

Overview of latest results with Quark Models

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Outline of the talk

- The hCQM
- Int q Diq M
- The helicity amplitudes
- The elastic e.m. form factors of the nucleon
- Strong decays
- The Unquenched Quark Model (higher Fock components in a systematic way)

The Model (hCQM)

hypercentral **C**onstituent **Q**uark **M**odel

different CQMs for bayons

	Kin. Energy	SU(6) inv	SU(6) viol	date
Isgur-Karl	non rel	h.o. + shift	OGE	1978-9
Capstick-Isgur	rel	string + coul-like	OGE	1986
U(7) B.I.L.	rel M^2	vibr+L	Guersey-R	1994
Hyp. O(6)	non rel/rel	hyp.coul+linear	OGE	1995
Glozman Riska	non rel/rel	Plessas h.o./linear	GBE	1996
Bonn	rel	linear 3-body	instanton	2001

Hypercentral Constituent Quark Model hCQM

free parameters fixed from the spectrum

Comment

The description of the spectrum is the first task of a model builder

Predictions for:

photocouplings

transition form factors

elastic form factors

.....

describe data (if possible)

understand what is missing

LQCD (De Rújula, Georgi, Glashow, 1975)

the quark interaction contains

a long range **spin-independent** term

a short range spin dependent term

Spin-independence \rightarrow SU(6) configurations

SU(6) configurations for three quark states

$$6 \times 6 \times 6 = 20 + 70 + 70 + 56$$

A M M S

Notation

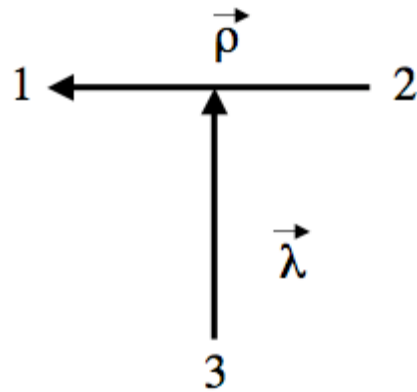
$$(d, L^\pi)$$

d = dim of SU(6) irrep

L = total orbital angular momentum

π = parity

Jacobi coordinates



$$L^2(\Omega)Y_{[\gamma]}(\Omega) = -\gamma(\gamma + 4)Y_{[\gamma]}(\Omega)$$

γ grand angular quantum number

$$\sum_{i < j} V(\mathbf{r}_{ij}) \approx V(\mathbf{x}) + \dots$$

Hyperspherical Coordinates

$$(\rho, \Omega_\rho, \lambda, \Omega_\lambda) \Rightarrow (\mathbf{x}, t, \Omega_\rho, \Omega_\lambda)$$

$$x = \sqrt{\rho^2 + \lambda^2} \quad \text{hyperradius}$$

$$t = \text{arctg} \frac{\rho}{\lambda} \quad \text{hyperangle}$$

$$\gamma = 2n + l_\rho + l_\lambda$$

$$L^2(\Omega) \Leftrightarrow C_2(O(6))$$

$$Y_{[\gamma]}(\Omega)$$

Hyperspherical harmonics

$$\gamma = 2n + l_\rho + l_\lambda$$

Hasenfratz et al. 1980:

$\sum V(\mathbf{r}_i, \mathbf{r}_j)$ is approximately hypercentral

Hypercentral Hypothesis

$$V = V(x)$$

Factorization

$$\psi(x, t, \Omega_\rho, \Omega_\lambda) = \underbrace{\psi_{\nu\gamma}(x)}_{\text{("dynamics")}} \underbrace{Y_{[\gamma, l_\rho, l_\lambda]}}_{\text{("geometry")}}$$

Only one differential equation in x (hyperradial equation)

Hypercentral Model

Phys. Lett. B, 1995

$$V(x) = -\tau/x + \alpha x$$

Hypercentral approximation of

$$V = -b/r + c r$$

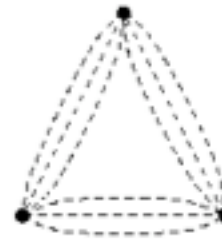
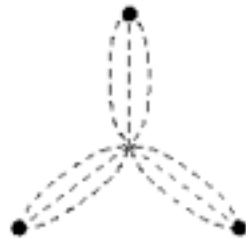
- QCD fundamental mechanism



3-body forces

Carlson et al, 1983
Capstick-Isgur 1986
hCQM 1995

- Flux tube model

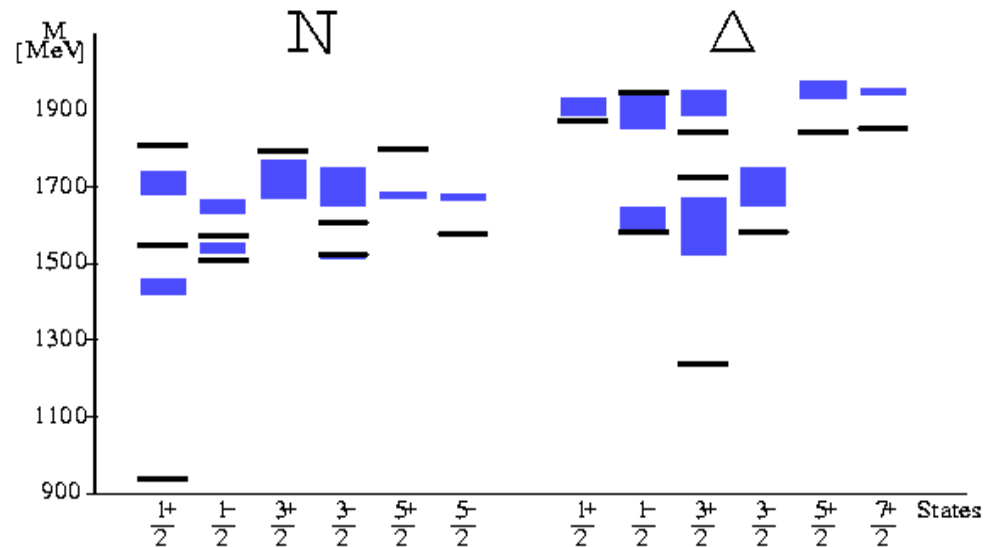


Hypercentral Model (1)

$$H_{3q} = 3m + \sum_{i=1}^3 \frac{\mathbf{p}_i^2}{2m} + V(\mathbf{x}) + H_{hyp}$$

M. Ferraris, M. M. Giannini, M. Pizzo, E. Santopinto, L. Tiator, Phys. Lett. B364 (1995), 231

- $V(\mathbf{x}) = -\frac{\tau}{x} + \alpha x$; $H_{hyp} = A \left[\sum_{i<j} V^S(\mathbf{r}_i, \mathbf{r}_j) \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j + \text{tensor} \right]$
- **3** parameters τ α $A \leftarrow$ fixed to the spectrum, $m = \frac{M}{3}$



$$\tau = 4.59$$

$$\alpha = 1.61 \text{ fm}^{-1}$$

$$A \leftarrow (N - \Delta)$$

$$x = \sqrt{\rho^2 + \lambda^2}$$

hyperradius

Results (predictions)
with the Hypercentral Constituent
Quark Model

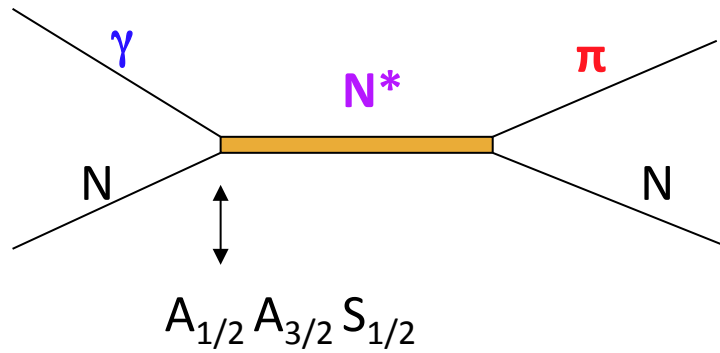
for

- Helicity amplitudes
- Elastic nucleon form factors

The helicity amplitudes

HELICITY AMPLITUDES

Extracted from electroproduction of mesons



Definition

$$A_{1/2} = \langle N^* J_z = 1/2 | H_{em}^T | N J_z = -1/2 \rangle \quad \S$$

$$A_{3/2} = \langle N^* J_z = 3/2 | H_{em}^T | N J_z = 1/2 \rangle \quad \S$$

$$S_{1/2} = \langle N^* J_z = 1/2 | H_{em}^L | N J_z = 1/2 \rangle$$

N, N^* nucleon and resonance as 3q states

H_{em}^T, H_{em}^L model transition operator

§ results for the negative parity resonances:

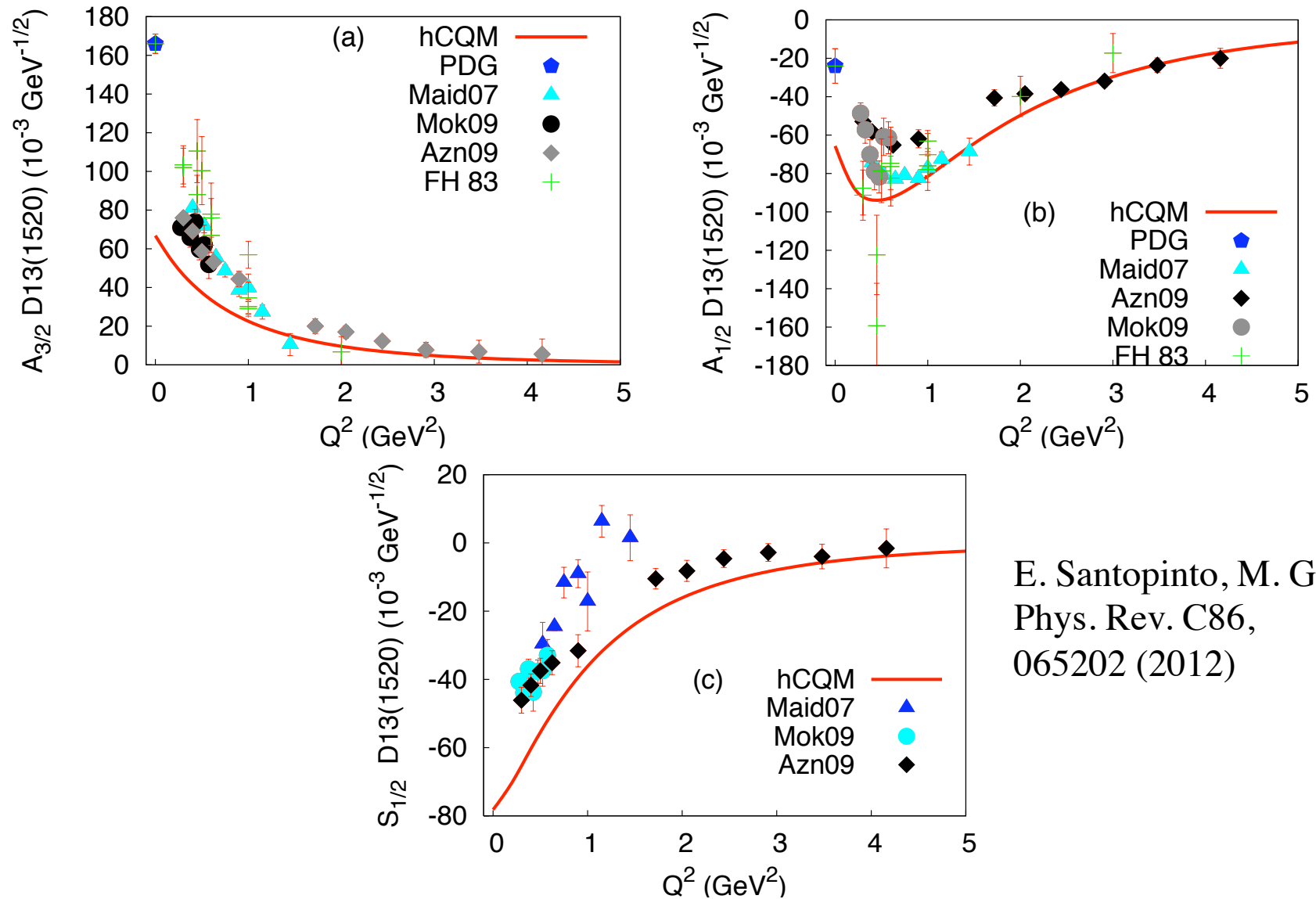
M. Aiello, M.G., E. Santopinto J. Phys. G24, 753 (1998)

Systematic predictions for transverse and longitudinal amplitudes

E. Santopinto et al. , Phys. Rev. C86, 065202 (2012)

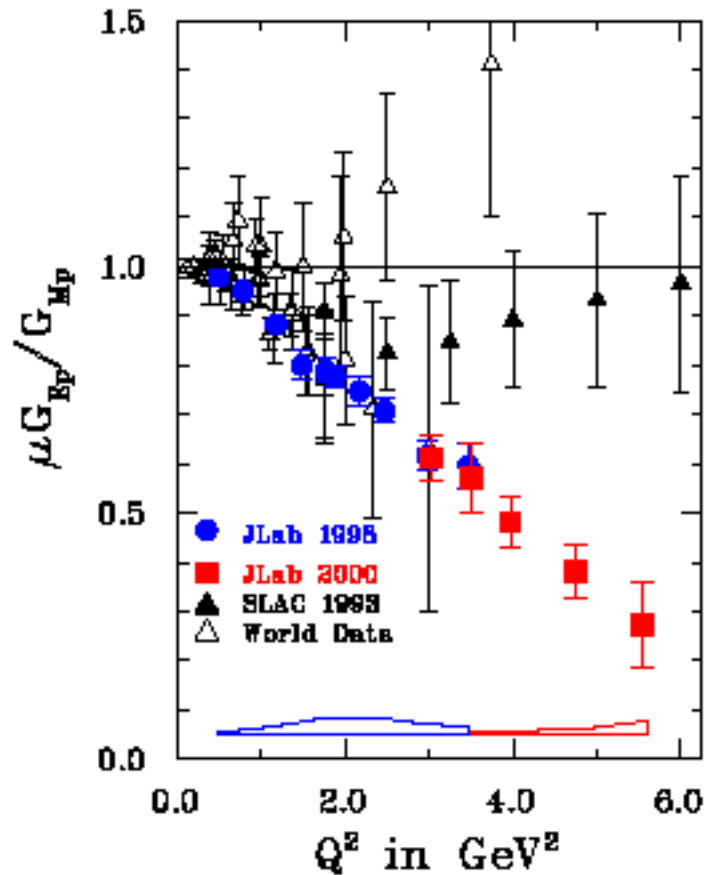
Proton and neutron electro-excitation to 14 resonances

N(1520) $3/2^-$ transition amplitudes



E. Santopinto, M. Giannini.
Phys. Rev. C86,
065202 (2012)

The nucleon elastic form factors



- elastic scattering of polarized electrons on polarized protons
- measurement of polarizations asymmetry gives directly the ratio G_E^p / G_M^p
- discrepancy with Rosenbluth data (?)
- linear and strong decrease
- pointing towards a zero (!)
- latest data seem to confirm the behaviour

RELATIVITY

Various levels

- relativistic kinetic energy
- Lorentz boosts
- Relativistic dynamics
- quark-antiquark pair effects (meson cloud)
- relativistic equations (BS, DS)

Point Form Relativistic Dynamics

Point Form is one of the Relativistic Hamiltonian Dynamics
for a fixed number of particles (Dirac)

