

A Relativistic Model for the Electromagnetic Structure of Baryons from the 3rd Resonance Region

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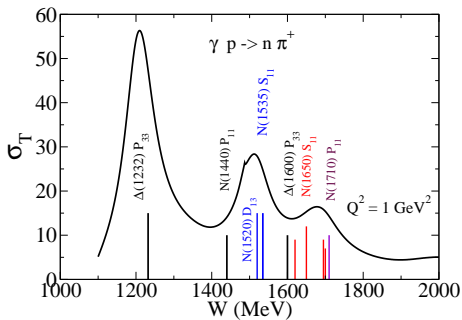
GR and K Tsushima, PRD 89, 073010 (2014); GR, PRD 90, 033010 (2014)

Collaborators: F. Gross (Jlab), M.T. Peña (Lisbon) and K. Tsushima (UCS/São Paulo)

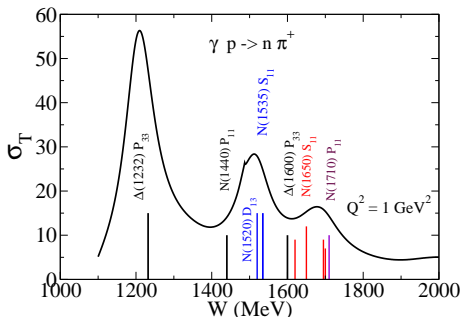
Nucleon Resonances: From Photoproduction to
High Photon Virtualities
ECT*, Trento, Italy

October 15, 2015

Motivation

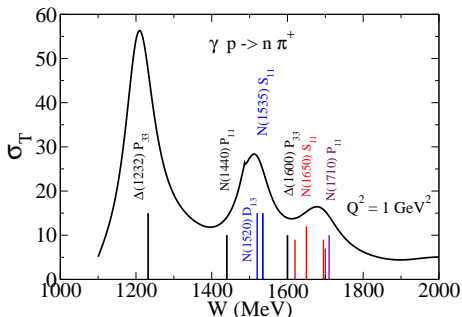


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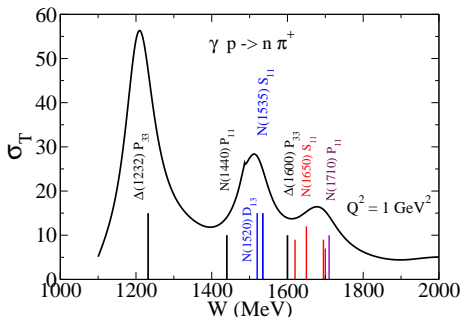
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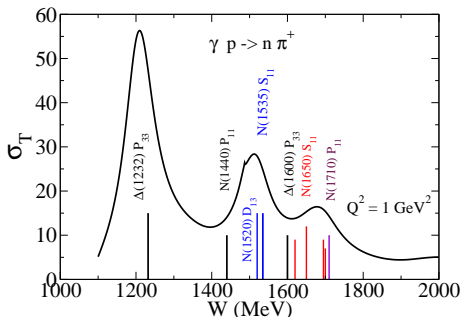
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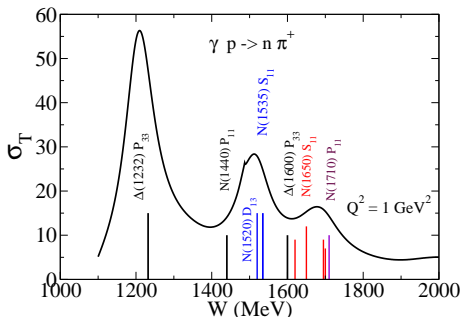
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 - Interpret the data (theory/models)

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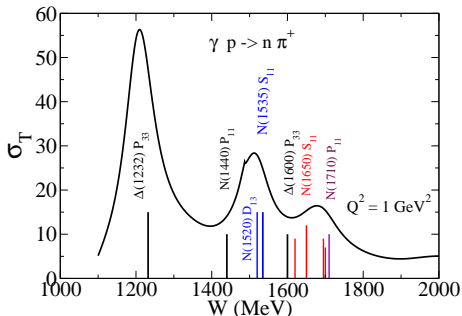
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- Jlab-12 GeV-upgrade**

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- **Challenges:**
 - Interpret the data (theory/models)
 - Provide predictions (higher Q^2 , higher W)**Jlab-12 GeV-upgrade**
- Improve description of the **3rd resonance region** & Extend calculations for higher Q^2

- **Study of $\gamma^* N \rightarrow N^*$ reactions**
- **Covariant Spectator Quark Model**
Wave functions, quark current, transition current
- **Predictions for the $N(1710)$** (2nd radial excitation of the nucleon)
- **Results for $N(1535), N(1520)$** (S_{11} and D_{13})
- **Single Quark Transition Model**
Simple relation between the helicity transition amplitudes of the same $SU(6)$ supermultiplet
- **Application:**
Input: amplitudes for the $N(1520)_{\frac{3}{2}^-}$ and $N(1535)_{\frac{1}{2}^-}$
Output: amplitudes for $N(1650), N(1700), \Delta(1620), \Delta(1700)$

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May fails for small Q^2 – meson cloud effects

Plan of the talk

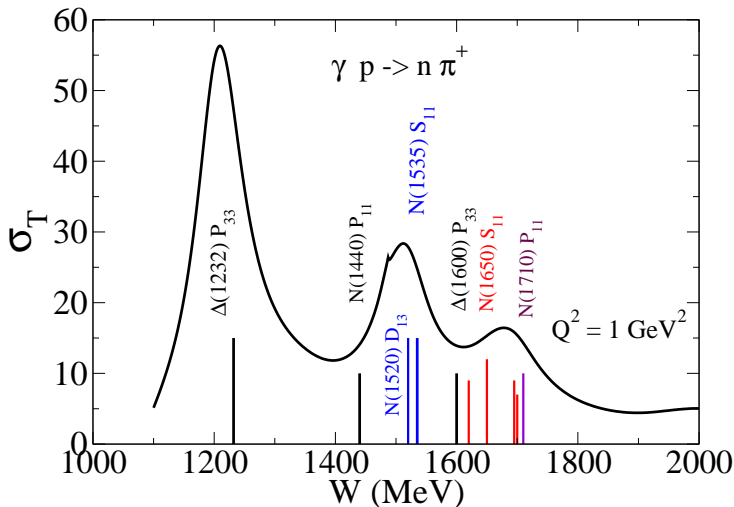
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SQTM has $SU_F(2)$; CSQM breaks $SU_F(2) \Rightarrow$ react. proton targets

Nucleon Resonance Structure



Methods to study the $\gamma^* N \rightarrow N^*$ reactions

- **QCD** (only practical at high Q^2)
- **Lattice QCD** (large m_π , euclidean space ...)
- **(Effective) Chiral Perturbation Theory**
(baryons and mesons and degrees of freedom)
small energy and momentum
- **Baryon-Meson coupled channel reaction models**
- **Dyson-Schwinger** (non-perturbative; quarks and gluons, euclidean)
- **Constituent quark models and chiral quark models**
quarks with structure, quark-quark interaction
- **Covariant Spectator Quark Model (Minkowski)**
Wave function determined phenomenologically (no dynamical eq.)
Parametrization of the wave function by FF (M_B not predicted)

F Gross, GR, MT Peña, K Tsushima, ...

