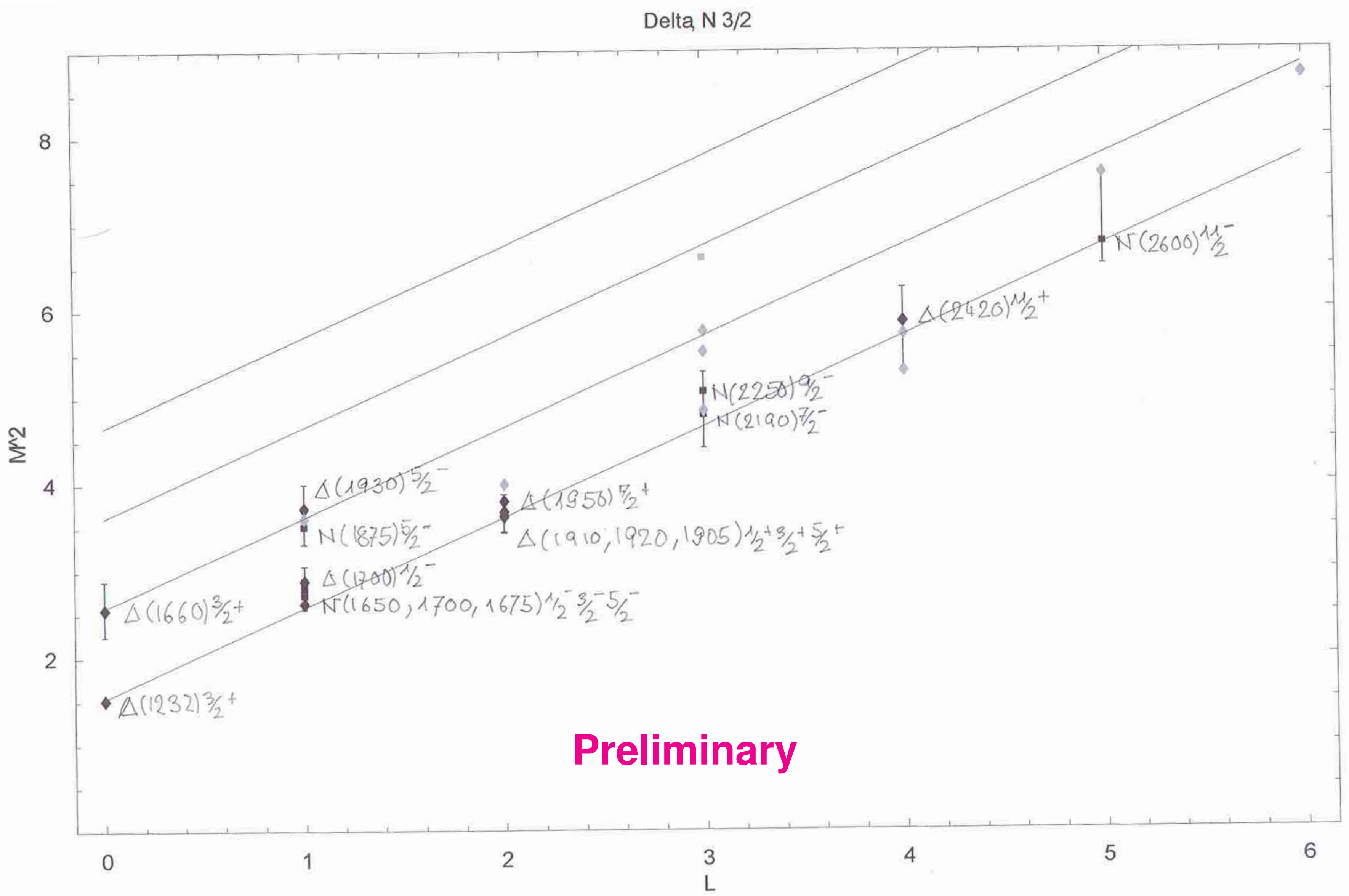
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L_M^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(L_M + s - 1)\right)\phi_{Meson} = M^2\phi_{Meson}$$