Construction of a representation of the Poincaré generators
 P_μ (tetramomentum), J_k (angular momenta), K_i (boosts)
obeying the Poincaré group commutation relations
in particular

$$[P_k, K_i] = i \delta_{kj} H$$

Three forms:

Light (LF), Instant (IF), Point (PF)

Differ in the number and type of (interaction) free generators

Point form: P_μ interaction dependent
 J_k and K_i free

Composition of angular momentum states as in the
non relativistic case

Mass operator $M = M_0 + M_I$

$$M_0 = \sum_i \sqrt{\vec{p}_i^2 + m^2} \quad \sum_i \vec{p}_i = 0$$

\vec{P}_i undergo the same Wigner rotation $\rightarrow M_0$ is invariant

Similar reasoning for the hyperradius

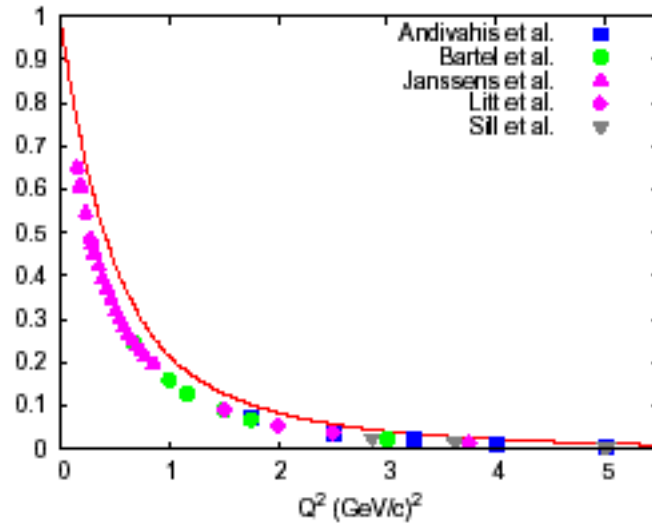
The eigenstates of the relativistic hCQM are interpreted as
eigenstates of the mass operator M

Moving three-quark states are obtained through
(interaction free) Lorentz boosts (velocity states)

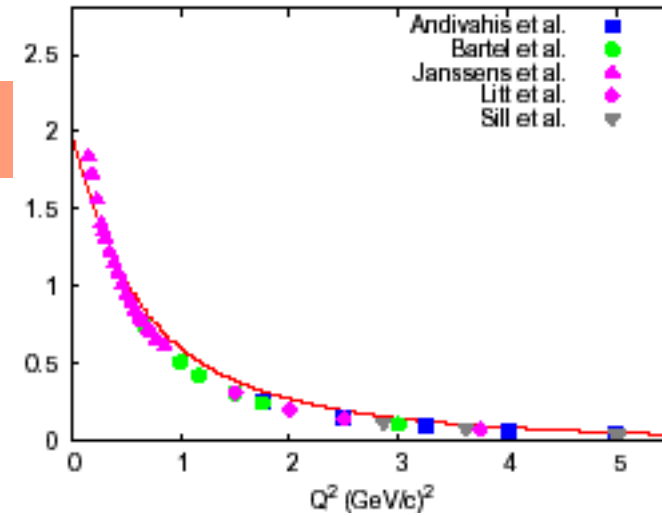
Calculated values!

- Boosts to initial and final states
- Expansion of current to any order
- Conserved current

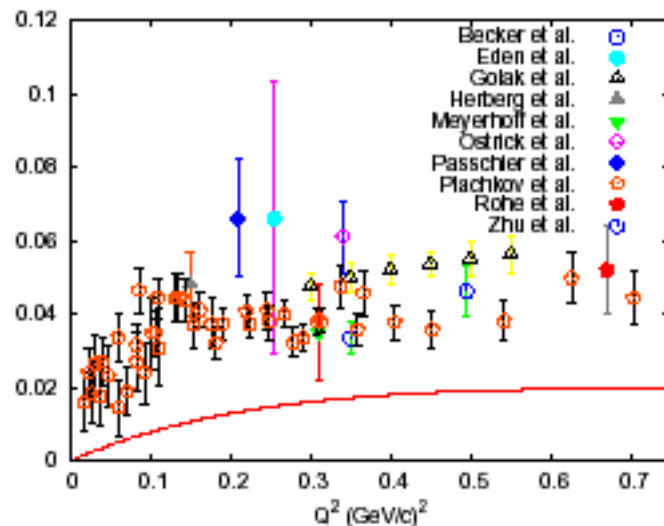
G_E^p



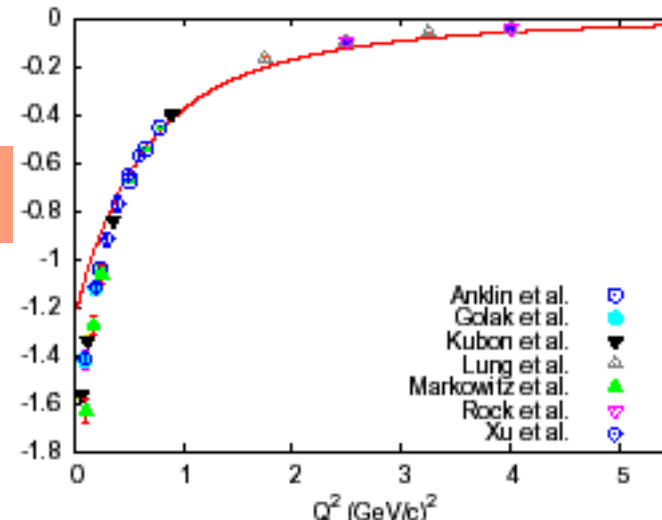
G_M^p



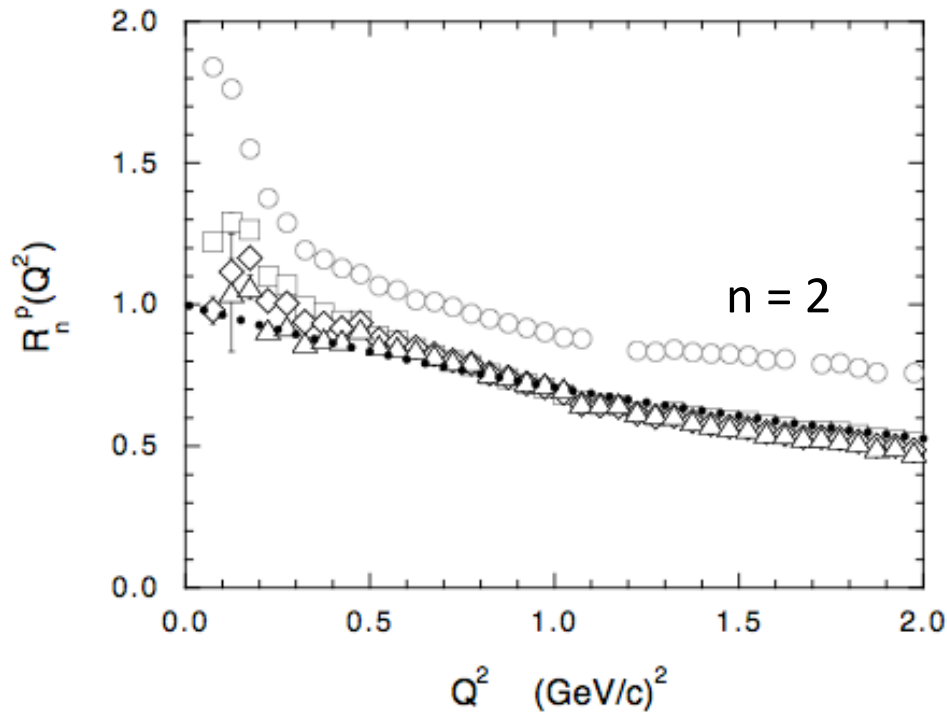
G_E^n



G_M^n



Further support 2



Ratio between
proton Nachtmann moments &
CQ distribution

Bloom-Gilman duality

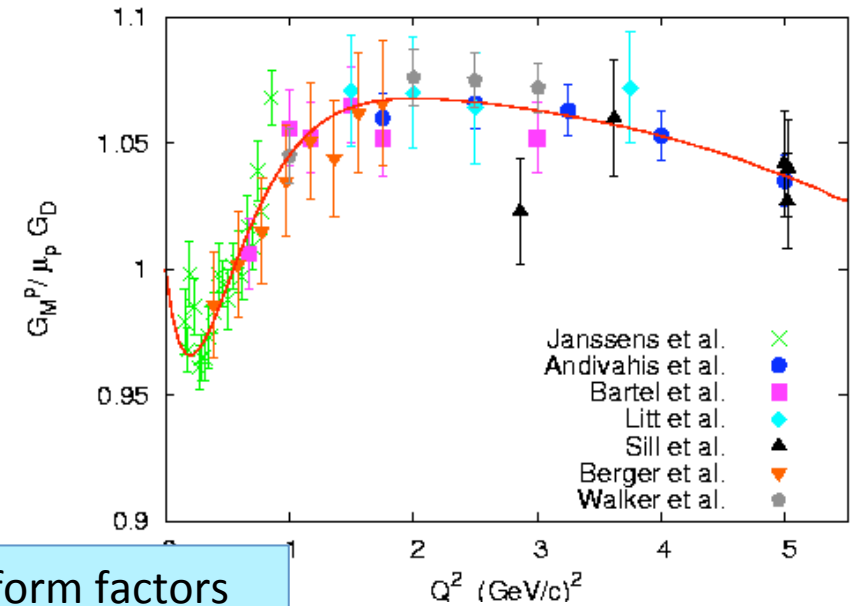
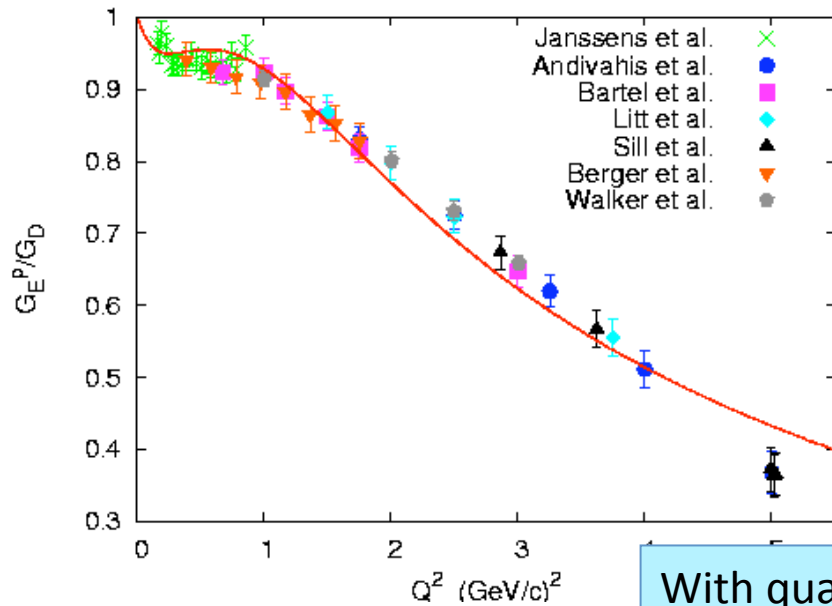
Inelastic proton scattering as elastic scattering on CQ

(approximate) scaling function \longrightarrow square of CQ ff

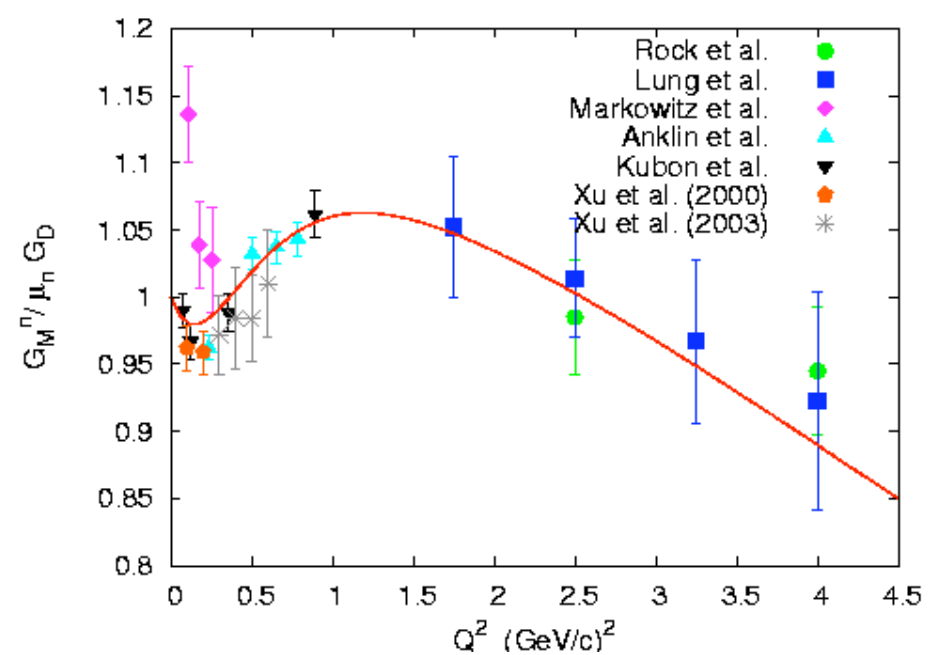
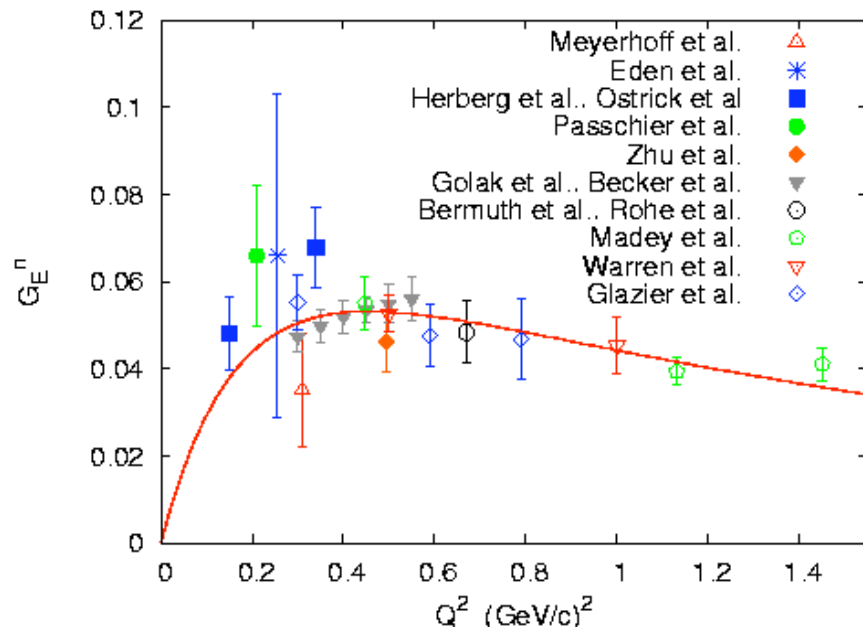
$$F(Q^2) = 1/(1 + 1/6 r_{\text{CQ}}^2 Q^2)$$