Int. J. Mod. Phys. E **22**, 1330015 (2013)– (pages 89-92); arXiv:1008.0371 [hep-ph]

- Nucleon and Δ electromagnetic form factors

PRC 77, 015202 (2008); PLB 678, 355 (2009); PLB 690, 183 (2010); JPG 36, 085004 (2009); PRD 86, 093022 (2012)

- Electromagnetic transition form factors $\gamma^* N \rightarrow N^*$

$N^* = \Delta(1232), N^*(1440), N^*(1520), N^*(1535), \Delta(1600), N^*(1710), \dots$

EPJA 36, 329 (2008); PRD 78, 114017 (2008); PRD 82, 073007 (2010); PRD 81, 074020 (2010); PRD 84, 051301 (2011); PRD 89, 073010 (2014)

- Octet baryon and decuplet baryon e.m. form factors:

physical regime, nuclear medium and **extension to lattice QCD**

PRD, 033004 80 (2009); JPG 36, 115011 (2009); PRD 80, 013008 (2009); PRD 83, 054011 (2011); PRD 84, 054014 (2011); PRD 87, 093011 (2013); JPG 40, 015102 (2013)

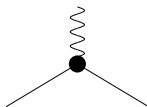
- $\Delta(1232)$ mass distribution for the Dalitz decay: $\Delta \rightarrow Ne^+e^-$ ($pp \rightarrow e^+e^-pp$)

PRD 85, 113014 (2012) **Timelike regime**

- Nucleon – Deep Inelastic Scattering – PRC 77, 015202 (2008); PRD 85 093006 (2012)

Covariant Spectator Quark Model – Introduction

- Quarks with electromagnetic structure
(**impulse approximation**)



$$j_q^\mu = \left(\frac{1}{6} f_{1+} + \frac{1}{2} f_{1-\tau_3} \right) \gamma^\mu + \left(\frac{1}{6} f_{2+} + \frac{1}{2} f_{2-\tau_3} \right) \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}$$

form factors $f_{i\pm}$ parametrized according with **vector meson dominance**
simulate structure associated with $q\bar{q}$ and gluon dressing

- Use **QM symmetries** to represent the **structure of the wave functions**
- **Shape** (radial structure) determined **phenomenologically**
by **experimental data** or **lattice data** of some ground state systems
- constraints from **valence quark d.o.f.** \Rightarrow **Calibrate model**
- **Make predictions** for $\gamma^* N \rightarrow N^*$ form factors/helicity amplitudes

Spectator QM: Transition currents

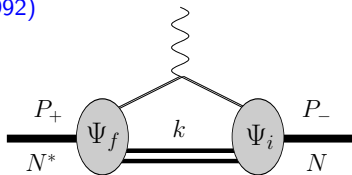
Quark current $j_q^\mu \oplus$ Baryon wave function $\Psi_B \Rightarrow J^\mu$

Transition current J^μ in **spectator formalism**

F Gross et al PR 186 (1969); PRC 45, 2094 (1992)

Relativistic impulse approximation:

$$J^\mu = 3 \sum_\lambda \int \bar{\Psi}_f(P_+, k) j_q^\mu \Psi_i(P_-, k)$$



integrate spectator q

$$q = P_+ - P_-, \quad P = \frac{1}{2}(P_+ + P_-), \quad Q^2 = -q^2$$

Spectator QM: Transition currents

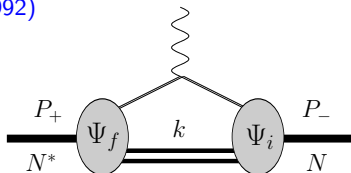
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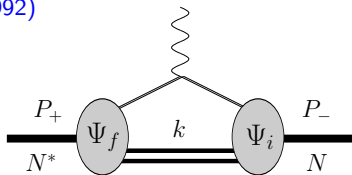
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If $q \cdot J \neq 0$: Landau prescription: $J^\mu \rightarrow J^\mu - \frac{q \cdot J}{q^2} q^\mu$

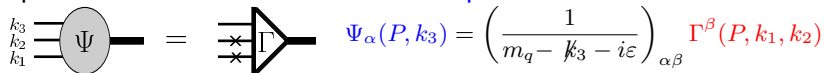
JJ Kelly, PRC 56, 2672 (1997); Z Batiz and F Gross, PRC 58, 2963 (1998)

Spectator QM: Baryon wave functions (1)

- Baryon: 3 constituent quark system

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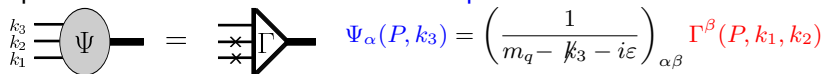
- Baryon: 3 constituent quark system
- **Covariant Spectator Theory**: wave function Ψ defined in terms of a 3-quark vertex Γ with 2 on-mass-shell quarks


$$\Psi_\alpha(P, k_3) = \left(\frac{1}{m_q - \not{k}_3 - i\varepsilon} \right)_{\alpha\beta} \Gamma^\beta(P, k_1, k_2)$$

Gross and Agbakpe PRC 73, 015203 (2006); Gross, GR and Peña PRC 77, 015202 (2008)

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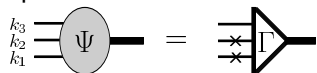

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- Ψ is **free** of singularities ($3q$ on-shell $\Gamma \equiv 0$) \Rightarrow parametrize Ψ
Stadler, Gross and Frank PRC 56, 2396 (1998); Savkli and Gross PRC 63, 035208 (2001)

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- On-shell integration (k_1, k_2) \Rightarrow $k = k_1 + k_2$, $r = \frac{1}{2}(k_1 - k_2)$
 \Rightarrow integration in \mathbf{k} and $s = (k_1 + k_2)^2$
Gross, GR and Peña, PRC 77, 015202 (2008); PRD 85, 093005 (2012)

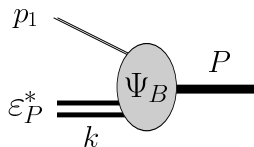
$$\int_{k_1} \int_{k_2} = \frac{\pi}{4} \int d\Omega_{\hat{\mathbf{r}}} \int_{4m_q^2}^{+\infty} ds \sqrt{\frac{s - 4m_q^2}{s}} \int \frac{d^3\mathbf{k}}{2\sqrt{s + \mathbf{k}^2}} \rightarrow \int \frac{d^3\mathbf{k}}{2\sqrt{m_D^2 + \mathbf{k}^2}}$$

Mean value theorem: $\sqrt{s} \rightarrow m_D$; cov. int. in diquark **on-shell** mom.

Spectator QM: Baryon wave functions (2)

- **Effective diquark** justified by the **Impulse approximation**
- **Baryon wave functions**: $B = \mathbf{diquark} \oplus \mathbf{quark}$
Combination of **diquark** (12) and single **quark** (3) states, using $SU(6) \otimes O(3)$:

$$\Psi_B = \sum (\text{color}) \otimes (\text{flavor}) \otimes (\text{spin-orbital}) \otimes \underbrace{\psi_B(P, k)}_{\text{radial}}$$



- Wave function Ψ_B expressed at the rest frame
- **Covariant** generalization of Ψ_B in terms **baryon properties** after *integration* on the diquark internal variables
- **Phenomenology** included on the **quark-diquark radial wave function**

$$\psi_N(\chi) = \frac{N_0}{m_D(\beta_1 + \chi)(\beta_2 + \chi)}, \quad \chi = \frac{(M - m_D)^2 - (P - k)^2}{Mm_D}$$

β_1, β_2 : momentum scale parameters

Spectator QM: Nucleon wave function †

Nucleon wave function: [PRC 77,015202 (2008); EPJA 36, 329 (2008)]

Simplest structure –**S-state** in quark-diquark system (rest frame)

$$\Psi_N(P, k) = \frac{1}{\sqrt{2}} [\Phi_I^0 \Phi_S^0 + \Phi_I^1 \Phi_S^1] \psi_N(P, k)$$

Isospin states: $\Phi_I^{0,1}$

Spin states: defined in terms of Nucleon-Dirac spinor $u(P)$; diquark polarization vector ε_λ