- LFWE for nucleons

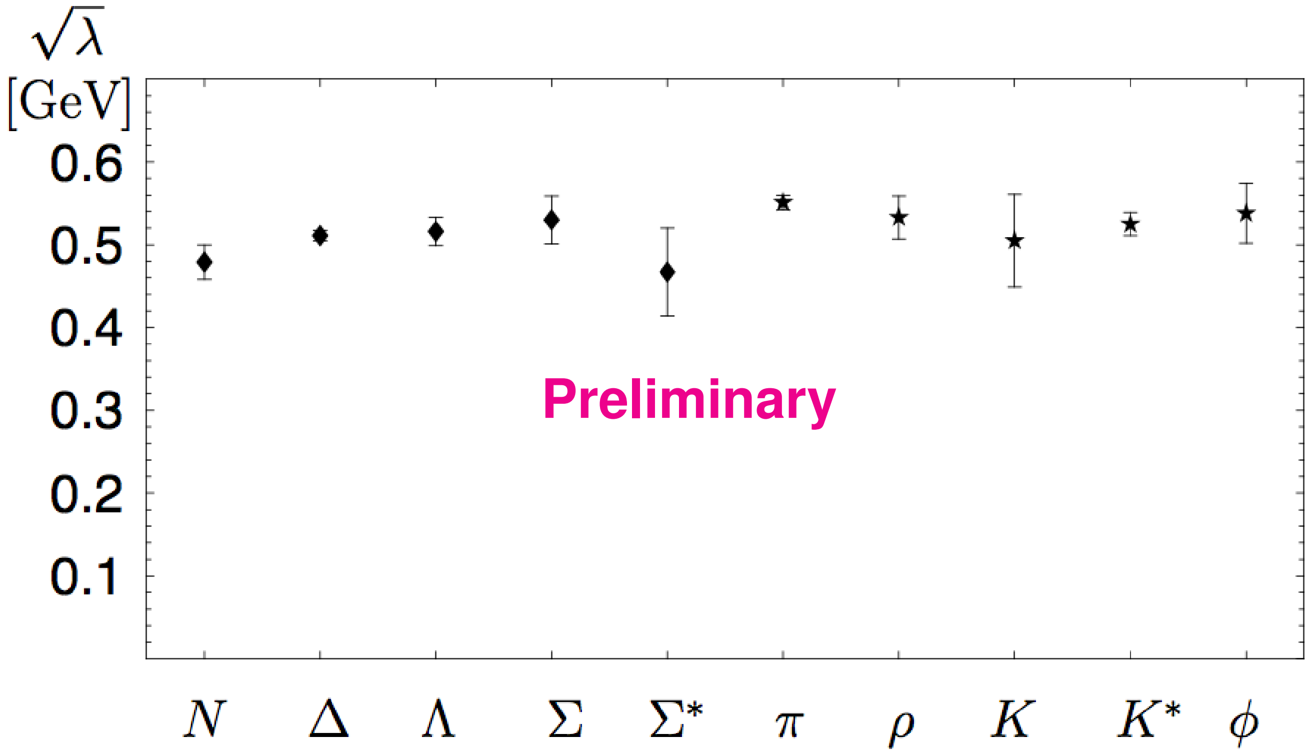
$$\left(-\frac{d^2}{d\zeta^2} - \frac{1 - 4L_B^2}{4\zeta^2} + \lambda^2\zeta^2 + 2\lambda(L_B + s + 1)\right)\phi_{Baryon} = M^2\,\phi_{Baryon}$$

with $L_M = L_B + 1$

- Spin of the spectator cluster s is the spin of the corresponding meson !


Delta, N 3/2
M^2
L
0
1
2
3
4
5
6
8
Δ(1232)3/2+
Δ(1660)3/2+
Δ(1700)1/2−
N(1650, 1700, 1675)1/2− 3/2− 5/2−
Δ(1930)5/2−
N(1875)5/2−
Δ(1950)7/2+
Δ(1910, 1920, 1905)1/2+ 3/2+ 5/2+
N(2250)9/2−
N(2190)7/2−
Δ(2420)11/2+
N(2600)11/2−
Preliminary

- How good is semiclassical approximation based on superconformal QM and LF clustering properties?