$$r_{\text{CQ}} \cong 0.2\text{-}0.4 \text{ fm}$$

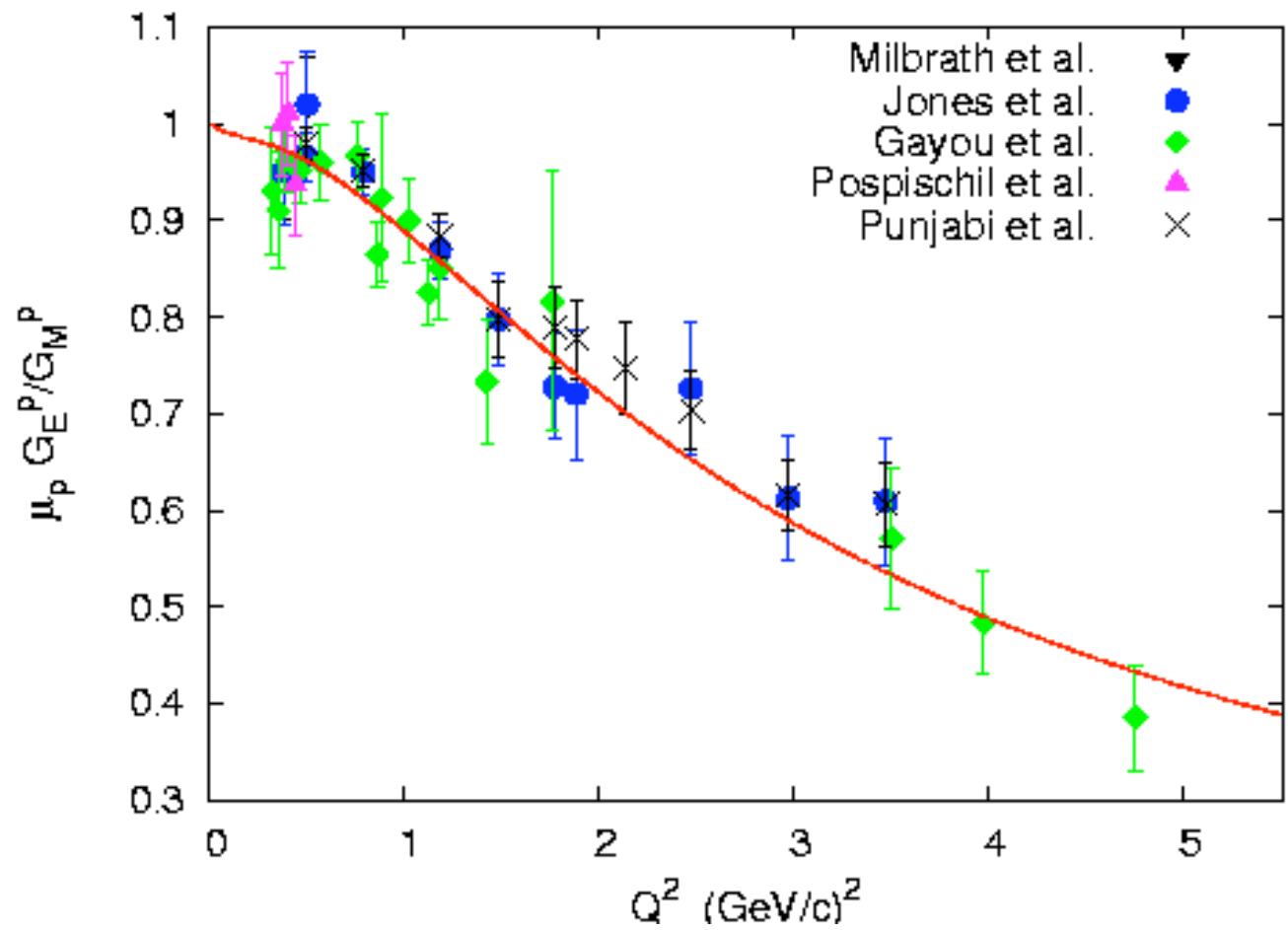
Ricco ,,et al., PR **D67**, 094004 (2003)

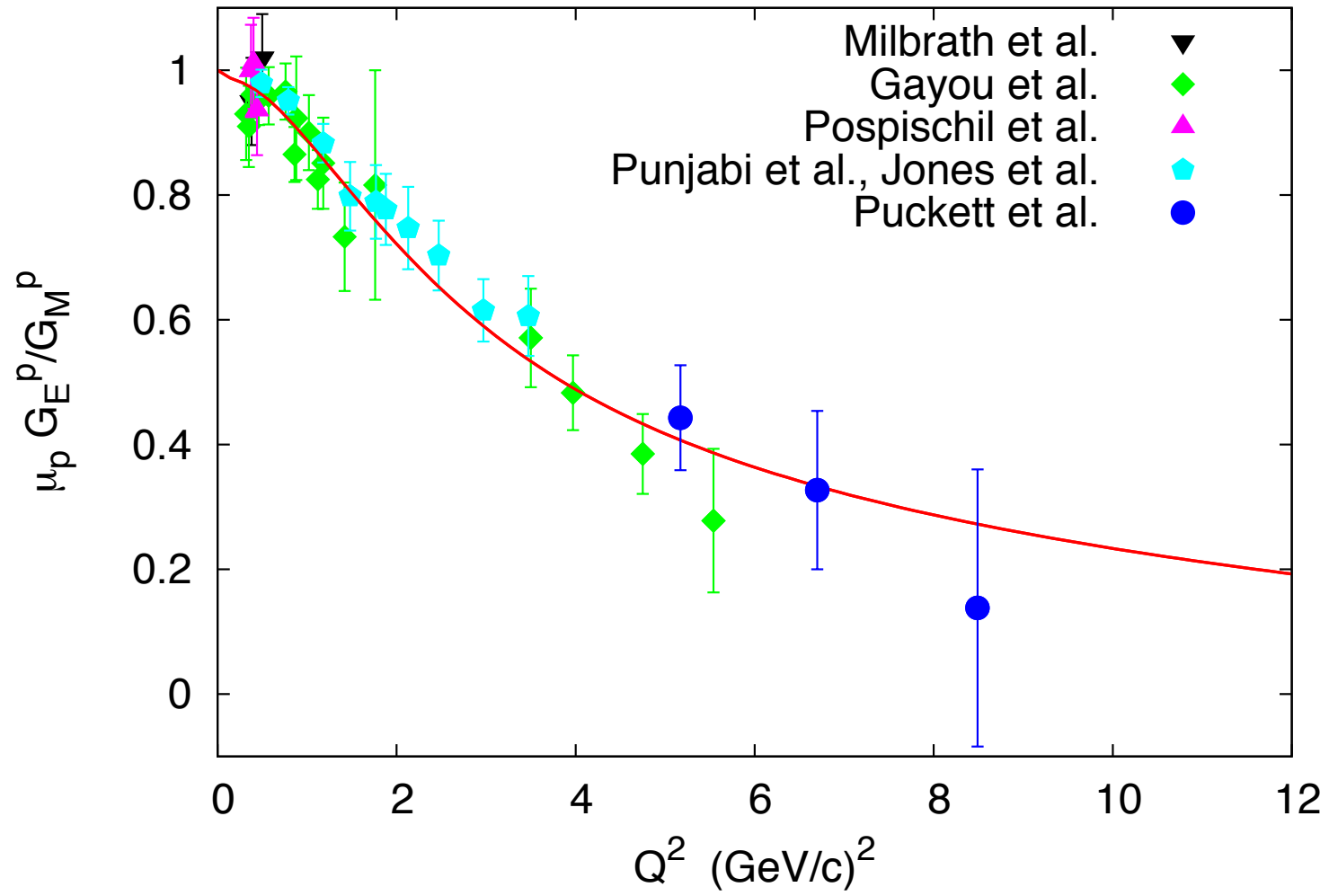


With quark form factors

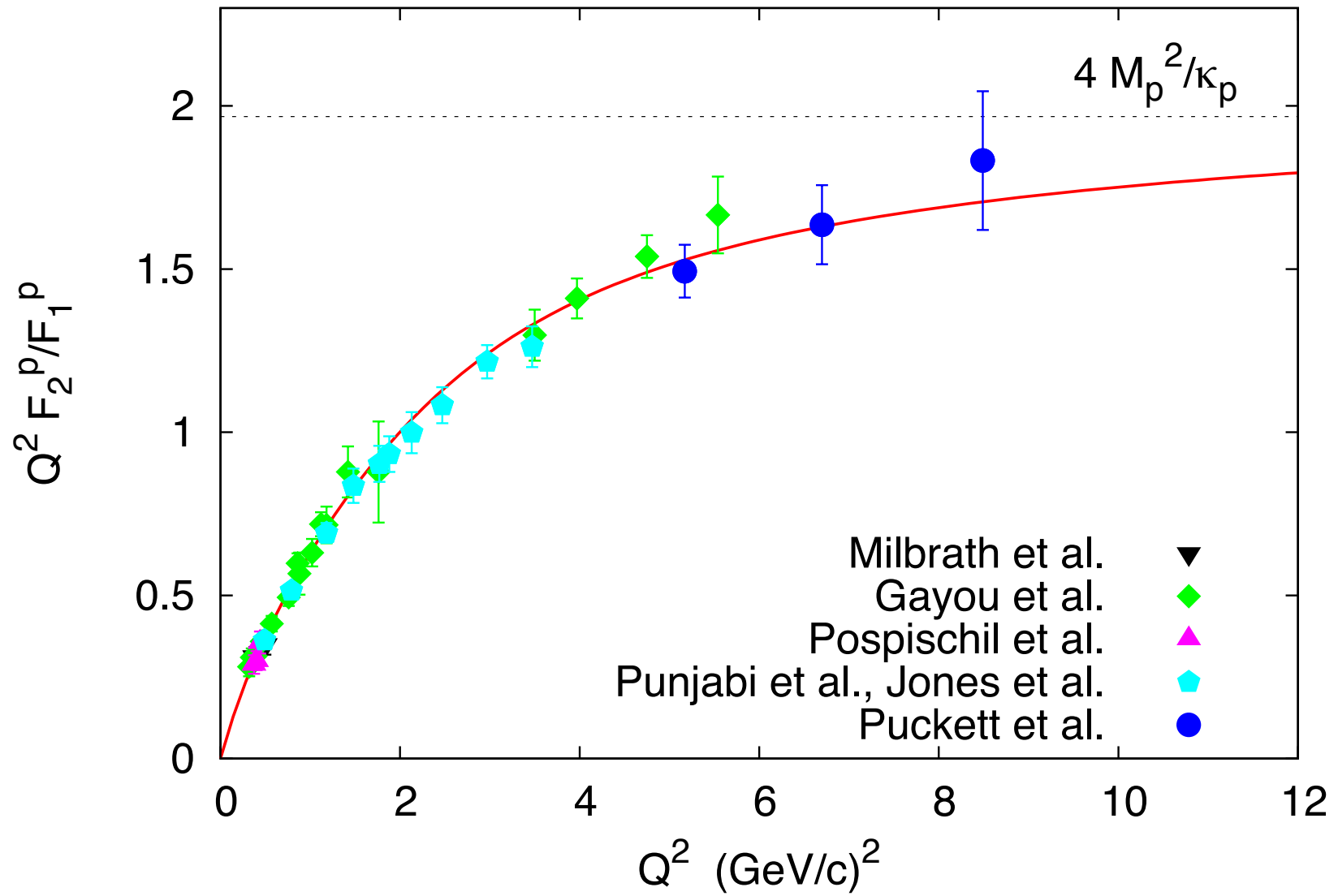


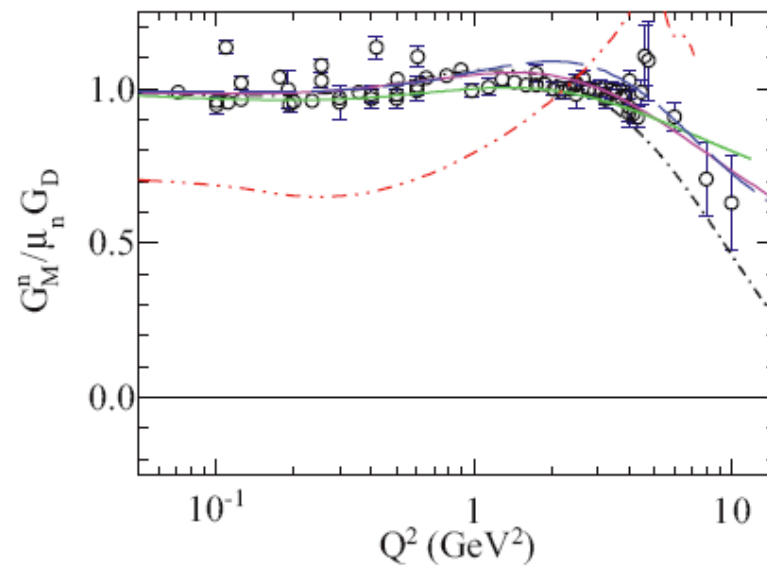
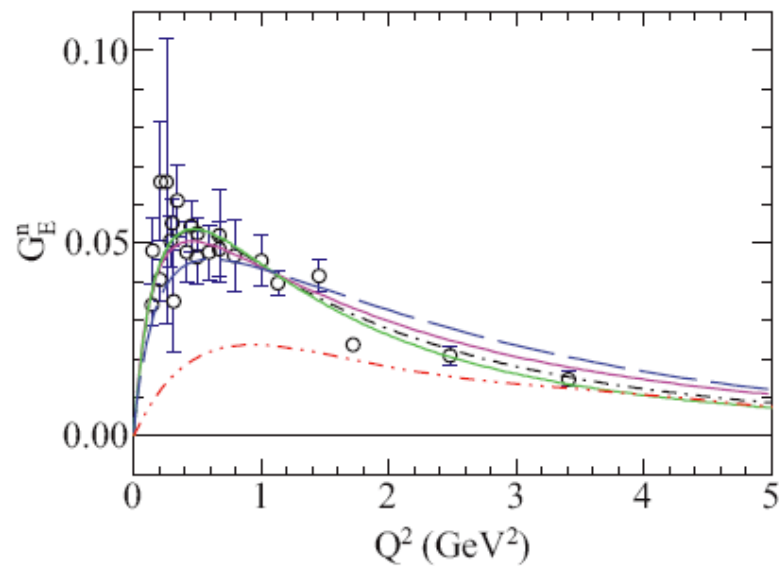
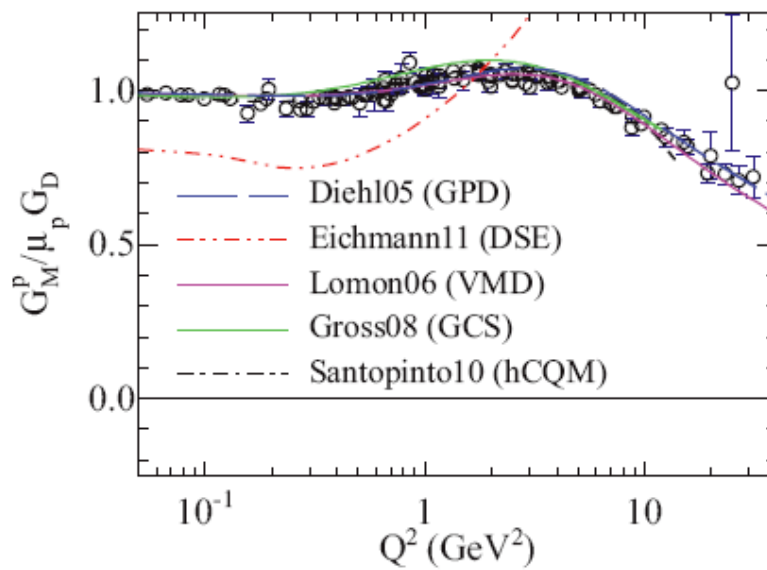
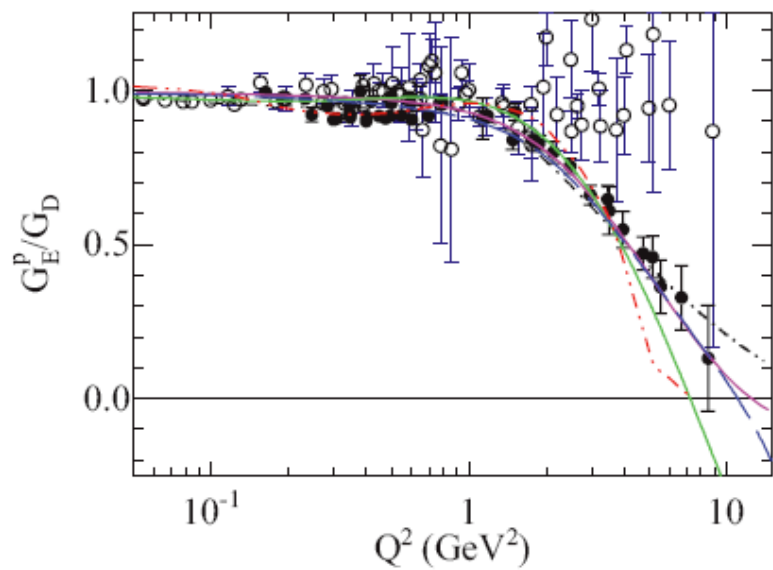
De Sanctis, Giannini, Santopinto, Vassallo Phys. Rev. C76, 062201 (2007)