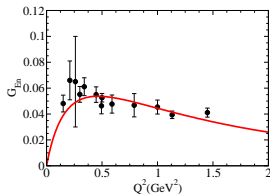
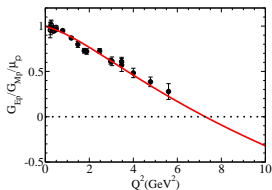
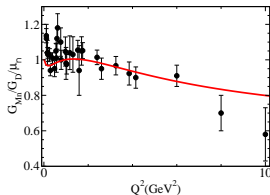
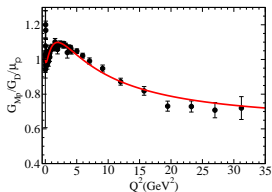
$$\Phi_S^0(s) \equiv u(P, s) \quad \Phi_S^1(s) \equiv -(\varepsilon_\lambda^*)_\alpha U^\alpha(P, s)$$

$$U^\alpha(P, s) = \sum_{\lambda s'} \langle \frac{1}{2} s'; 1\lambda | \frac{1}{2} s \rangle \varepsilon_\lambda^\alpha u(P, s') \rightarrow \frac{1}{\sqrt{3}} \gamma_5 \left(\gamma^\alpha - \frac{P^\alpha}{M} \right) u(P, s)$$

$\varepsilon_\lambda = \varepsilon_{\lambda P}$ **function of nucleon momentum**

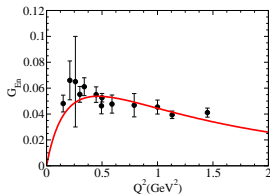
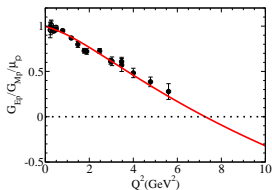
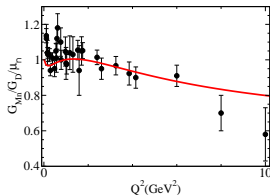
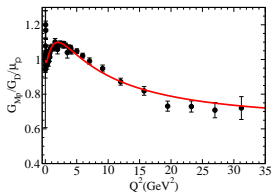
F Gross, GR and MT Peña, PRC 77, 035203 (2008)

Nucleon form factors [F Gross, GR and MT Peña, PRC 77, 015202 (2008)]



- Model calibrated by Nucleon form factor data
- Quark current fix 4 parameters; Scalar wave function (2 parameters)
- No pion cloud (explicit);

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- Model calibrated by Nucleon form factor data
- Quark current fix 4 parameters; Scalar wave function (2 parameters)
- No pion cloud (explicit); can be extended to the lattice QCD regime

GR and MT Peña, JPG 36, 115011 (2009); PRD 80 (2009) 013008; GR, K Tushima and AW Thomas, JPG 40 015102 (2013)

$\gamma^* N \rightarrow R$, $R =$ radial excitation of the nucleon

- $N0 =$ Nucleon
 $N1 = N(1440) \equiv$ Roper, 1st radial excitation
 $N2 = N(1710) \approx$ 2nd radial excitation

Same spin and isospin structure as the nucleon

- States distinguished by radial wave function:
 ψ_{N0} , ψ_{N1} , ψ_{N2} (and masses)
- **Orthogonality** given at $Q^2 = 0$ by

$$\int_k \psi_{N1} \psi_{N0} = 0, \quad \int_k \psi_{N2} \psi_{N0} = 0, \quad \int_k \psi_{N2} \psi_{N1} = 0,$$

- \Rightarrow Define ψ_{N1} , ψ_{N2} , from ψ_{N0}
with the same short-range structure: $\psi_{Nj} \propto \frac{1}{\beta_2 + \chi}$
- **No adjustable parameters** \rightarrow predictions

GR and K Tsushima, PRD 81, 074020 (2010); PRD 89 073010 (2014)

$\gamma^* N \rightarrow R$, $R =$ radial excitation of the nucleon (2)

Radial wave functions $\beta_2 > \beta_1$ (β_2 – short range)

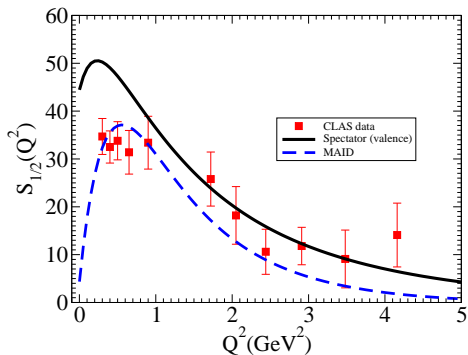
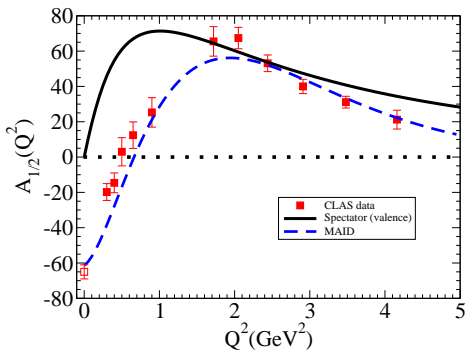
$$\psi_{N0}(\chi_{N0}) = N_0 \times \frac{1}{m_D(\beta_1 + \chi_N)(\beta_2 + \chi_N)}$$

$$\psi_{N1}(\chi_{N1}) = N_1 \frac{\beta_3 - \chi_{N1}}{\beta_1 + \chi_{N1}} \times \frac{1}{m_D(\beta_1 + \chi_{N1})(\beta_2 + \chi_{N1})}$$

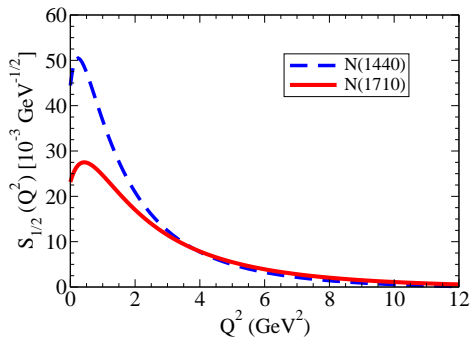
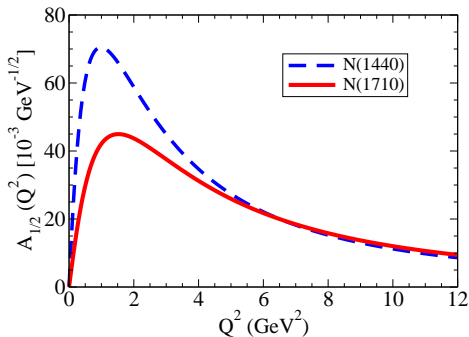
$$\psi_{N2}(\chi_{N2}) = N_2 \frac{\chi_{N2}^2 - \beta_4 \chi_{N2} + \beta_5}{(\beta_1 + \chi_{N2})^2} \times \frac{1}{m_D(\beta_1 + \chi_{N2})(\beta_2 + \chi_{N2})}$$

$\beta_3, \beta_4, \beta_5 \Leftarrow$ Orthogonality conditions

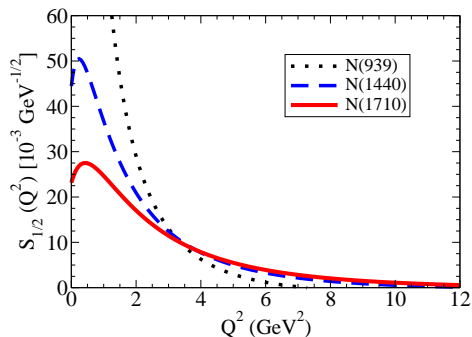
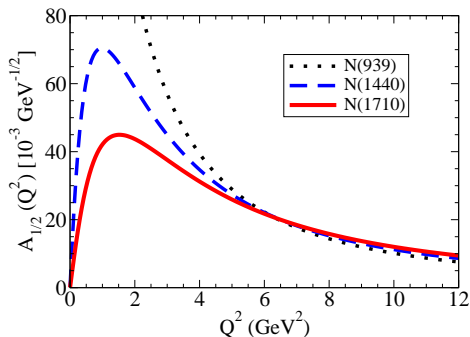
$\gamma^* N \rightarrow N(1440)$: Helicity amplitudes [PRD 81, 074020 (2010)] †



- CLAS data - Aznauryan et al PRC 80, 055203 (2009), MAID fit
- Good agreement for $Q^2 > 1.5 \text{ GeV}^2$
- Difference for $Q^2 < 1.5 \text{ GeV}^2$ –manifestation of meson cloud
- Good description also of lattice data Valence q d.o.f.