Best fit for the hadronic scale $\sqrt{\lambda}$ from the different sectors including radial and orbital excitations

(6) Light-front holographic cluster decomposition and form factors

[S. J. Brodsky, GdT, H. G. Dosch, C. Lorcé] **Work in progress**

- LF Holographic FF $F_{\tau=N}(Q^2)$ expressed as the $N-1$ product of poles for twist $\tau=N$

  S. J. Brodsky and GdT, PRD **77**, 056007 (2008)

$$
\begin{aligned}
F_{\tau=2}(Q^2) &= \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)} \\
F_{\tau=3}(Q^2) &= \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)} \\
&\cdots \\
F_{\tau=N}(Q^2) &= \frac{1}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1+\frac{Q^2}{\mathcal{M}_{\rho^{N-2}}^2}\right)}
\end{aligned}
$$

- Spectral formula

$$
M_{\rho^n}^2 \to 4\kappa^2\,(n+1/2)
$$

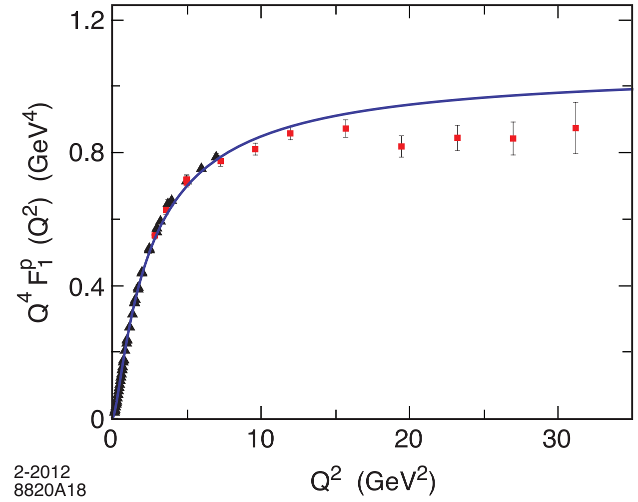
- Cluster decomposition in terms of twist $\tau=2$ FFs !

$$
F_{\tau=N}(Q^2) = F_{\tau=2}\left(Q^2\right) F_{\tau=2}\left(\frac{1}{3}Q^2\right)\cdots F_{\tau=2}\left(\frac{1}{2N-3}Q^2\right)
$$

- Example: Dirac proton FF F_1^p
  in terms of the pion form factor F_π:

$$F_1^p(Q^2) = F_\pi\left(Q^2\right) F_\pi\left(\frac{1}{3}Q^2\right)$$

(equivalent to $\tau = 3$ FF)



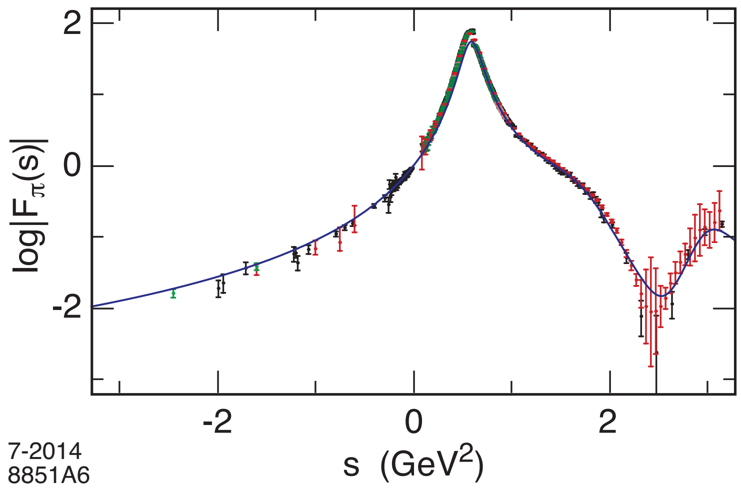
- But . . . we know that higher Fock components are required.
  Example time-like pion FF:

$$|\pi\rangle = \psi_{q\bar{q}/\pi}|q\bar{q}\rangle_{\tau=2} + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle_{\tau=4} + \cdots$$

$$F_\pi(q^2) = (1-\gamma)F_{\tau=2}(q^2) + \gamma F_{\tau=4}(q^2)$$

$$P_{q\bar{q}q\bar{q}} = 12.5\%$$

S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, PR **584**, 1 (2015)