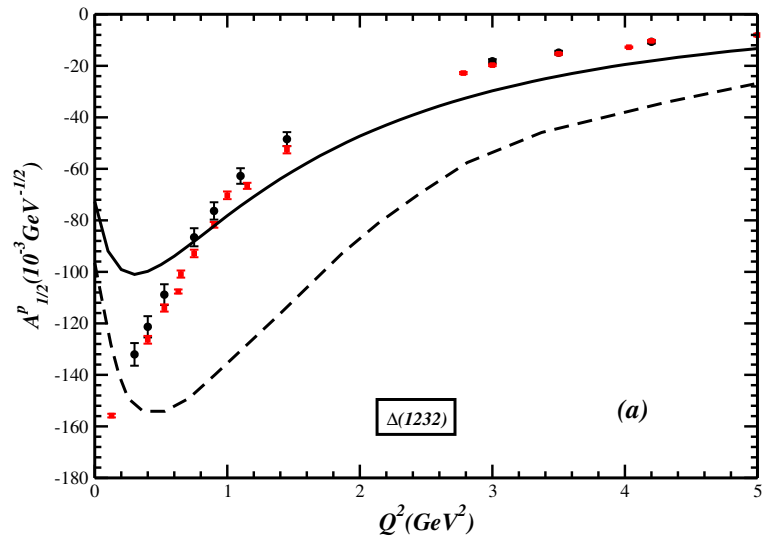




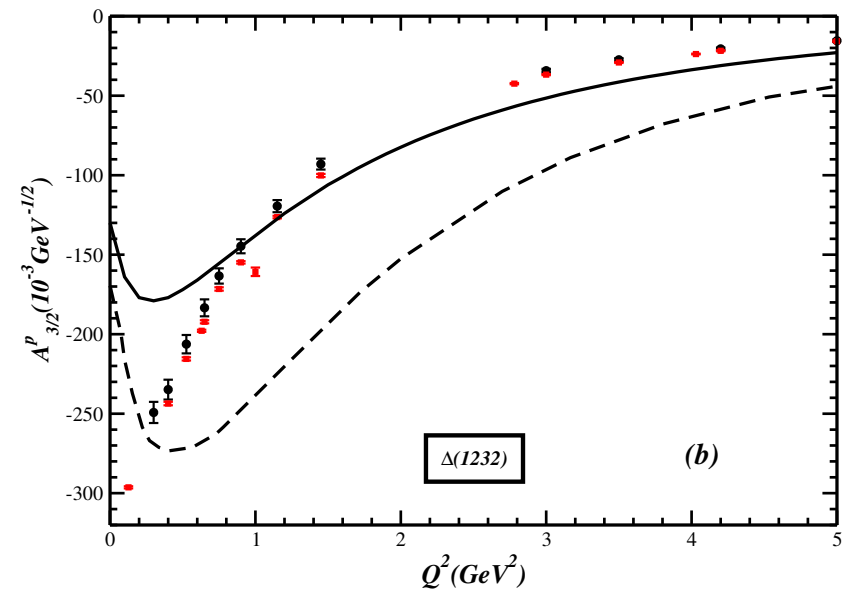
E. Santopinto, A. Vassallo, M. M. Giannini, and M. De Sanctis, Phys. Rev. C 82, 065204 (2010)







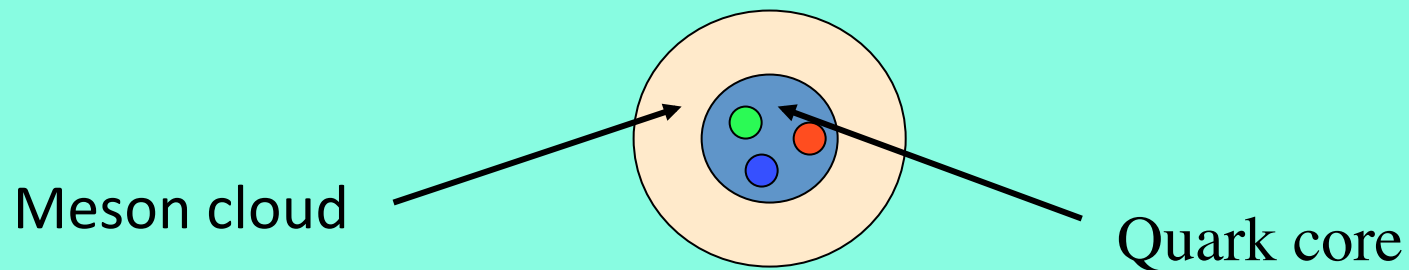
Relativistic hCQM In Point Form



Y.B. Dong, M.Giannini., E. Santopinto,
A. Vassallo,
Few-Body Syst. **55** (2014) 873-876

please note

- the medium Q^2 behaviour is fairly well reproduced
- there is lack of strength at **low** Q^2 (outer region) in the e.m. transitions
- emerging picture:
 quark core plus (meson or sea-quark) **cloud**



The Interacting Quark Diquark Model

Interacting qD model

E. Santopinto, PRC72, 022201 (2005)

I part: Construction of the states

▪ Diquark

- Two correlated quarks in S wave:symm.

- Baryon in 1_c color representation \rightarrow diquark in $\bar{3}_c$ (A)

- Diquark WF: $\begin{array}{c} \square \otimes \square = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \\ 6 \otimes 6 = 15 \oplus 21 \end{array}$ Ψ_D (spin-flavor) symmetric \rightarrow 15 (A) repr. not present

• $SU(6)_{sf}$ representations for baryons

~~$$\begin{array}{c} \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \square \\ \hline \end{array} = \begin{array}{|c|} \hline \square \\ \hline \square \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \\ 15 \otimes 6 = 20(A) \oplus 70(MA) \end{array}$$~~

$$\begin{array}{c} \begin{array}{|c|c|} \hline \square & \square \\ \hline \end{array} \otimes \begin{array}{|c|} \hline \square \\ \hline \end{array} = \begin{array}{|c|c|} \hline \square & \square \\ \hline \square & \square \\ \hline \end{array} \oplus \begin{array}{|c|c|c|} \hline \square & \square & \square \\ \hline \end{array} \\ 21 \otimes 6 = 70(MS) \oplus 56(S) \end{array}$$

- Problem of missing resonances

Scalar & axial-vector diquarks

- **21 $SU(6)_{sf}$ representation**

- Decomposed in $SU(2)_s \times SU(3)_f$

- $[\bar{3},0]$ & $[6,1]$ representations. Notation: [flavor,spin]

- **“Good” & “bad” diquarks**

- According to OGE-calculations, $[\bar{3},0]$ is energetically favored
[Wilczek, Jaffe]

- $[\bar{3},0]$: good (scalar) diquark

- $[6,1]$: bad (axial-vector) diquark

Evidences of diquark correlations

- **Regge behavior of hadrons**

Baryons arranged in rotational Regge trajectories ($J=\alpha+\alpha'M^2$) with the same slope of the mesonic ones.

- **$\Delta = 1/2$ rule in weak nonleptonic decays**

Neubert and Stech, Phys. Lett. B **231** (1989) 477; Phys. Rev. D **44** (1991) 775

- **Regularities in parton distribution functions and in spin-dependent structure functions**

Close and Thomas, Phys. Lett. B **212** (1988) 227

- **Regularities in $\Lambda(1116)$ and $\Lambda(1520)$ fragmentation functions**

Jaffe, Phys. Rept. **409** (2005) 1 [Nucl. Phys. Proc. Suppl. **142** (2005) 343]

Wilczek, hep-ph/0409168

- **Any interaction that binds π and ρ mesons in the rainbow-ladder approximation of the DSE will produce diquarks**

Cahill, Roberts and Praschifka, Phys. Rev. D **36** (1987) 2804

- **Indications of diquark confinement**

Bender, Roberts and Von Smekal, Phys. Lett. B **380** (1996) 7

the Interacting qD model

E. Santopinto, PRC72, 022201 (2005)

▪ Hamiltonian

$$H = \frac{p^2}{2m} - \frac{\tau}{r} + \beta r + [B\delta_{S_{12},1} + C\delta_0] +$$
$$+ (-1)^{l+1} 2Ae^{-\alpha r} [(\vec{s}_{12} \cdot \vec{s}_3) + (\vec{t}_{12} \cdot \vec{t}_3) + (\vec{s}_{12} \cdot \vec{s}_3)(\vec{t}_{12} \cdot \vec{t}_3)]$$

- Non-rel. Kinetic energy + Coulomb + linear confining terms
- Splitting between scalar & axial-vector diquarks
- Exchange potential
-

Rel. Interacting qD model

J. Ferretti, E. Santopinto & A. Vassallo, PRC83, 065204 (2011)

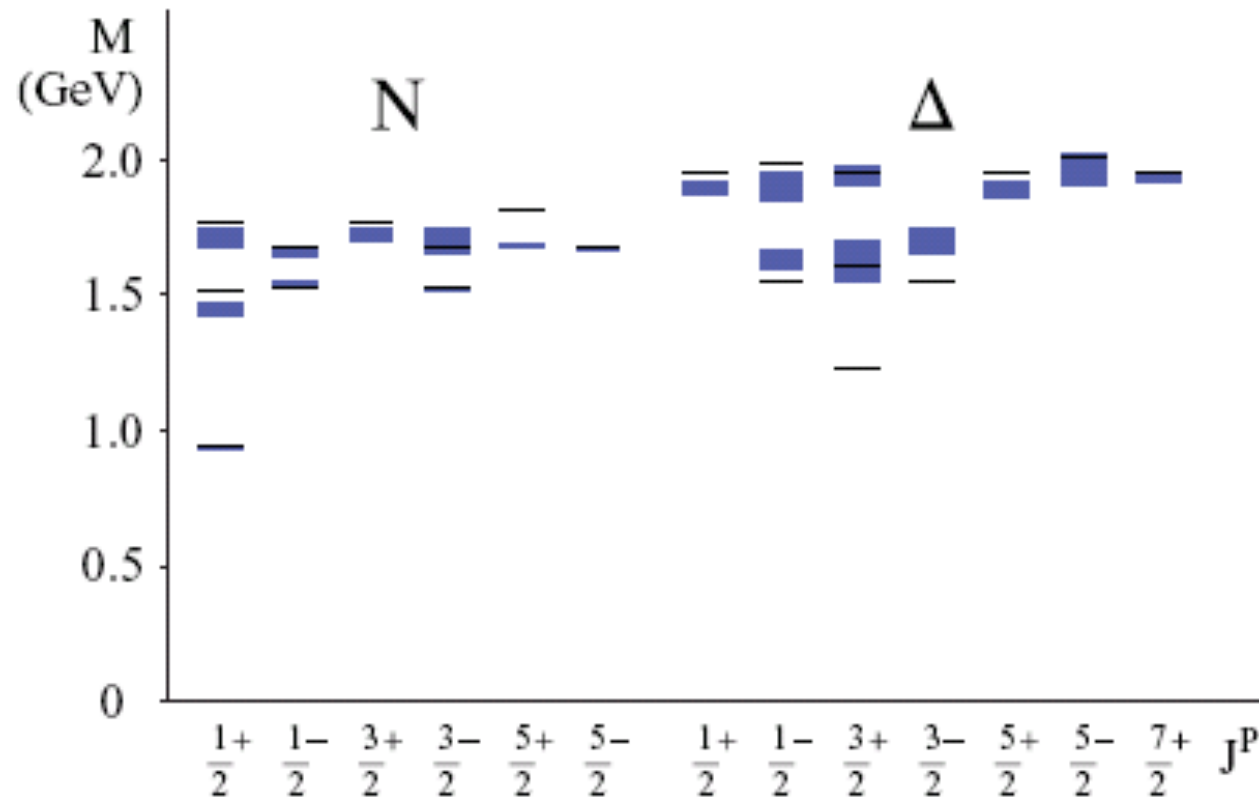
- Relativistic extension of the previous model (point-form formalism).

$$M = E_0 + \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} + M_{\text{dir}}(r) + M_{\text{cont}}(r) + M_{\text{ex}}(r),$$
$$M_{\text{dir}}(r) = -\frac{\tau}{r}(1 - e^{-\mu r}) + \beta r.$$
$$M_{\text{ex}}(r) = (-1)^{l+1} e^{-\sigma r} [A_S(\vec{s}_1 \cdot \vec{s}_2) + A_I(\vec{t}_1 \cdot \vec{t}_2) + A_{SI}(\vec{s}_1 \cdot \vec{s}_2)(\vec{t}_1 \cdot \vec{t}_2)],$$
$$M_{\text{cont}} = \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon} \frac{\eta^3 D}{\pi^{3/2}} e^{-\eta^2 r^2} \delta_{L,0} \delta_{s_1,1} \left(\frac{m_1 m_2}{E_1 E_2}\right)^{1/2+\epsilon}$$

- Numerical solution with variational program
- Parameters fitted to nonstrange baryon spectrum