- Prediction of $N(1710)$ compared with Roper amplitudes
- **Results similar with Roper for $Q^2 > 4 \text{ GeV}^2$**
Same short-range structure
- **Low Q^2** : no prediction – **dominance of meson cloud**

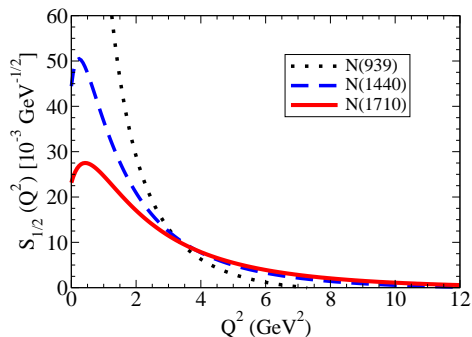
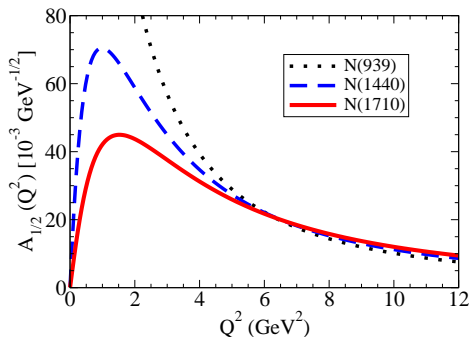


- Compare with nucleon form factors (\mathcal{R} - Roper)

$$\mathcal{R} = \frac{e}{2} \sqrt{\frac{(M_R - M)^2 + Q^2}{M_R M K}}, \quad K = \frac{M_R^2 - M^2}{2M_R}, \quad \tau \rightarrow \frac{Q^2}{4M^2}$$

- *Equivalent* amplitudes (extra factor $\sqrt{2}$)

$$A_{1/2} \rightarrow \sqrt{2} \mathcal{R} G_M, \quad S_{1/2} \rightarrow \sqrt{2} \frac{\mathcal{R}}{\sqrt{2}} \sqrt{\frac{1+\tau}{\tau}} G_E,$$



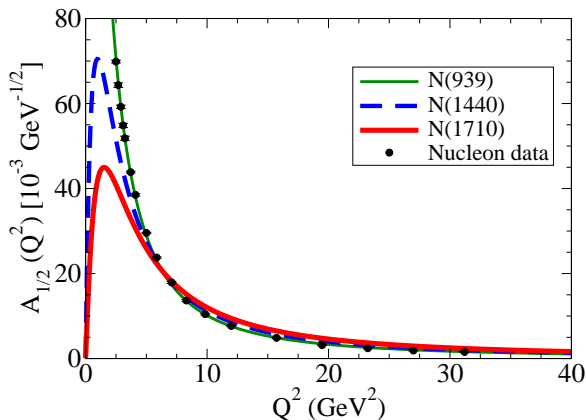
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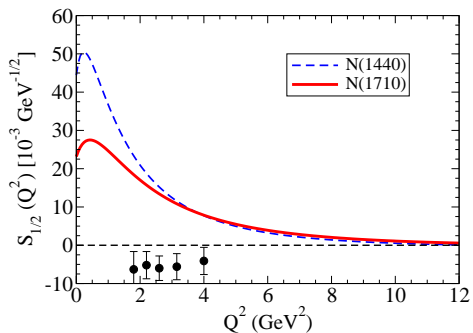
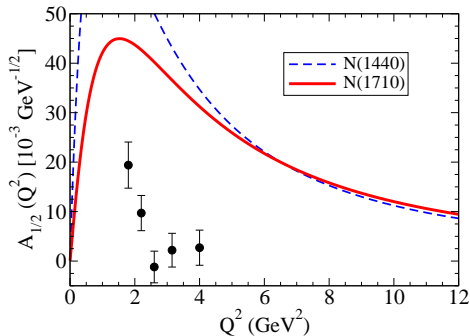
- *Equivalent* amplitudes (extra factor $\sqrt{2}$) Prediction

$$A_{1/2} \rightarrow \sqrt{2} \mathcal{R} G_M, \quad S_{1/2} \rightarrow \sqrt{2} \frac{\mathcal{R}}{\sqrt{2}} \sqrt{\frac{1+\tau}{\tau}} G_E,$$

Amplitude $A_{1/2}$: Roper, $N(1710) \approx G_M(\text{Nucleon})$



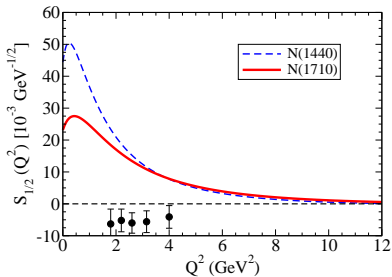
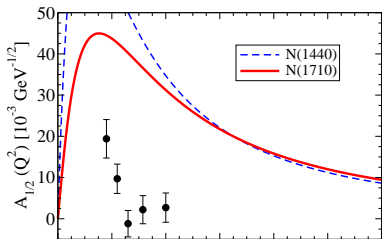
Data: J. Arrington, W. Melnitchouk and J. A. Tjon, PRC **76**, 035205 (2007)



CLAS data: K Park et al, **PRC 91, 045203 (2015)**

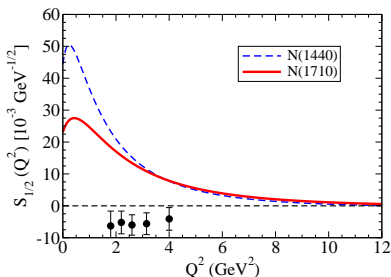
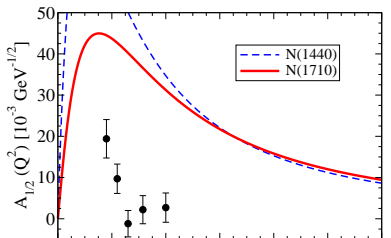
— model predictions fail for intermediate Q^2

Amplitude $S_{1/2}$: difference of sign ...



Possible interpretations:

- Our results are valid only for larger Q^2
- $N(1710)$ it is not just a radial excitation
- There are mixture of (close) states
- $N(1710)$ is a dynamically generated resonance (EBAC): $N(1820)$
N Suzuki *et al*, PRL 104, 042302 (2010)



Discussion

- We can test if there is a dominance of the **valence quark** effects: large Q^2 :

$$A_{1/2} \propto \frac{1}{Q^3}, \quad S_{1/2} \propto \frac{1}{Q^3}$$

- Dominance of meson cloud $qqq - (q\bar{q})$:
suppression $\propto 1/Q^4$ – stronger falloff

Baryon-meson molecule

($\pi N - \pi\pi N$; $\pi N - \sigma N$; $\sigma_v N$, gN , ...)