- Transition form factors for the radial transition $n = 0 \to n = 1$:

$$
\begin{aligned}
F_{\tau=2}^{n=0\to1}(Q^2) &= \frac{1}{2}\frac{\frac{Q^2}{M_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)} \\
F_{\tau=3}^{n=0\to1}(Q^2) &= \frac{\sqrt{2}}{3}\frac{\frac{Q^2}{M_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho''}^2}\right)} \\
&\cdots \\
F_{\tau=N}^{n=0\to1}(Q^2) &= \frac{\sqrt{N-1}}{N}\frac{\frac{Q^2}{M_\rho^2}}{\left(1+\frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1+\frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)\cdots\left(1+\frac{Q^2}{\mathcal{M}_{\rho^{N-1}}^2}\right)}
\end{aligned}
$$

where $F_{\tau=N}^{n=0\to1}(Q^2)$ is expressed as the N product of poles

- LF cluster decomposition: Express the transition form factor as the product of the pion transition form factor times the $N-1$ product of pion elastic form factors evaluated at different scales

$$
F_{\tau=N}^{n=0\to1}(Q^2) = \frac{\sqrt{N-1}}{N} F_{\tau=2}^{n=0\to1}\left(Q^2\right) F_{\tau=2}\left(\frac{1}{3}Q^2\right)\cdots F_{\tau=2}\left(\frac{1}{2(N+1)-3}Q^2\right). \tag{1}
$$

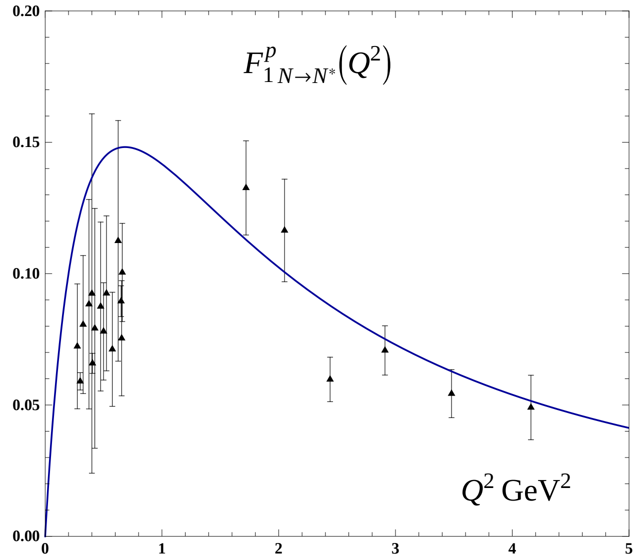
- Example: Dirac transition form factor of the proton to a Roper state $F^p_{1\,N\to N^*}$:

$$F^p_{1\,N\to N^*}(Q^2) = \frac{2\sqrt{2}}{3} F_{\pi\to\pi'}(Q^2) F_\pi\left(\frac{1}{5}Q^2\right)$$

(equivalent to $\tau = 3$ TFF)

[GdT and S. J. Brodsky, AIP Conf. Proc. **1432**, 168 (2012)]

Old JLab data

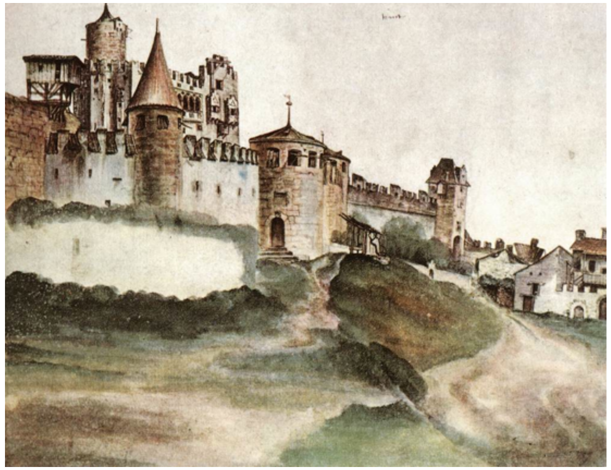


- Holographic QCD computation including $A^p_{1/2}$ and $S^p_{1/2}$:

  T. Gutsche, V. E. Lyubovitskij, I. Schmidt and A. Vega, PRD **87**, 016017 (2013)

- Data confirmed by recent JLab data. Possible solution to describe small ($Q^2 < 1$ GeV^2) data: Include $\tau = 5$ higher Fock component $|qqq\bar{q}q\rangle$ in addition to $\tau = 3$ valence $|qqq\rangle$

$$F_{\tau=3} \sim \frac{1}{Q^4}, \qquad F_{\tau=5} \sim \frac{1}{Q^8}$$



Thanks !

For a review: S. J. Brodsky, GdT, H. G. Dosch and J. Erlich, Phys. Rept. **584**, 1 (2015)