Rel. Interacting qD model

J. Ferretti, E. Santopinto & A. Vassallo, PRC83, 065204 (2011)



Resonance	Status	M^{expt} (MeV)	J^P	L^P	S	s_1	n_r	M^{calc} (MeV)
$N(939) P_{11}$	****	939	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	0	939
$N(1440) P_{11}$	****	1420–1470	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	1	1513
$N(1520) D_{13}$	****	1515–1525	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	0	1527
$N(1535) S_{11}$	****	1525–1545	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	0	1527
$N(1650) S_{11}$	****	1645–1670	$\frac{1}{2}^-$	1^-	$\frac{1}{2}, \frac{3}{2}$	1	0	1671
$N(1675) D_{15}$	****	1670–1680	$\frac{5}{2}^-$	1^-	$\frac{3}{2}$	1	0	1671
$N(1680) F_{15}$	****	1680–1690	$\frac{5}{2}^+$	2^+	$\frac{1}{2}$	0	0	1808
$N(1700) D_{13}$	***	1650–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}, \frac{3}{2}$	1	0	1671
$N(1710) P_{11}$	***	1680–1740	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	1	0	1768
$N(1720) P_{13}$	****	1700–1750	$\frac{3}{2}^+$	0^+	$\frac{1}{2}$	1	0	1768
$\Delta(1232) P_{33}$	****	1231–1233	$\frac{3}{2}^+$	0^+	$\frac{1}{2}$	1	0	1233
$\Delta(1600) P_{33}$	***	1550–1700	$\frac{3}{2}^+$	0^+	$\frac{1}{2}$	1	1	1602
$\Delta(1620) S_{31}$	****	1600–1660	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	0	1554
$\Delta(1700) D_{33}$	****	1670–1750	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	0	1554
$\Delta(1900) S_{31}$	**	1850–1950	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	1	1	1986
$\Delta(1905) F_{35}$	****	1865–1915	$\frac{7}{2}^+$	2^+	$\frac{1}{2}$	1	0	1952
$\Delta(1910) P_{31}$	****	1870–1920	$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	1	0	1952
$\Delta(1920) P_{33}$	***	1900–1970	$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	1	0	1952
$\Delta(1930) D_{35}$	***	1900–2020	$\frac{5}{2}^-$	1^-	$\frac{1}{2}$	1	0	2005
$\Delta(1950) F_{37}$	****	1915–1950	$\frac{7}{2}^+$	2^+	$\frac{1}{2}$	1	0	1952
$N(2100) P_{11}$	*	1855–1915	$\frac{1}{2}^+$	0^+	$\frac{1}{2}$	0	2	1893
$N(2090) S_{11}$	*	1869–1987	$\frac{1}{2}^-$	1^-	$\frac{1}{2}$	0	1	1882
$N(1900) P_{13}$	**	1820–1974	$\frac{3}{2}^+$	2^+	$\frac{1}{2}$	0	0	1808
$N(2080) D_{13}$	**	1740–1940	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	0	1	1882
$\Delta(1750) P_{31}$	*	1708–1780	$\frac{3}{2}^+$	0^+	$\frac{1}{2}$	1	0	1858
$\Delta(1940) D_{33}$	*	1947–2167	$\frac{3}{2}^-$	1^-	$\frac{1}{2}$	1	1	1986

**0 missing resonances
below 2 GeV**

Model Parameters

$m_q = 200 \text{ MeV}$	$m_S = 600 \text{ MeV}$	$m_{AV} = 950 \text{ MeV}$
$\tau = 1.25$	$\mu = 75.0 \text{ fm}^{-1}$	$\beta = 2.15 \text{ fm}^{-2}$
$A_S = 375 \text{ MeV}$	$A_I = 260 \text{ MeV}$	$A_{SI} = 375 \text{ MeV}$
$\sigma = 1.71 \text{ fm}^{-1}$	$E_0 = 154 \text{ MeV}$	$D = 4.66 \text{ fm}^2$
$\eta = 10.0 \text{ fm}^{-1}$	$\epsilon = 0.200$	

Rel. Interacting qD model – strange B.

E. Santopinto & J. Ferretti, Phys.Rev. C92 (2015) , 025202

▪ Model

- Model extended to the strange sector
- Mass operator

$$M = E_0 + \sqrt{\vec{q}^2 + m_1^2} + \sqrt{\vec{q}^2 + m_2^2} + M_{\text{dir}}(r) + M_{\text{ex}}(r)$$

$$M_{\text{ex}}(r) = (-1)^{L+1} e^{-\sigma r} [A_S \vec{s}_1 \cdot \vec{s}_2 + A_F \vec{\lambda}_1^f \cdot \vec{\lambda}_2^f + A_I \vec{t}_1 \cdot \vec{t}_2]$$

$$M_{\text{dir}}(r) = -\frac{\tau}{r}(1 - e^{-\mu r}) + \beta r.$$

- Gursej-Radicati inspired exchange interaction
- Parameters fitted to strange baryon spectrum

Rel. Interacting qD model – strange B.

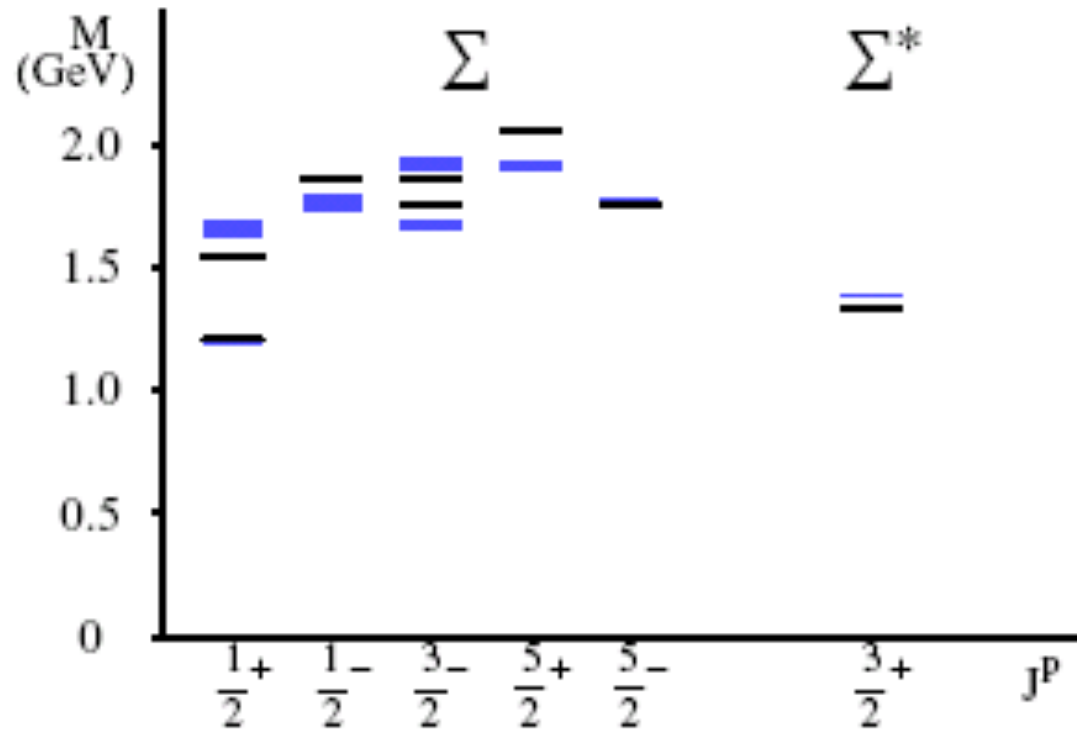
E. Santopinto & J. Ferretti, Phys.Rev. C92 (2015) , 025202

■ Parameters

Parameter	Value (Fit 1)	Value (Fit 2)	Parameter	Value (Fit 1)	Value (Fit 2)
m_n	200 MeV	159 MeV	m_s	550 MeV	213 MeV
$m_{[n,n]}$	600 MeV	607 MeV	$m_{[n,s]}$	900 MeV	856 MeV
$m_{\{n,n\}}$	950 MeV	963 MeV	$m_{\{n,s\}}$	1200 MeV	1216 MeV
$m_{\{s,s\}}$	1580 MeV	1352 MeV	τ	1.20	1.02
μ	75.0 fm ⁻¹	28.4 fm ⁻¹	β	2.15 fm ⁻²	2.36 fm ⁻²
A_S	350 MeV	-436 MeV	A_F	100 MeV	193 MeV
A_I	250 MeV	791 MeV	σ	2.30 fm ⁻¹	2.25 fm ⁻¹
E_0	141 MeV	150 MeV	ϵ	0.37	–
D	6.13 fm ²	–	η	11.0 fm ⁻¹	–

Rel. Interacting qD model – strange B.

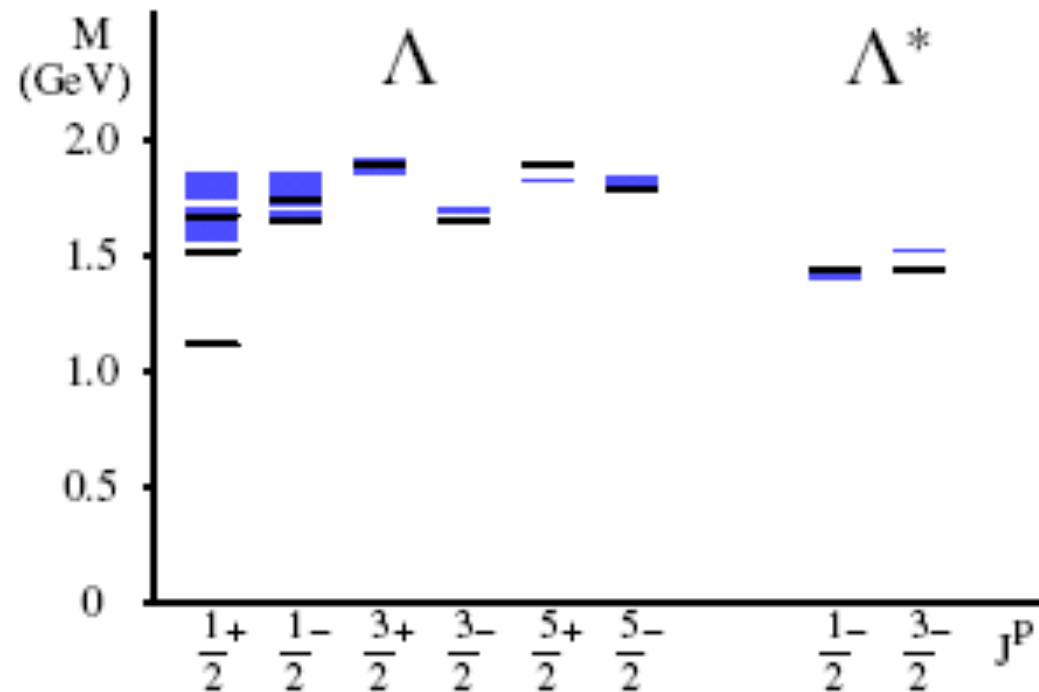
E. Santopinto & J. Ferretti, Phys.Rev. C92 (2015) , 025202



Rel. Interacting qD model

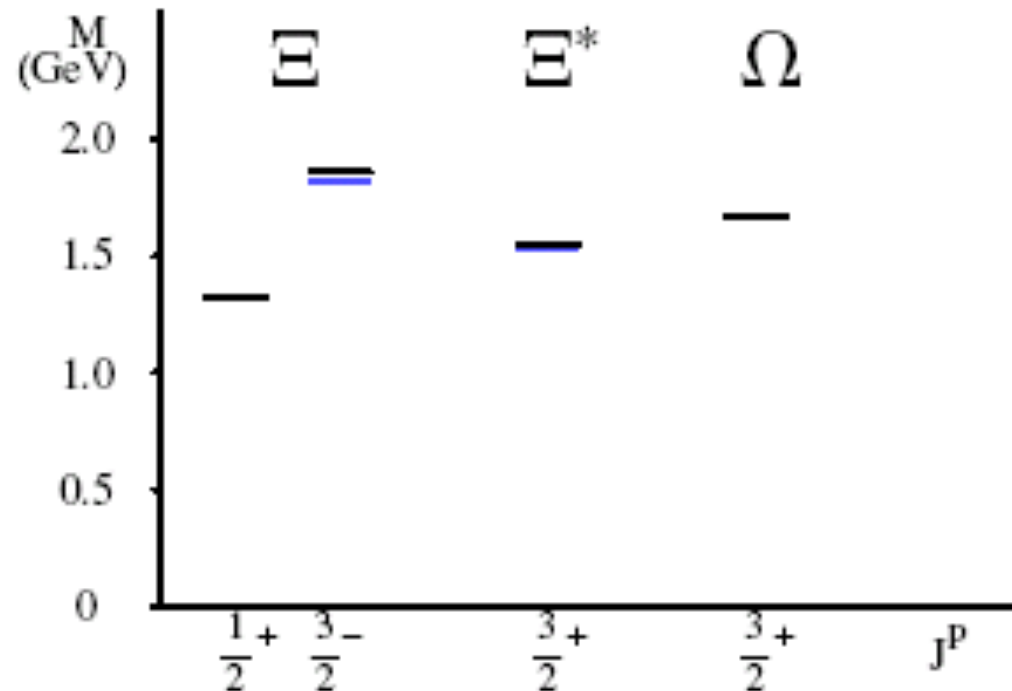
E. Santopinto & J. Ferretti, Phys.Rev. C92 (2015) , 025202

■ Lambda & Lambda* states



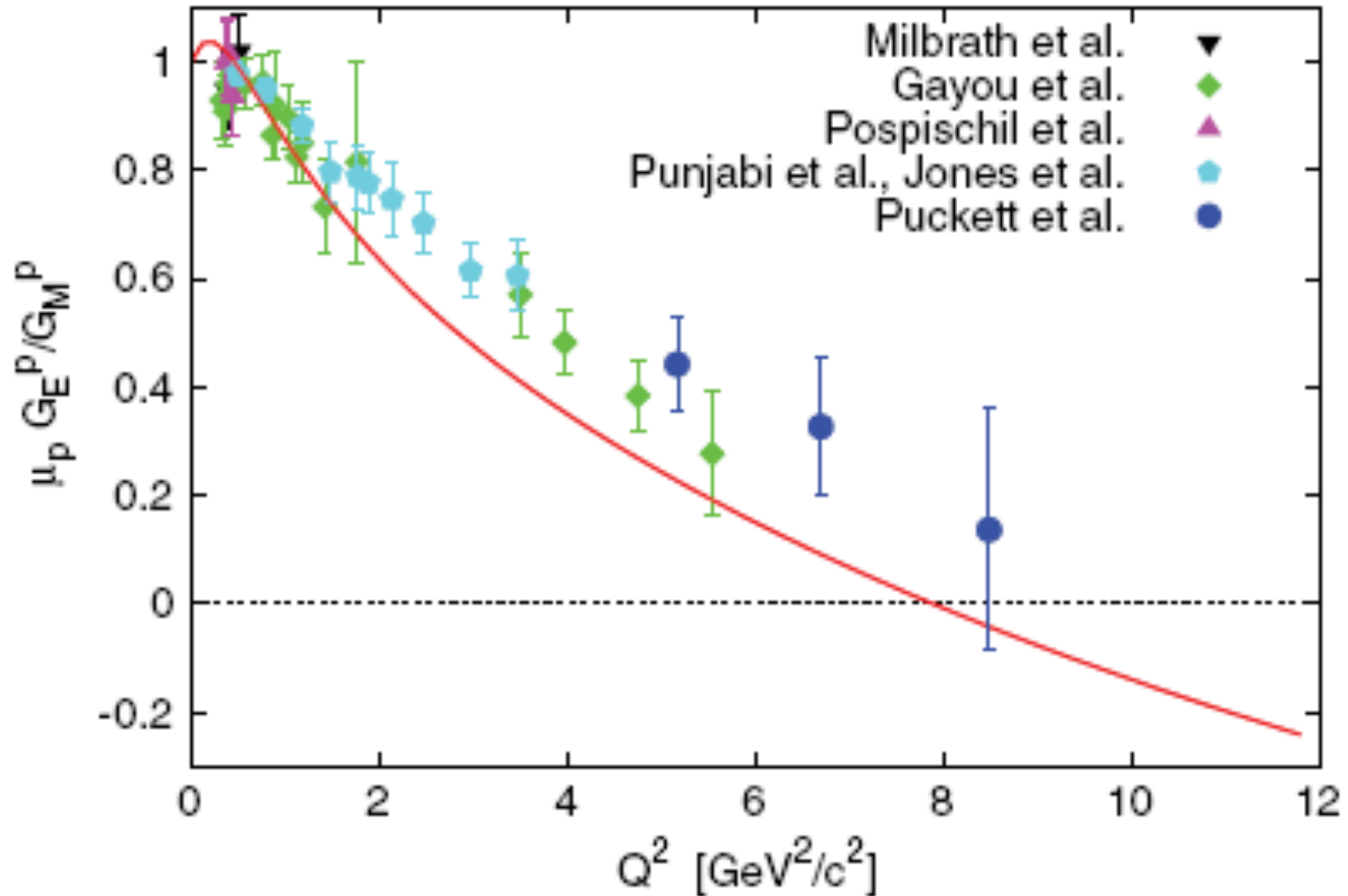
Rel. Interacting qD model – strange sector

■ E. Santopinto & J. Ferretti, Phys.Rev. C92 (2015) , 025202



Ratio $\mu_p G_E^p/G_M^p$

De Sanctis, Ferretti, Santopinto, Vassallo, Phys. Rev. C 84, 055201 (2011)



Interacting Quark Diquark model , *E. Santopinto, Phys. Rev. C 72, 022201(R) (2005)*

Unquenching the quark model
for the MESONS & Why Unquenching?