- Most quark models predict

$$A_{1/2} > 0, \quad S_{1/2} > 0$$

T Melde *et al*, PRD 77 114002 (2008);

M Ronniger *et al*, EPJA 49, 8 (2013)

- Hyperspherical QM predicts $S_{1/2} < 0$

Santopinto and Giannini, PRC 86, 065202 (2012)

good description of the data

- Results for $N(1535)\frac{1}{2}^-$, $N(1520)\frac{3}{2}^-$
- Single Quark Transition Model
- Predictions for $N(1650)\frac{1}{2}^-$, $N(1700)\frac{3}{2}^-$, $\Delta(1620)\frac{1}{2}^-$, $\Delta(1700)\frac{1}{2}^-$

Wave functions $N(1520), N(1535)$ [$N^2 : s_q = 1/2; N^4 : s_q = 3/2$] †

Using the $SU(6) \otimes O(3)$ structure; flavor wf: $\Phi_I^{0,1}$; spin wf: $X_{\lambda,\rho}^S, S = \frac{1}{2}, \frac{3}{2}$

λ = symmetric ρ = anti – symmetric

$$\left| N^2, \frac{1}{2}^- \right\rangle = N_{1/2} \left[\Phi_I^0 X_\rho^{1/2} + \Phi_I^1 X_\lambda^{1/2} \right] \psi_{S11}$$

$$\left| N^2, \frac{3}{2}^- \right\rangle = N_{3/2} \left[\Phi_I^0 X_\rho^{3/2} + \Phi_I^1 X_\lambda^{3/2} \right] \psi_{D13}$$

$$\left| N^4, S^- \right\rangle = \dots$$

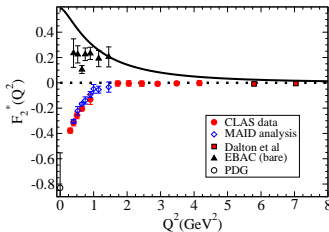
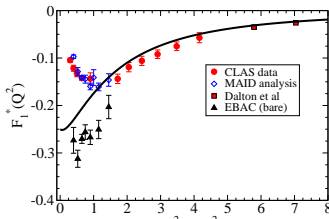
diquark: $k_\lambda = k_1 + k_2$, diquark internal momentum $k_\rho = \frac{1}{2}(k_1 - k_2)$,

$$X_\rho^S(s) = \sum_{ms'} \langle 1\frac{1}{2}; ms' | Ss \rangle \left[Y_{1m}(k_\rho) |s'\rangle_\lambda + Y_{1m}(k_\lambda) |s'\rangle_\rho \right]$$

$$X_\lambda^S(s) = \sum_{ms'} \langle 1\frac{1}{2}; ms' | Ss \rangle \left[Y_{1m}(k_\rho) |s'\rangle_\rho - Y_{1m}(k_\lambda) |s'\rangle_\lambda \right],$$

\Rightarrow covariant generalization: $\mathbf{k} \rightarrow k - \frac{k \cdot P}{P^2} P; Y_{lm}(k_\rho) \rightarrow \xi^m; |s'\rangle_{\rho,\lambda}$

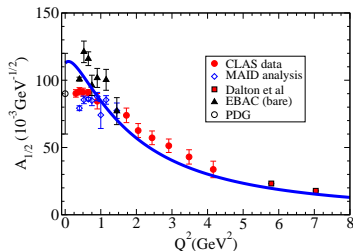
- Spin 1/2 dominance
 $\cos \theta_S \simeq 0.85 \rightarrow 1$
 $|N(1535)\rangle \simeq |N^2, \frac{1}{2}^-\rangle$
- Pointlike diquark $Y_{10}(k_\rho) \rightarrow 0$
 $k_\rho = \frac{1}{2}(k_1 - k_2) \rightarrow 0$
- $\psi_{S11} \approx \psi_N$
 $\mathcal{I}_{S11}(Q^2) = \int_k \frac{k_z}{|\mathbf{k}|} \psi_{S11} \psi_N$
- **Approximated orthogonality**
 $\mathcal{I}_{S11}(0) \neq 0 (\Rightarrow F_1^*(0) \neq 0)$
- F_1^* – good model for large Q^2
- F_2^* – model fails, ...
describes EBAC data
(quark core)
- Data ($Q^2 > 1.5 \text{ GeV}^2$): $F_2^* \approx 0$



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 describes EBAC data
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- Data ($Q^2 > 1.5 \text{ GeV}^2$): $F_2^* \approx 0$

When $F_2^* = 0$ ($A_{1/2} \propto F_1^*$)

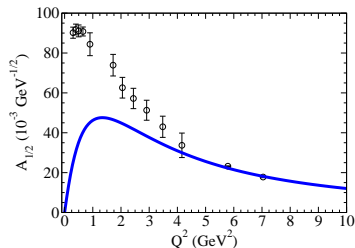
$$A_{1/2} = -\frac{2}{3} F_S (f_{1+} + 2f_{1-} - \tau_3) \mathcal{I}_{S11}$$



- No pointlike diquark
change normalization
- Orthogonality imposed
 $\mathcal{I}_{S11}(0) \equiv 0$
Redefine ψ_{S11}
(new parameter β_3)
- ψ_{S11} adjusted
to high Q^2 data
- Then $F_1^*(0) = 0$
But cannot
describe low Q^2
(meson cloud !!)

When $F_2^* = 0$ ($A_{1/2} \propto F_1^*$)

$$A_{1/2} = -\frac{\sqrt{2}}{3} F_S (f_{1+} + 2f_{1-} - \tau_3) \mathcal{I}_{S11}$$



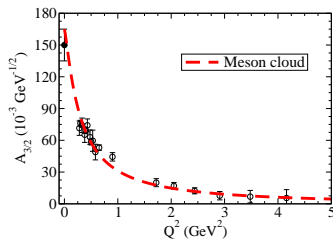
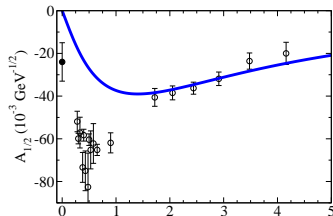
- Spin 1/2 dominance
($\cos \theta_D \simeq 1$)

- Amplitudes:

$$\mathcal{I}_{D13} = \int_k \frac{k_z}{|\mathbf{k}|} \psi_{D13} \psi_N$$

$$A_{1/2} \propto (f_{1+} + 2f_{1-} - \tau_3) \mathcal{I}_{D13} \\ + (f_{2+} + 2f_{2-} - \tau_3) \mathcal{I}_{D13} \\ \text{(valence)}$$

- ψ_{D13} fitted to high Q^2 data
- Orthogonality: $\mathcal{I}_{D13}(0) = 0$



- Spin 1/2 dominance
($\cos \theta_D \simeq 1$)

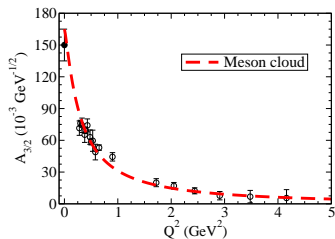
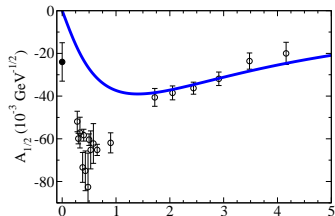
- Amplitudes:

$$\mathcal{I}_{D13} = \int_k \frac{k_z}{|\mathbf{k}|} \psi_{D13} \psi_N$$

$$A_{1/2} \propto (f_{1+} + 2f_{1-} - \tau_3) \mathcal{I}_{D13} \\ + (f_{2+} + 2f_{2-} - \tau_3) \mathcal{I}_{D13} \\ \text{(valence)}$$

$$A_{3/2} = \frac{\sqrt{3}}{4} F_D G_4^\pi \\ \text{(meson cloud)}$$

- ψ_{D13} fitted to high Q^2 data
- Orthogonality: $\mathcal{I}_{D13}(0) = 0$



N^* radial wave function (optional)

$$\psi_R(P, k) = \frac{N_R}{m_D(\beta_2 + \chi)} \left\{ \frac{1}{\beta_1 + \chi} - \frac{\lambda_R}{\beta_3 + \chi} \right\}$$

New short range parameter β_3 – determined by large Q^2 data

- Model for $N(1520)$, $N(1535)$ that include diquark structure
- Model describes the high Q^2 regime
(adding a adjustable parameter β_i for resonance)
- For $N(1520)$ the amplitude $A_{3/2}$ is the consequence of **meson cloud**
(zero contribution from **valence quarks**)
 $\Rightarrow A_{3/2}$ phenomenological parametrization
- Small Q^2 : failure of the model;
no meson cloud effects included (except for $A_{3/2}^{D13}$)

Single Quark Transition Model

- Wave function given by $SU(6) \otimes O(3)$ group:
supermultiplets $[SU(6), L^P]$ - $SU(6)$: number of particles (inc. spin proj.)
 Hey and Weyers, PL 48B, 69(1974); Cottingham and Dunbar, ZPC 2, 41 (1979);
 Burkert et al, PRC 67, 035204 (2003)
- Photon interaction with the quarks in **impulse approximation**
- Transverse current:

$$J^+ = AL^+ + B\sigma^+L_z + C\sigma_zL^+ + D\sigma^-L^+L^-$$

A, B, C, D functions of Q^2 for the same $[SU(6), L^P]$

- supermultiplet $[70, 1^-]$ (**negative parity**):
 $N(1520), N(1535), N(1650), N(1700), \Delta(1620), \Delta(1700)$
 – only 3 independent coefficients: A, B, C
- $SU(6)$ breaking: $\theta_S \approx 31^\circ, \theta_D \approx 6^\circ$ ($1/2^- = S11, 3/2^- = D13$)

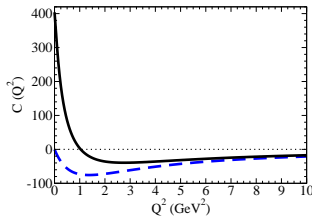
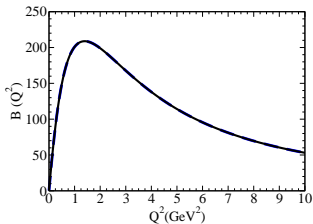
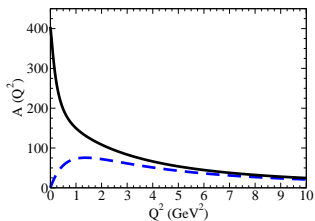
$$\begin{aligned}
 |N(1535)\rangle &= \cos\theta_S \overbrace{\left|N^2, \frac{1}{2}^-\right\rangle}^{s_q=1/2} - \sin\theta_S \overbrace{\left|N^4, \frac{1}{2}^-\right\rangle}^{s_q=3/2}, & |N(1520)\rangle &= \cos\theta_D \overbrace{\left|N^2, \frac{3}{2}^-\right\rangle}^{s_q=1/2} - \sin\theta_D \overbrace{\left|N^4, \frac{3}{2}^-\right\rangle}^{s_q=3/2} \\
 |N(1650)\rangle &= \sin\theta_S \left|N^2, \frac{1}{2}^-\right\rangle + \cos\theta_S \left|N^4, \frac{1}{2}^-\right\rangle, & |N(1700)\rangle &= \sin\theta_D \left|N^2, \frac{3}{2}^-\right\rangle + \cos\theta_D \left|N^4, \frac{3}{2}^-\right\rangle
 \end{aligned}$$

SQTM: $[70, 1^-]$ amplitudes

State	Amplitude	
$S_{11}(1535)$	$A_{1/2}$	$\frac{1}{6}(A + B - C) \cos \theta_S$
$D_{13}(1520)$	$A_{1/2}$	$\frac{1}{6\sqrt{2}}(A - 2B - C) \cos \theta_D$
	$A_{3/2}$	$\frac{1}{2\sqrt{6}}(A + C) \cos \theta_D$
$S_{11}(1650)$	$A_{1/2}$	$\frac{1}{6}(A + B - C) \sin \theta_S$
$S_{31}(1620)$	$A_{1/2}$	$\frac{1}{18}(3A - B + C)$
$D_{13}(1700)$	$A_{1/2}$	$\frac{1}{6\sqrt{2}}(A - 2B - C) \sin \theta_D$
	$A_{3/2}$	$\frac{1}{2\sqrt{6}}(A + C) \sin \theta_D$
$D_{33}(1700)$	$A_{1/2}$	$\frac{1}{18\sqrt{2}}(3A + 2B + C)$
	$A_{3/2}$	$\frac{1}{6\sqrt{6}}(3A - C)$

SQTM: Functions A , B and C

$$A = 2 \frac{A_{1/2}^{S11}}{\cos \theta_S} + \sqrt{2} A_{1/2}^{D13} + \sqrt{6} A_{3/2}^{D13}, \quad B = 2 \frac{A_{1/2}^{S11}}{\cos \theta_S} - 2\sqrt{2} A_{1/2}^{D13}, \quad C = -2 \frac{A_{1/2}^{S11}}{\cos \theta_S} - \sqrt{2} A_{1/2}^{D13} + \sqrt{6} A_{3/2}^{D13}$$



- - - only valence quark contributions ($A + C = 0$) – Model 1
- include meson cloud ($A_{3/2}^{D13}$) – Model 2

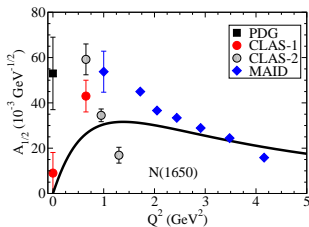
Based in results for $S11$ and $D13$: \Rightarrow predictions for $Q^2 > 2 \text{ GeV}^2$

Data:

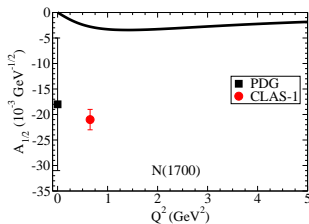
- **CLAS:**
I G Aznauryan, et al, PRC 72, 045201 (2005);
M. Dugger et al. (CLAS Collaboration), PRC 79, 065206 (2009)
- **CLAS-2:** preliminary CLAS data
V Mokeev et al, arXiv:1509.05460; V Mokeev, NSTAR 2015
- **MAID:**
D. Drechsel et al EJPA, 34, 69 (2007); L. Tiator et al, Chin. Phys. C 33, 1069 (2009); Eur. Phys. J. Spec. Top. 198, 141 (2011)
<http://wwwkph.kph.unimainz.de/MAID//maid2007/data.html>.
- **NSTAR:**
V D Burkert et al PRC 67, 035204 (2003);
V. Burkert, T.-S. H. Lee, R. Gothe, and V. Mokeev, Electromagnetic N-N* Transition Form Factors Workshop, Jlab, Newport News, 2008 (unpublished)

Results for $N(1650)$, $N(1700)$

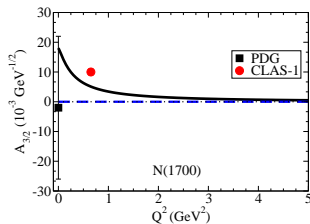
$A_{1/2}$



$A_{1/2}$



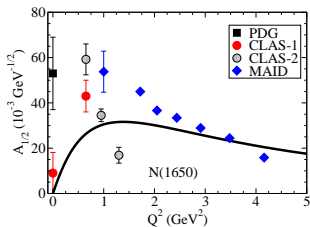
$A_{3/2}$



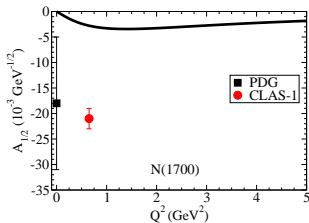
Data from **CLAS**, preliminary CLAS (CLAS-2), **MAID** and PDG