Santopinto, Galatà, Ferretti, Vassallo

Formalism

$$|\psi_A\rangle = \mathcal{N} \left[|A\rangle + \sum_{BCI} \int dk k^2 |BCkIJ\rangle \frac{\langle BCkIJ|T^\dagger|A\rangle}{M_A - E_B - E_C} \right]$$

The 3P_0 operator

$$T^\dagger = -3\gamma \sum_{i,j} \int d\vec{p}_i d\vec{p}_j \delta(\vec{p}_i + \vec{p}_j) C_{ij} F_{ij} e^{-\alpha_d^2 (p_i - p_j)^2} \\ \times [\chi_{ij} \times \mathcal{Y}_1(\vec{p}_i - \vec{p}_j)]_0^{(0)} b_i^\dagger(\vec{p}_i) d_j^\dagger(\vec{p}_j)$$

$L=S=1, J=0$, color singlet, flavor singlet



UQM: Meson Self Energies & couple channels

- Hamiltonian:

$$H = H_0 + V$$

- H_0 act only in the bare meson space and it is chosen the Godfray and Isgur model
- V couples $|A\rangle$ to the continuum $|BC\rangle$

- Dispersive equation

$$\Sigma(E_a) = \sum_{BC} \int_0^\infty q^2 dq \frac{|V_{a,bc}(q)|^2}{E_a - E_{bc}}$$

- from non-relativistic Schrödinger equation

- Bare energy E_a (H_0 eigenvalue) satisfies:

$$M_a = E_a + \Sigma(E_a)$$

- M_a = physical mass of meson A
- $\Sigma(E_a)$ = self energy of meson A

UQM: Meson Self Energies -- UQM I

- Coupling $V_{a,bc}(q)$ in $\Sigma(E_a)$ calculated as:

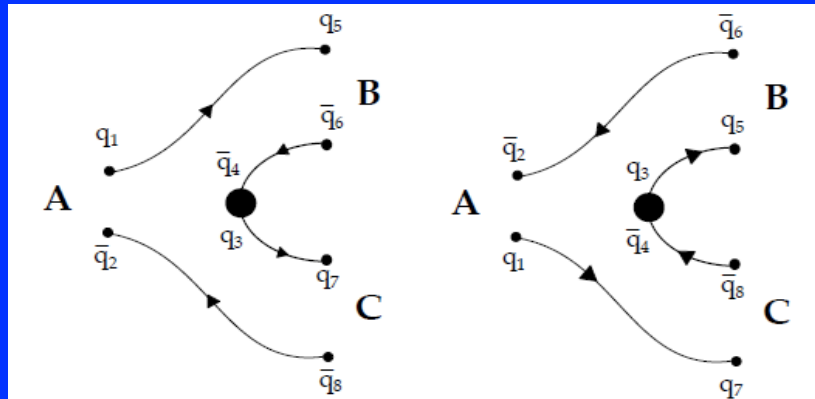
Sum over a complete set of accessible $SU_f(5) \otimes SU_{\text{spin}}(2)$

ground state (1S) mesons

Coupling calculated in the 3P_0 model

$$V_{a,bc}(q) = \sum_{\ell J} \langle BC \bar{q} \ell J | T^\dagger | A \rangle$$

- Two possible diagrams contribute:



- Self energy in the UQM:

$$\Sigma(E_a) = \sum_{BC\ell J} \int_0^\infty q^2 dq \frac{|\langle BC \bar{q} \ell J | T^\dagger | A \rangle|^2}{E_a - E_b - E_c}$$

Godfrey and Isgur model as bare mass

- Bare energies E_a calculated in the relativized G.I. Model for mesons

- Hamiltonian:

$$H = \sqrt{q^2 + m_1^2} + \sqrt{q^2 + m_2^2} + V_{\text{conf}} + V_{\text{hyp}} + V_{\text{so}}$$

- Confining potential:

$$V_{\text{conf}} = - \left(\frac{3}{4} c + \frac{3}{4} br - \frac{\alpha_s(r)}{r} \right) \vec{F}_1 \cdot \vec{F}_2$$

- Hyperfine interaction:

$$V_{\text{hyp}} = - \frac{\alpha_s(r)}{m_1 m_2} \left[\frac{8\pi}{3} \vec{S}_1 \cdot \vec{S}_2 \delta^3(\vec{r}) + \frac{1}{r^3} \left(\frac{3 \vec{S}_1 \cdot \vec{r} \vec{S}_2 \cdot \vec{r}}{r^2} - \vec{S}_1 \cdot \vec{S}_2 \right) \right] \vec{F}_i \cdot \vec{F}_j$$

- Spin-orb. :

$$V_{\text{so,cm}} = - \frac{\alpha_s(r)}{r^3} \left(\frac{1}{m_i} + \frac{1}{m_j} \right) \left(\frac{\vec{S}_i}{m_i} + \frac{\vec{S}_j}{m_j} \right) \cdot \vec{L} \vec{F}_i \cdot \vec{F}_j$$

$$V_{\text{so,tp}} = - \frac{1}{2r} \frac{\partial H_{ij}^{\text{conf}}}{\partial r} \left(\frac{\vec{S}_i}{m_i^2} + \frac{\vec{S}_j}{m_j^2} \right) \cdot \vec{L}$$

UQM or couple channel Quark Model

- Parameters of the relativized QM fitted to

$$M_a = E_a + \Sigma(E_a)$$

- Recursive fitting procedure
- M_a = calculated physical masses of q bar-q mesons → reproduce experimental spectrum [PDG]
- Intrinsic error of QM/UQM calculations: 30-50 MeV

UQM: charmonium with self-energy corr.

- Parameters of the UQM (3P_0 vertices)

Parameter	Value
γ_0	0.510
α	0.500 GeV
r_q	0.335 fm
m_n	0.330 GeV
m_s	0.550 GeV
m_c	1.50 GeV

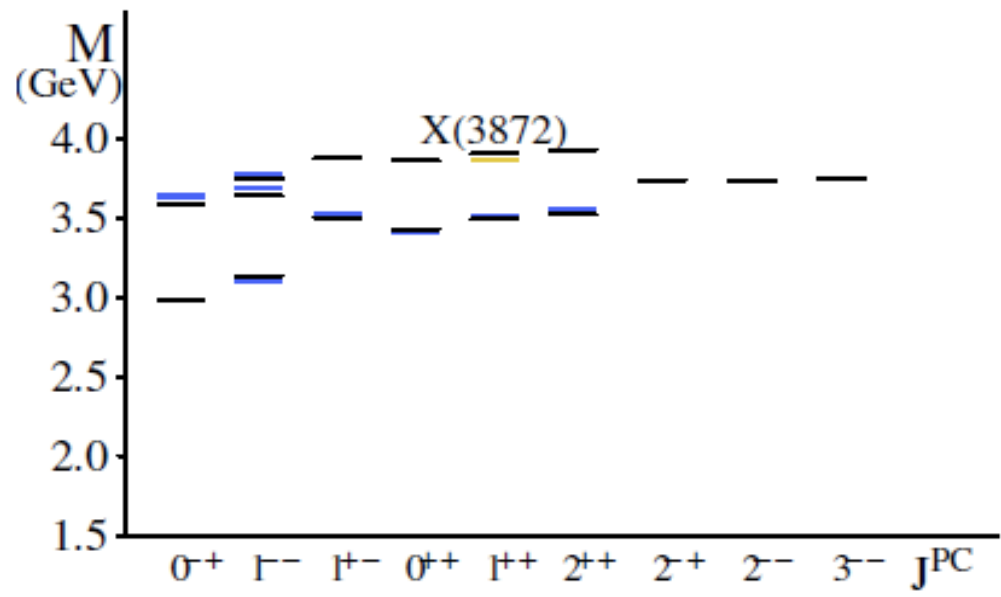
-
-
- fitted to:

State	DD	DD^*	D^*D^*	D_sD_s	$D_sD_s^*$	$D_s^*D_s^*$	Total	Exp.
$\eta_c(3^1S_0)$	–	38.8	52.3	–	–	–	91.1	–
$\Psi(4040)(3^3S_1)$	0.2	37.2	39.6	3.3	–	–	80.3	80 ± 10
$h_c(2^1P_1)$	–	64.6	–	–	–	–	64.6	–
$\chi_{c0}(2^3P_0)$	97.7	–	–	–	–	–	97.7	–
$\chi_{c2}(2^3P_2)$	27.2	9.8	–	–	–	–	37.0	–
$\Psi(3770)(1^3D_1)$	27.7	–	–	–	–	–	27.7	27.2 ± 1.0
$c\bar{c}(1^3D_3)$	1.7	–	–	–	–	–	1.7	–
$c\bar{c}(2^1D_2)$	–	62.7	46.4	–	8.8	–	117.9	–
$\Psi(4160)(2^3D_1)$	11.2	0.4	39.4	2.1	5.6	–	58.7	103 ± 8
$c\bar{c}(2^3D_2)$	–	43.5	49.3	–	11.3	–	104.1	–
$c\bar{c}(2^3D_3)$	17.2	58.3	48.1	3.6	2.6	–	129.8	–

UQM: charmonium spectrum with self-energy corr.

Ferretti, Galata' and Santopinto, Phys. Rev. C 88, 015207 (2013)

State	J^{PC}	$D\bar{D}$	$\bar{D}D^*$ $D\bar{D}^*$	\bar{D}^*D^*	$D_s\bar{D}_s$	$D_s\bar{D}_s^*$ $\bar{D}_sD_s^*$	$D_s^*\bar{D}_s^*$	$\eta_c\eta_c$	$\eta_c J/\Psi$	$J/\Psi J/\Psi$	$\Sigma(E_a)$	E_a	M_a	$M_{exp.}$
$\eta_c(1^1S_0)$	0^{-+}	-	-34	-31	-	-8	-8	-	-	-2	-83	3062	2979	2980
$J/\Psi(1^3S_1)$	1^{--}	-8	-27	-41	-2	-6	-10	-	-2	-	-96	3233	3137	3097
$\eta_c(2^1S_0)$	0^{-+}	-	-52	-41	-	-9	-8	-	-	-1	-111	3699	3588	3637
$\Psi(2^3S_1)$	1^{--}	-18	-42	-54	-2	-7	-10	-	-1	-	-134	3774	3640	3686
$h_c(1^1P_1)$	1^{+-}	-	-59	-48	-	-11	-10	-	-2	-	-130	3631	3501	3525
$\chi_{c0}(1^3P_0)$	0^{++}	-31	-	-72	-4	-	-15	0	-	-3	-125	3555	3430	3415
$\chi_{c1}(1^3P_1)$	1^{++}	-	-54	-53	-	-9	-11	-	-	-2	-129	3623	3494	3511
$\chi_{c2}(1^3P_2)$	2^{++}	-17	-40	-57	-3	-8	-10	0	-	-2	-137	3664	3527	3556
$h_c(2^1P_1)$	1^{+-}	-	-55	-76	-	-12	-8	-	-1	-	-152	4029	3877	-
$\chi_{c0}(2^3P_0)$	0^{++}	-23	-	-86	-1	-	-13	0	-	-1	-124	3987	3863	-
$\chi_{c1}(2^3P_1)$	1^{++}	-	-30	-66	-	-11	-9	-	-	-1	-117	4025	3908	3872
$\chi_{c2}(2^3P_2)$	2^{++}	-2	-42	-54	-4	-8	-10	0	-	-1	-121	4053	3932	3927
$c\bar{c}(1^1D_2)$	2^{-+}	-	-99	-62	-	-12	-10	-	-	-	-	-	-	-
$\Psi(3770)(1^3D_1)$	1^{--}	-11	-40	-84	-4	-2	-16	-	-	-	-	-	-	-
$c\bar{c}(1^3D_2)$	2^{--}	-	-106	-61	-	-11	-11	-	-	-	-	-	-	-
$c\bar{c}(1^3D_3)$	3^{--}	-25	-49	-88	-4	-8	-10	-	-	-	-	-	-	-



UQM: charmonium with self-energy corr.