Results for $N(1650)$, $N(1700)$

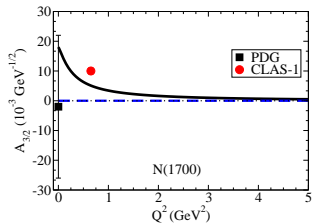
$A_{1/2}$



$A_{1/2}$



$A_{3/2}$

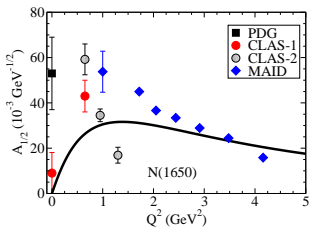


Data from **CLAS**, preliminary CLAS (CLAS-2), **MAID** and PDG

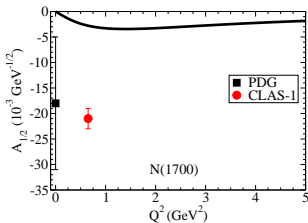
- Model 2: better for $A_{3/2} - N(1700)$

Results for $N(1650)$, $N(1700)$

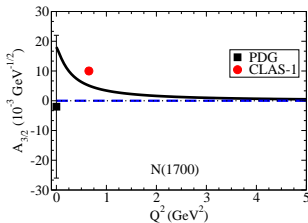
$A_{1/2}$



$A_{1/2}$



$A_{3/2}$

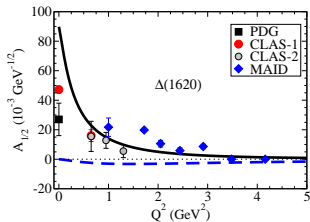


Data from **CLAS**, preliminary CLAS (CLAS-2), **MAID** and PDG

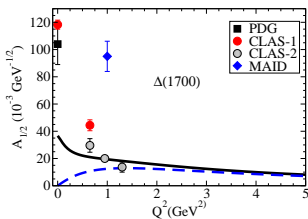
- Model 2: better for $A_{3/2} - N(1700)$
- Both models: good for $N(1650)$: $Q^2 > 2 \text{ GeV}^2$

Results for $\Delta(1620)$, $\Delta(1700)$

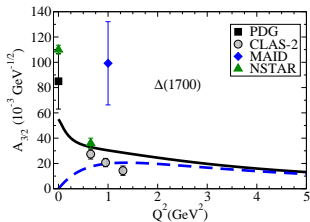
$A_{1/2}$



$A_{1/2}$



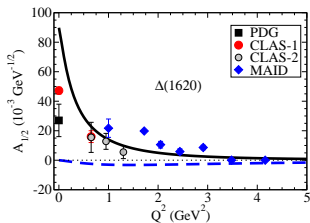
$A_{3/2}$



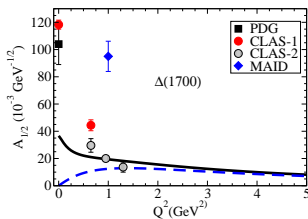
Data from **CLAS**, preliminary CLAS (CLAS-2), **MAID**, PDG and **NSTAR** (proceedings and conferences)

Results for $\Delta(1620)$, $\Delta(1700)$

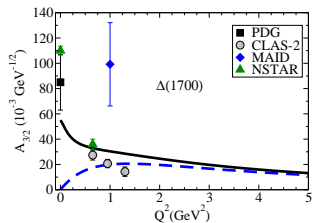
$A_{1/2}$



$A_{1/2}$



$A_{3/2}$

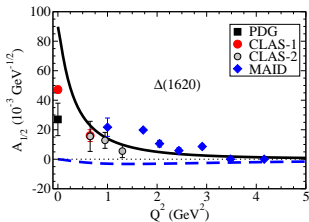


Data from **CLAS**, preliminary CLAS (CLAS-2), **MAID**, PDG and **NSTAR** (proceedings and conferences)

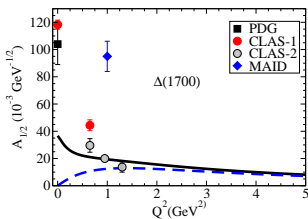
- $\Delta(1700)$: both models with similar results $Q^2 \gtrsim 1 \text{ GeV}^2$

Results for $\Delta(1620)$, $\Delta(1700)$

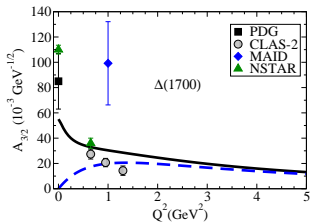
$A_{1/2}$



$A_{1/2}$



$A_{3/2}$



Data from **CLAS**, preliminary CLAS (CLAS-2), **MAID**, PDG and **NSTAR** (proceedings and conferences)

- $\Delta(1700)$: both models with similar results $Q^2 \gtrsim 1 \text{ GeV}^2$
- $\Delta(1620)$: Model 2 –good for $Q^2 > 2 \text{ GeV}^2$

Simple parametrization for large Q^2

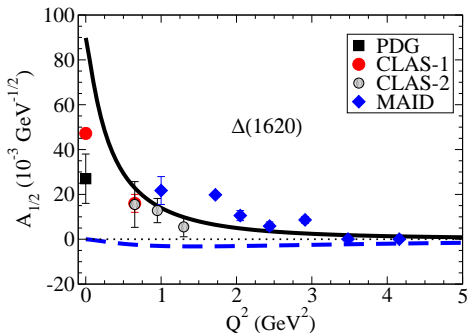
Facilitate comparison with future data – powers from pQCD

$$A_{1/2}(Q^2) = D \left(\frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^{3/2}, \quad A_{3/2}(Q^2) = D \left(\frac{\Lambda^2}{\Lambda^2 + Q^2} \right)^{5/2}$$

State	Amplitude	$D(10^{-3}\text{GeV}^{-1/2})$	$\Lambda^2(\text{GeV}^2)$
$S_{11}(1650)$	$A_{1/2}$	68.90	3.35
$S_{31}(1620)$	$A_{1/2}$
$D_{13}(1700)$	$A_{1/2}$	-8.51	2.82
	$A_{3/2}$	4.36	3.61
$D_{33}(1700)$	$A_{1/2}$	39.22	2.69
	$A_{3/2}$	42.15	8.42

$\gamma^* N \rightarrow \Delta(1620)$

$$A_{1/2}^{S31} \propto \left(2 \frac{A_{1/2}^{S11}}{\cos \theta_S} + 4\sqrt{2}A_{1/2}^{D13} + 4\sqrt{6}A_{3/2}^{D13} \right)$$

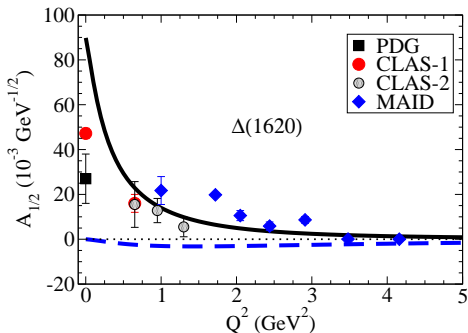


- - - only valence quark contributions

— include meson cloud ($A_{3/2}^{D13}$)

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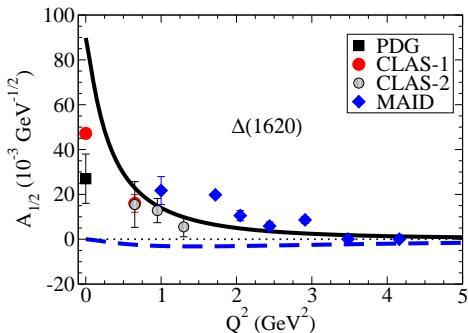