Ferretti, Galata' and Santopinto, Phys. Rev. C 88

- Experimental mass: 3871.68 ± 0.17 MeV [PDG]
- Several predictions for X(3872)'s mass. Here: **c bar-c + continuum effects**

$\chi_{c1}(2^3P_1)$'s mass (MeV)	Reference
3908	This paper
4007.5	[20]
3990 [1]	[2]
3920.5	[3]
3896 [3]	[4]
	[5]

- [1] Ferretti, Galata' and Santopinto, Phys. Rev. C 88, 015207 (2013);
- [2] Eichten et al., Phys. Rev. D 69,(2004)
- [3] Kalashnikova, Phys. Rev. D 72, 034010 (2005)
- [4] Eichten et al., Phys. Rev. D 73, 014014 (2008)
- [5] Pennington and Wilson, Phys. Rev. D 76, 077502 (2007)

☒ Interpretation of the X(3872) as a charmonium state plus an extra component due to the coupling to the meson-meson continuum

Ferretti, Galatà, Santopinto, **Phys.Rev. C88 (2013) 1, 015207**

- UCQM results used to study the problem of the **X(3872)** mass, meson with $J^{PC} = 1^{++}$, 2^3P_1 quantum numbers
- Experimental mass: 3871.68 ± 0.17 MeV [PDG]
- X(3872) very close to $D \bar{D}^*$ decay threshold
- Possible importance of continuum coupling effects?
- **Several interpretations:**
 - pure $c \bar{c}$
 - $D \bar{D}^*$ molecule
 - tetraquark
 - $c \bar{c} +$ continuum effects
- necessary to study strong and radiative decays to understand the situation

Radiative decays

Ferretti, Galatà, Santopinto, Phys.Rev. D90 (2014) 5, 054010

Transition	E_γ [MeV]	$\Gamma_{c\bar{c}}$ [KeV] present paper	$\Gamma_{D\bar{D}^*}$ [KeV] Ref. [7]	$\Gamma_{D\bar{D}^*}$ [KeV] Ref. [9]	$\Gamma_{D\bar{D}^*}$ [KeV] Ref. [59]	$\Gamma_{c\bar{c}+D\bar{D}^*}$ [KeV] Ref. [60]	$\Gamma_{exp.}$ [KeV] PDG [43]
$X(3872) \rightarrow J/\Psi\gamma$	697	11	8	64 – 190	125 – 251	2 – 17	≈ 7
$X(3872) \rightarrow \Psi(2S)\gamma$	181	70	0.03			7 – 59	≈ 36
$X(3872) \rightarrow \Psi(3770)\gamma$	101	4.0	0				
$X(3872) \rightarrow \Psi_2(1^3D_2)\gamma$	34	0.35	0				

[7] Swanson: molecular interpretation

[9] Oset: molecular interpretation

[59]-[60] Faessler : molecular ; $c\bar{c}$ + molecular

The Molecular model does not predict radiative decays into $\Psi(3770)$ and $\Psi_2(1^3D_2)$ - \rightarrow Possible way to distinguish between the two interpretations

- **Prompt production from CDF collaboration in high-energy hadron collisions incompatible with a molecular interpretation**
- meson-meson molecule: large (a few fm) and fragile
- See: Bignamini et al., Phys. Rev. Lett. **103**, 162001 (2009); Bauer, Int. J. Mod. Phys. A **20**, 3765 (2005)

Bottomonium spectrum (in a couple channel calculations)

Ferretti, Santopintio, Phys.Rev. D90, 094022 (2014)

- Parameters of the UQM (3P_0 vertices)

Parameter	Value
γ_0	0.732
α	0.500 GeV
r_q	0.335 fm
m_n	0.330 GeV
m_s	0.550 GeV
m_c	1.50 GeV
m_b	4.70 GeV

- Pair-creation strength γ_0 fitted to:

$$\begin{aligned}
 \Gamma_{\Upsilon(4S) \rightarrow B\bar{B}} &= 2\Phi_{A \rightarrow BC} \left| \langle BC \bar{q}_0 \ell J | T^\dagger | A \rangle \right|^2 \\
 &= 2\Phi_{\Upsilon(4S) \rightarrow B\bar{B}} \\
 &\quad \left| \langle B\bar{B} \bar{q}_0 11 | T^\dagger | \Upsilon(4S) \rangle \right|^2 \\
 &= 21 \text{ MeV} ,
 \end{aligned}$$

Bottomonium Strong Decays

Ferretti, Santopinto, Phys.Rev. D90 094022 (2014)

- Two-body strong decays. Results:

State	Mass [MeV]	J^{PC}	BB	BB^* $\bar{B}B^*$	B^*B^*	B_sB_s	$B_sB_s^*$ $\bar{B}_sB_s^*$	$B_s^*B_s^*$
$\Upsilon(4^3S_1)$	10.595 $10579.4 \pm 1.2^\dagger$	1^{--}	21	–	–	–	–	–
$\chi_{b2}(2^3F_2)$	10585	2^{++}	34	–	–	–	–	–
$\Upsilon(3^3D_1)$	10661	1^{--}	23	4	15	–	–	–
$\Upsilon_2(3^3D_2)$	10667	2^{--}	–	37	30	–	–	–
$\Upsilon_2(3^1D_2)$	10668	2^{-+}	–	55	57	–	–	–
$\Upsilon_3(3^3D_3)$	10673	3^{--}	15	56	113	–	–	–
$\chi_{b0}(4^3P_0)$	10726	0^{++}	26	–	24	–	–	–
$\Upsilon_3(2^3G_3)$	10727	3^{--}	3	43	39	–	–	–
$\chi_{b1}(4^3P_1)$	10740	1^{++}	–	20	1	–	–	–
$h_b(4^1P_1)$	10744	1^{+-}	–	33	5	–	–	–
$\chi_{b2}(4^3P_2)$	10751	2^{++}	10	28	5	1	–	–
$\chi_{b2}(3^3F_2)$	10800	2^{++}	5	26	53	2	2	–
$\Upsilon_3(3^1F_3)$	10803	3^{+-}	–	28	46	–	3	–
$\Upsilon(10860)$	$10876 \pm 11^\dagger$	1^{--}	1	21	45	0	3	1
$\Upsilon_2(4^3D_2)$	10876	2^{--}	–	28	36	–	4	4
$\Upsilon_2(4^1D_2)$	10877	2^{-+}	–	22	37	–	4	3
$\Upsilon_3(4^3D_3)$	10881	3^{--}	1	4	49	0	1	2
$\Upsilon_3(3^3G_3)$	10926	3^{--}	7	0	13	2	0	5
$\Upsilon(11020)$	$11019 \pm 8^\dagger$	1^{--}	0	8	26	0	0	2

Bottomonium spectrum (in couple channel calculations)

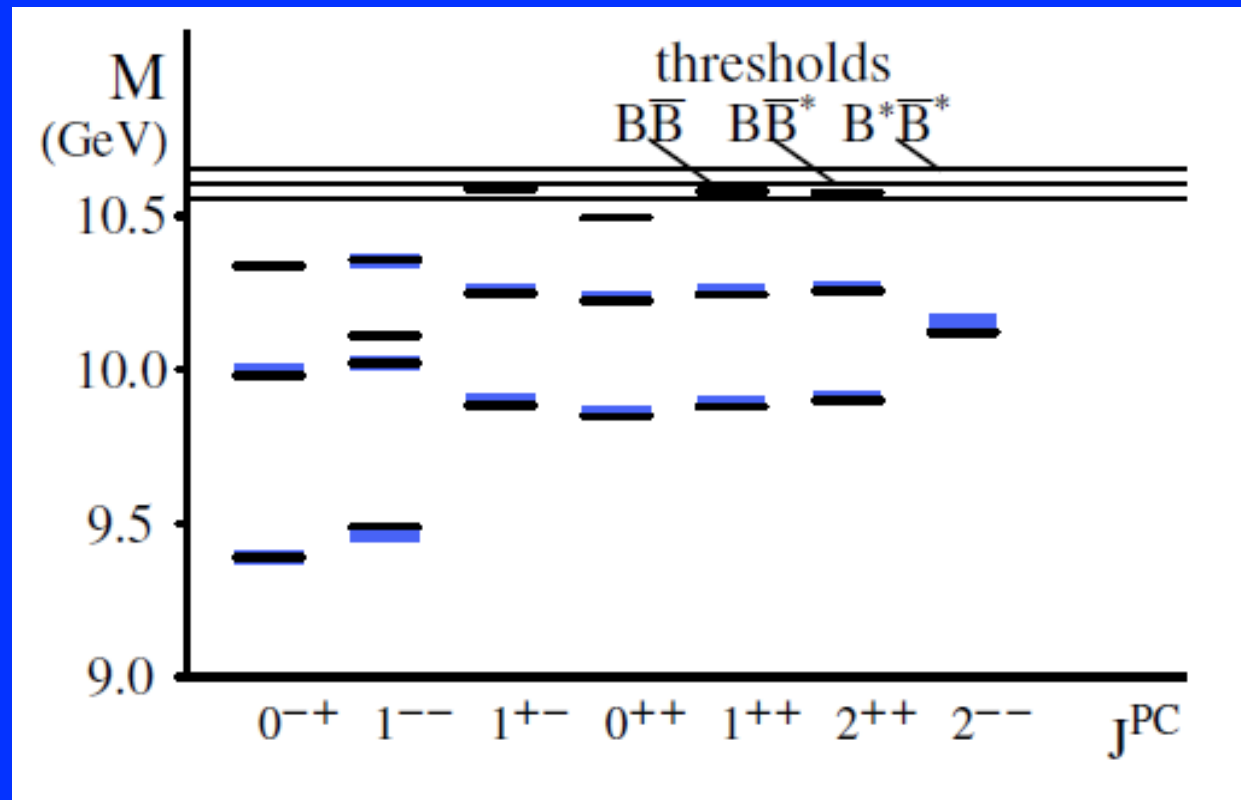
Ferretti, Santopintio, Phys.Rev. D90, 094022 (2014)

State	J^{PC}	BB	BB^* $\bar{B}B^*$	B^*B^*	B_sB_s	$B_sB_s^*$ $\bar{B}_sB_s^*$	$B_s^*B_s^*$	B_cB_c	$B_cB_c^*$ $\bar{B}_cB_c^*$	$B_c^*B_c^*$	$\eta_b\eta_b$	$\eta_b\Upsilon$	$\Upsilon\Upsilon$	$\Sigma(E_a)$	E_a	M_a	$M_{exp.}$
$\eta_b(1^1S_0)$	0^{-+}	-	-26	-26	-	-5	-5	-	-1	-1	-	-	0	-64	9455	9391	9391
$\Upsilon(1^3S_1)$	1^{--}	-5	-19	-32	-1	-4	-7	0	0	-1	-	0	-	-69	9558	9489	9460
$\eta_b(2^1S_0)$	0^{-+}	-	-43	-41	-	-8	-7	-	-1	-1	-	-	0	-101	10081	9980	9999
$\Upsilon(2^3S_1)$	1^{--}	-8	-31	-51	-2	-6	-9	0	0	-1	-	0	-	-108	10130	10022	10023
$\eta_b(3^1S_0)$	0^{-+}	-	-59	-52	-	-8	-8	-	-1	-1	-	-	0	-129	10467	10338	-
$\Upsilon(3^3S_1)$	1^{--}	-14	-45	-68	-2	-6	-10	0	0	-1	-	0	-	-146	10504	10358	10355
$h_b(1^1P_1)$	1^{+-}	-	-49	-47	-	-9	-8	-	-1	-1	-	0	-	-115	10000	9885	9899
$\chi_{b0}(1^3P_0)$	0^{++}	-22	-	-69	-3	-	-13	0	-	-1	0	-	0	-108	9957	9849	9859
$\chi_{b1}(1^3P_1)$	1^{++}	-	-46	-49	-	-8	-9	-	-1	-1	-	-	0	-114	9993	9879	9893
$\chi_{b2}(1^3P_2)$	2^{++}	-11	-32	-55	-2	-6	-9	0	-1	-1	0	-	0	-117	10017	9900	9912
$h_b(2^1P_1)$	1^{+-}	-	-66	-59	-	-10	-9	-	-1	-1	-	0	-	-146	10393	10247	10260
$\chi_{b0}(2^3P_0)$	0^{++}	-33	-	-85	-4	-	-14	0	-	-1	0	-	0	-137	10363	10226	10233
$\chi_{b1}(2^3P_1)$	1^{++}	-	-63	-60	-	-9	-10	-	-1	-1	-	-	0	-144	10388	10244	10255
$\chi_{b2}(2^3P_2)$	2^{++}	-16	-42	-72	-2	-6	-10	0	0	-1	0	-	0	-149	10406	10257	10269
$h_b(3^1P_1)$	1^{+-}	-	-18	-73	-	-11	-10	-	-1	-1	-	0	-	-114	10705	10591	-
$\chi_{b0}(3^3P_0)$	0^{++}	-4	-	-160	-6	-	-15	0	-	-1	0	-	0	-186	10681	10495	-
$\chi_{b1}(3^3P_1)$	1^{++}	-	-25	-74	-	-11	-10	-	0	-1	-	-	0	-121	10701	10580	-
$\chi_{b2}(3^3P_2)$	2^{++}	-19	-16	-79	-3	-8	-12	0	0	-1	0	-	0	-138	10716	10578	-
$\Upsilon_2(1^1D_2)$	2^{-+}	-	-72	-66	-	-11	-10	-	-1	-1	-	-	0	-161	10283	10122	-
$\Upsilon(1^3D_1)$	1^{--}	-24	-22	-90	-3	-3	-16	0	0	-1	-	0	-	-159	10271	10112	-
$\Upsilon_2(1^3D_2)$	2^{--}	-	-70	-68	-	-10	-11	-	-1	-1	-	0	-	-161	10282	10121	10164
$\Upsilon_3(1^3D_3)$	3^{--}	-18	-43	-78	-3	-8	-11	0	-1	-1	-	0	-	-163	10290	10127	-

Bottomonium

Ferretti, Santopinto, Phys.Rev. D90 (2014) 9, 094022

- Results:



There Is an analogous of the X(3872) in the $\chi_b(3P)$ system?

Ferretti, Santopinto, Phys.Rev. D90 (2014) 9, 094022

- Results used to study some properties of the $\chi_b(3P)$ system, meson multiplet with $N=3$, $L=1$ quantum numbers
- $\chi_b(3P)$ states close to first open bottom decay thresholds
- Possible importance of continuum coupling effects?
- **Pure $b\bar{b}$** and **$b\bar{b}$ + continuum effects** interpretations
- Necessary to study decays (strong, e.m., hadronic, ...) to confirm one interpretation
-

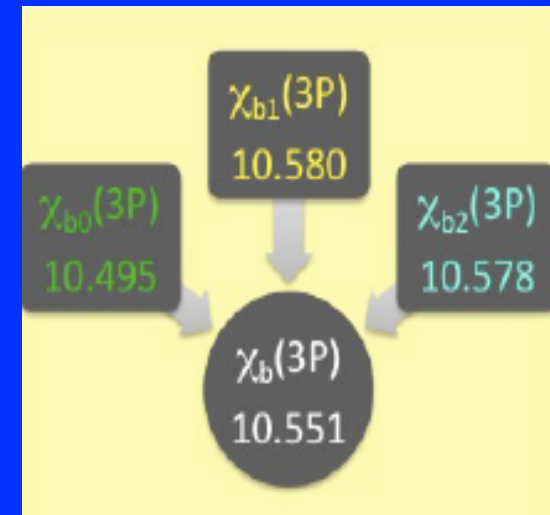
Couple Channels corrections to Bottomonium , the $\chi_b(3P)$ system