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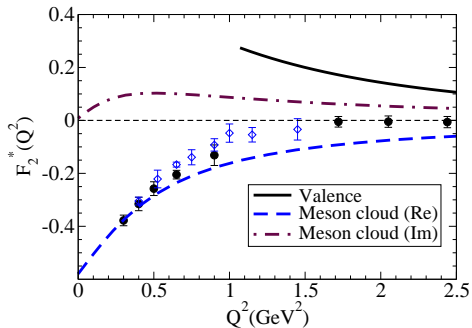
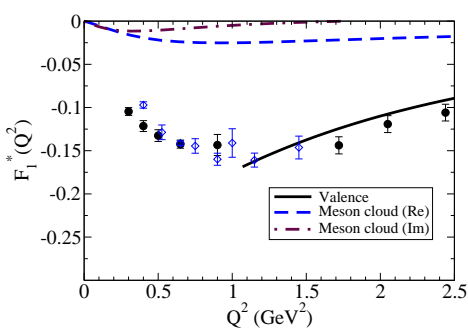
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$$A_{1/2}^{S31} \propto \left(\frac{1}{1 + \frac{Q^2}{1\text{GeV}^2}} \right)^{5/2}$$

$\gamma^* N \rightarrow N(1535)$: Meson cloud

GR, D Jido and K Tsushima, PRD 85, 093014 (2012)

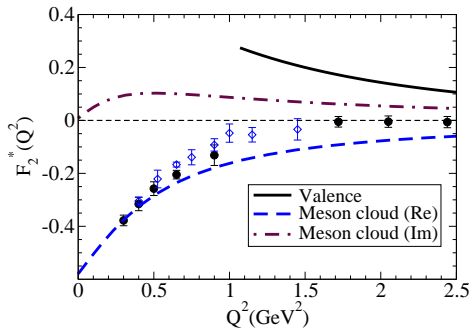
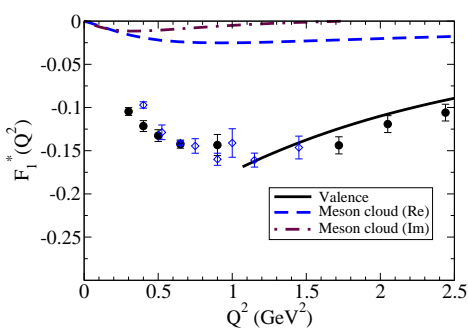


— Spectator quark model → Valence

--- D Jido, M Doring and E Oset, PRC 77, 065207 (2008) - χ Unitary Model
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$\gamma^* N \rightarrow N(1535)$: Meson cloud

GR, D Jido and K Tsushima, PRD 85, 093014 (2012) $(F_2^*)^{mc} \approx -(F_2^*)^B$



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Implications of $F_2^* = 0$?

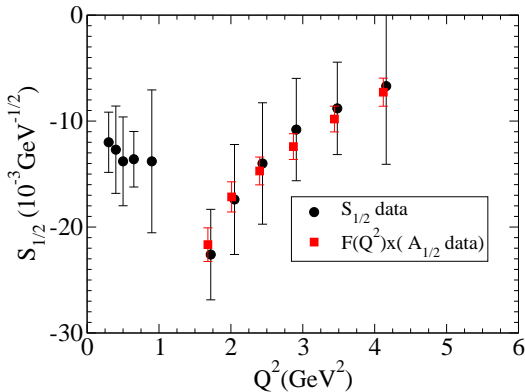
$$\tau = \frac{Q^2}{(M_R + M)^2} \quad Q^2 > 1.5 \text{ GeV}^2$$

$$S_{1/2} \simeq -\frac{\sqrt{1 + \tau} M_S^2 - M^2}{\sqrt{2} 2M_S Q} A_{1/2}$$

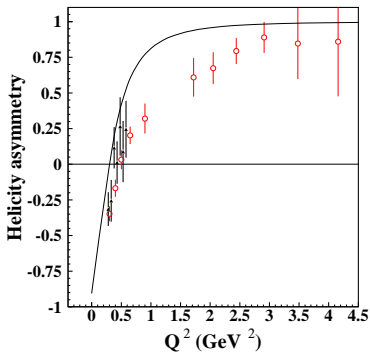
GR, K Tsushima
PRD 84, 051301 (2011)

GR, D Jido, K Tsushima
PRD 85, 093014 (2012)

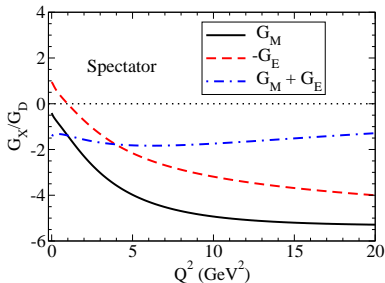
Cancellation between
valence and meson cloud



$\gamma^* N \rightarrow N(1520)$ form factors – large Q^2



$$A_h = \frac{|A_{1/2}|^2 - |A_{3/2}|^2}{|A_{1/2}|^2 + |A_{3/2}|^2}$$



$$A_h = 1 - \frac{3(G_M + G_E)^2}{2(3G_M^2 + G_E^2)}$$

$$G_M + G_E \rightarrow 0 \text{ very slowly}$$

Conclusions

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Thank you



Selected bibliography (part 1)

- $\gamma^* N \rightarrow N(1710)$ **transition at high momentum transfer**,
G. Ramalho and K. Tsushima, Phys. Rev. D **89**, 073010 (2014)
[arXiv:1402.3234 [hep-ph]].
- **Using the Single Quark Transition Model to predict nucleon resonance amplitudes**
G. Ramalho, Phys. Rev. D **90**, 033010 (2014) [arXiv:1407.0649 [hep-ph]].
- $\gamma^* N \rightarrow N^*(1520)$ **form factors in the spacelike region**
G. Ramalho and M. T. Peña, Phys. Rev. D **89**, 094016 (2014)
[arXiv:1309.0730 [hep-ph]].
- **A covariant model for the $\gamma N \rightarrow N(1535)$ transition at high momentum transfer**,
G. Ramalho and M. T. Peña, Phys. Rev. D **84**, 033007 (2011)
[arXiv:1105.2223 [hep-ph]].
- **A simple relation between the $\gamma N \rightarrow N(1535)$ helicity amplitudes**,
G. Ramalho and K. Tsushima, Phys. Rev. D **84**, 051301 (2011)
[arXiv:1105.2484 [hep-ph]].

Selected bibliography (part 2)

- **A covariant formalism for the N^* electroproduction at high momentum transfer, [Review](#)**
G. Ramalho, F. Gross, M. T. Peña and K. Tsushima, **Exclusive Reactions and High Momentum Transfer IV, 287 (2011)** [arXiv:1008.0371 [hep-ph]].
- **Studies of Nucleon Resonance Structure in Exclusive Meson Electroproduction, [Review \(pages 87-92\)](#)**
I. G. Aznauryan et al, Int. J. Mod. Phys. E **22**, 1330015 (2013) [arXiv:1212.4891 [nucl-th]].
- **A pure S-wave covariant model for the nucleon,**
F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. C **77**, 015202 (2008) [arXiv:nucl-th/0606029].
- **Covariant nucleon wave function with S, D, and P-state components,**
F. Gross, G. Ramalho and M. T. Peña, Phys. Rev. D **85**, 093005 (2012) [arXiv:1201.6336 [hep-ph]].
- **Valence quark contributions for the $\gamma^*NP11(1440)$ form factor,**
G. Ramalho and K. Tsushima, Phys. Rev. D **81**, 074020 (2010)