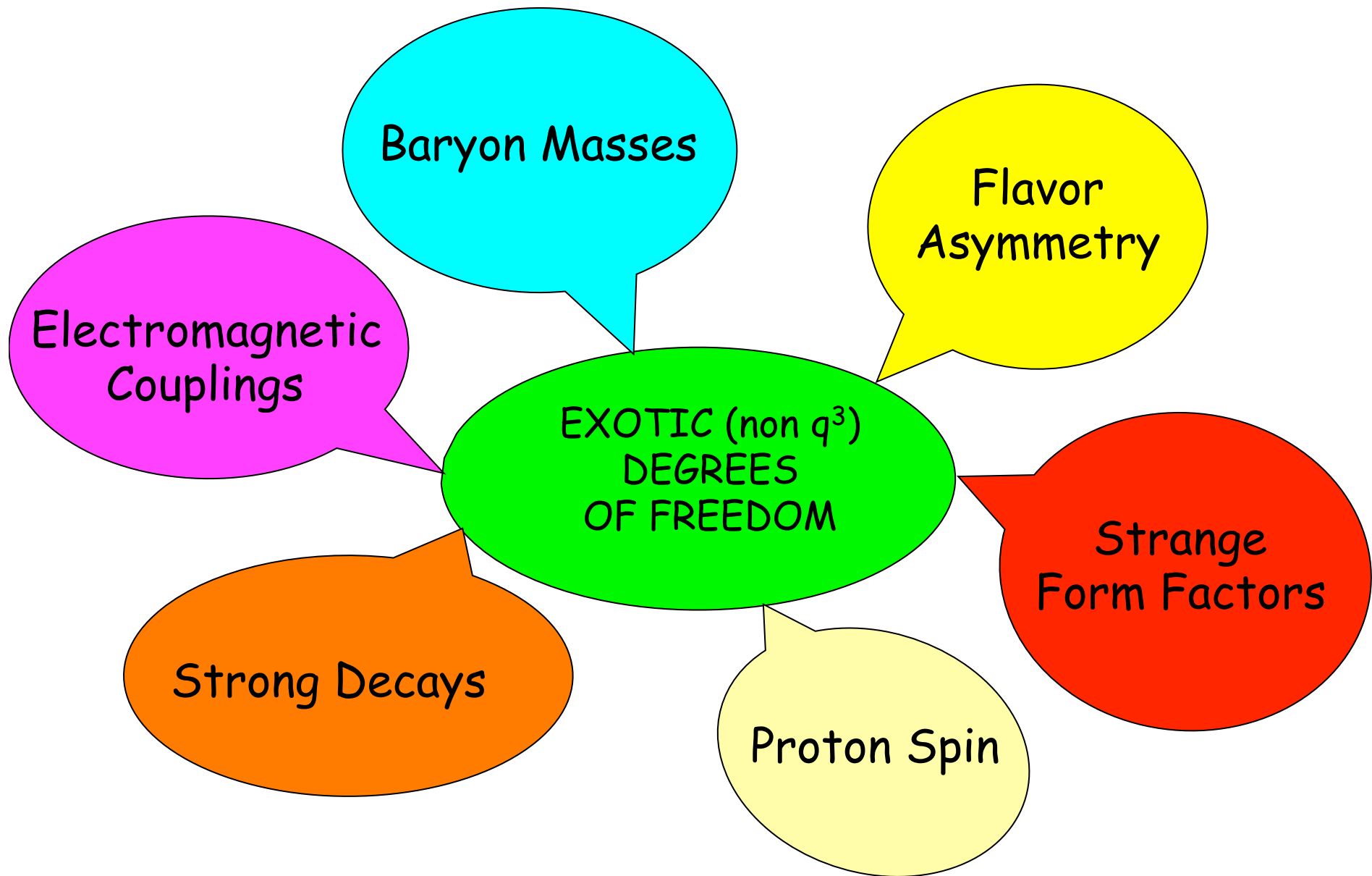
Ferretti, Santopinto, Phys.Rev. D90 (2014) 9, 094022

- **Some experimental results for the mass barycenter of the system:**

- $M[\chi_b(3P)] = 10.530 \pm 0.005$ (stat.) ± 0.009 (syst.) GeV
- Aad et al. [ATLAS Coll.], Phys. Rev. Lett. **108**, 152001 (2012)
- $M[\chi_b(3P)] = 10.551 \pm 0.014$ (stat.) ± 0.017 (syst.) GeV
- Abazov et al. [D0 Coll.], Phys. Rev. D 86, 031103 (2012)

- **Mass barycenter in the UQM:**





Unquenching the baryon quark model & Why Unquenching?

R.Bijker ,E. .Santopinto,. PRC 80, 065210 (2009),
E. Santopintom, R. Bijker,PRC 82, 062202 (2010);J. Ferrettii,Santopinto, Bijker
Phys. Rev. C 85, 035204 (2012)

Many versions of CQMs have been developed
(IK, CI, GBE, U(7), hCQM, Bonn, etc.)

non relativistic and relativistic

While these models display peculiar features,
they share the following main features :

the effective degrees of freedom of $3q$ and a confining potential

the underlying $O(3)$ $SU(3)$ symmetry

All of them are able to give a good description of the 3 and 4 stars
spectrum

CQMs:

S

Good description of the spectrum and magnetic moments

Predictions of many quantities:

strong couplings

photocouplings

helicity amplitudes

elastic form factors

structure functions

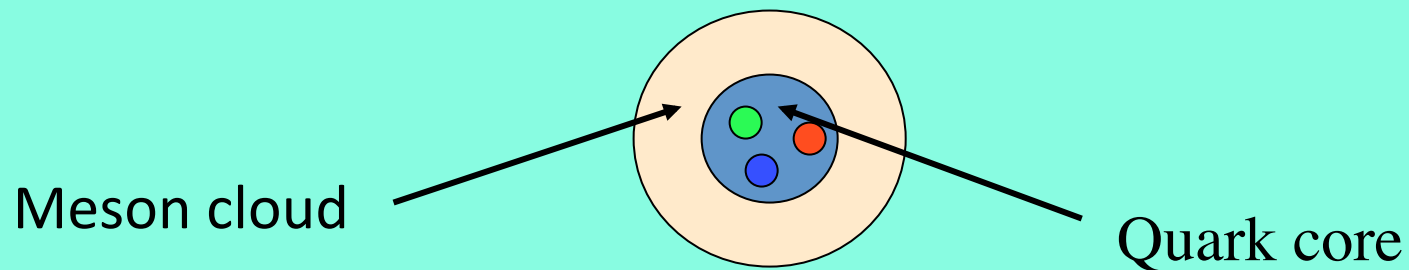
Based on the effective degrees of freedom of 3 constituent quarks

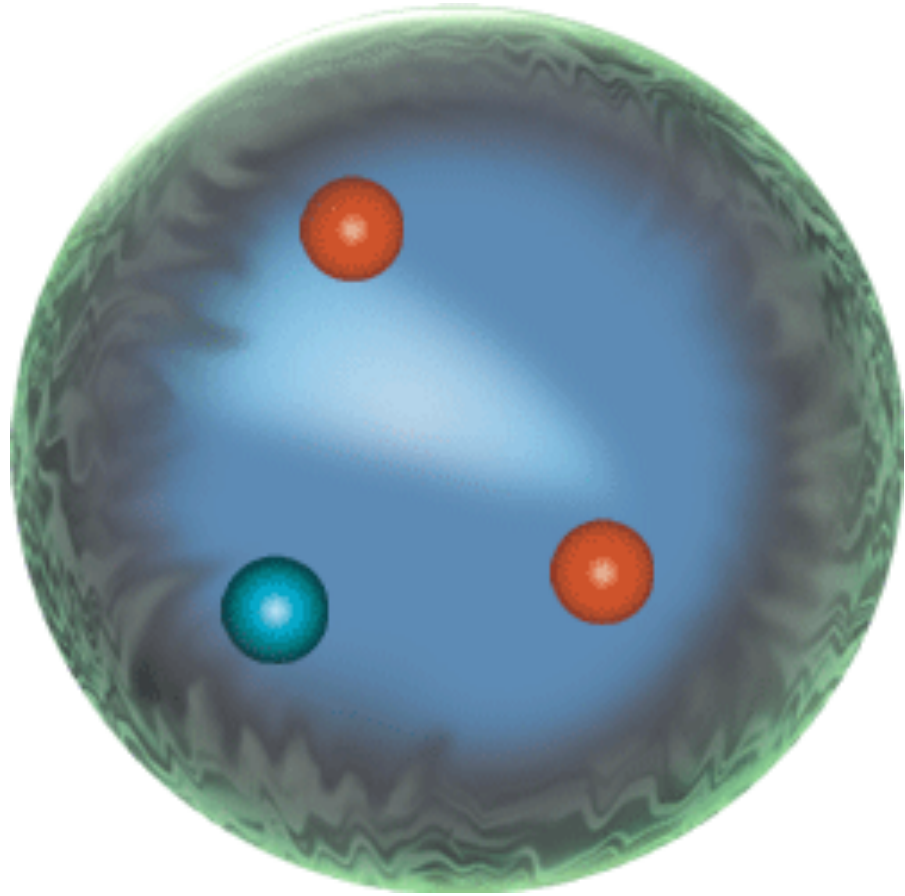
Considering also CQMs for mesons, CQMs able to reproduce the **overall trend of hundred of data**

- ... but they show very similar deviations for **observables** such as
- photocouplings
- helicity amplitudes,

please note

- the medium Q^2 behaviour is fairly well reproduced
- there is lack of strength at **low** Q^2 (outer region) in the e.m. transitions
- emerging picture:
 quark core plus (meson or sea-quark) **cloud**





Formalism

$$|\psi_A\rangle = \mathcal{N} \left\{ |A\rangle + \sum_{BClJ} \int d\vec{K} k^2 dk |BC\vec{K}klJ\rangle \frac{\langle BC\vec{K}klJ | T^\dagger | A\rangle}{M_A - E_B - E_C} \right\}$$

Three-quark configuration
SU(3) flavor symmetry

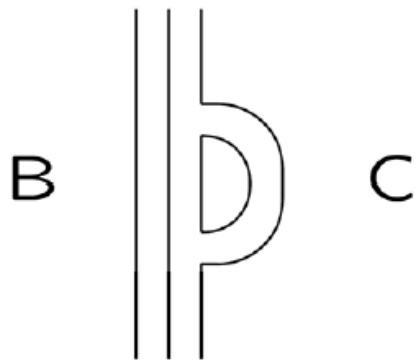
Five-quark component
Isospin symmetry

Pair-creation operator: $T^\dagger = T^\dagger(^3P_0)$
L=S=1, J=0, color singlet, flavor singlet

Unquenched Quark Model

- Harmonic oscillator quark model
- Sum over intermediate meson-baryon states includes for each oscillator shell all possible spin-flavor states
- Oscillator size parameters taken for baryons and mesons taken from literature (Capstick, Isgur, Karl)
- Smearing of the pair-creation vertex (Geiger, Isgur)
- Strength of 3P_0 coupling taken from literature on strong decays of baryons (Capstick, Roberts)
- No attempt to optimize the parameters

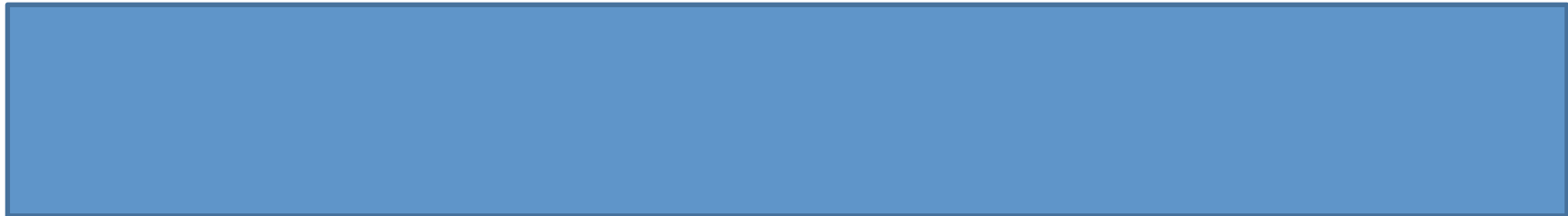
Unquenched Quark Model



Strange quark-antiquark
pairs in the proton with
h.o. wave functions

Tornqvist & Zenczykowski (1984)
Geiger & Isgur, PRD 55, 299 (1997)
Isgur, NPA 623, 37 (1997)

- Pair-creation operator with 3P_0 quantum numbers of vacuum



The good magnetic moment results of the CQM are preserved by the UCQM

Bijker, Santopinto, Phys.Rev.C80:065210,2009.

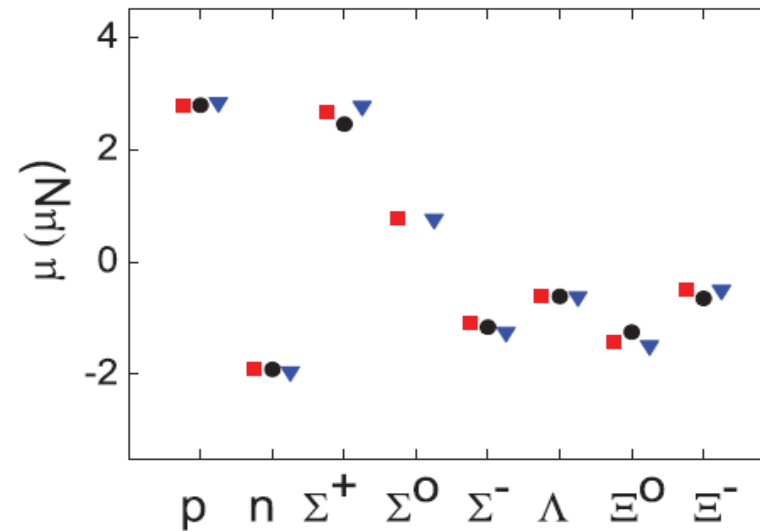


FIG. 3. (Color online) Magnetic moments of octet baryons: experimental values from the Particle Data Group [34] (circles), CQM (squares), and unquenched quark model (triangles).

Flavor Asymmetry

Gottfried sum rule

$$S_G = \int_0^1 dx \frac{F_{2p}(x) - F_{2n}(x)}{x} = \frac{1}{3} - \frac{2}{3} \int_0^1 dx [\bar{d}(x) - \bar{u}(x)]$$

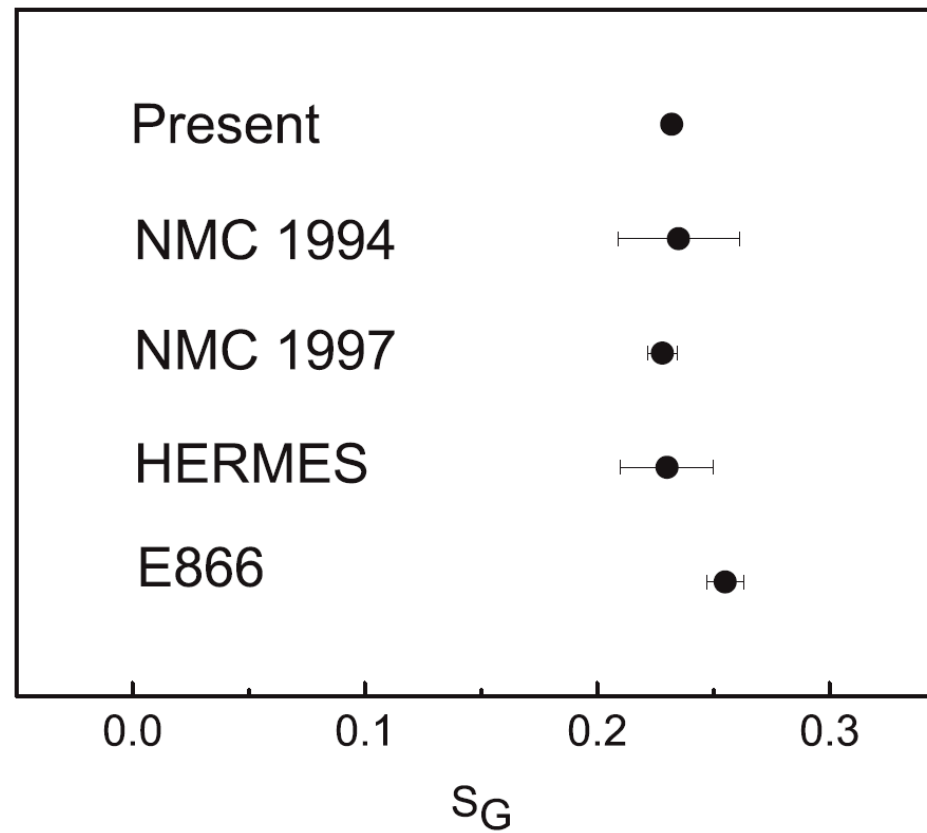
$$S_G \neq \frac{1}{3} \Rightarrow N_{\bar{d}} \neq N_{\bar{u}}$$

$$S_G = 0.2281 \pm 0.0065$$

$$\int_0^1 dx [\bar{d}(x) - \bar{u}(x)] \\ = 0.16 \pm 0.01$$

Proton Flavor asymmetry

Santopinto, Bijker, PRC 82,062202(R) (2010)



Flavor asymmetry of the octet baryons in the UCQM

Santopinto, Bijker, PRC 82,062202(R) (2010)

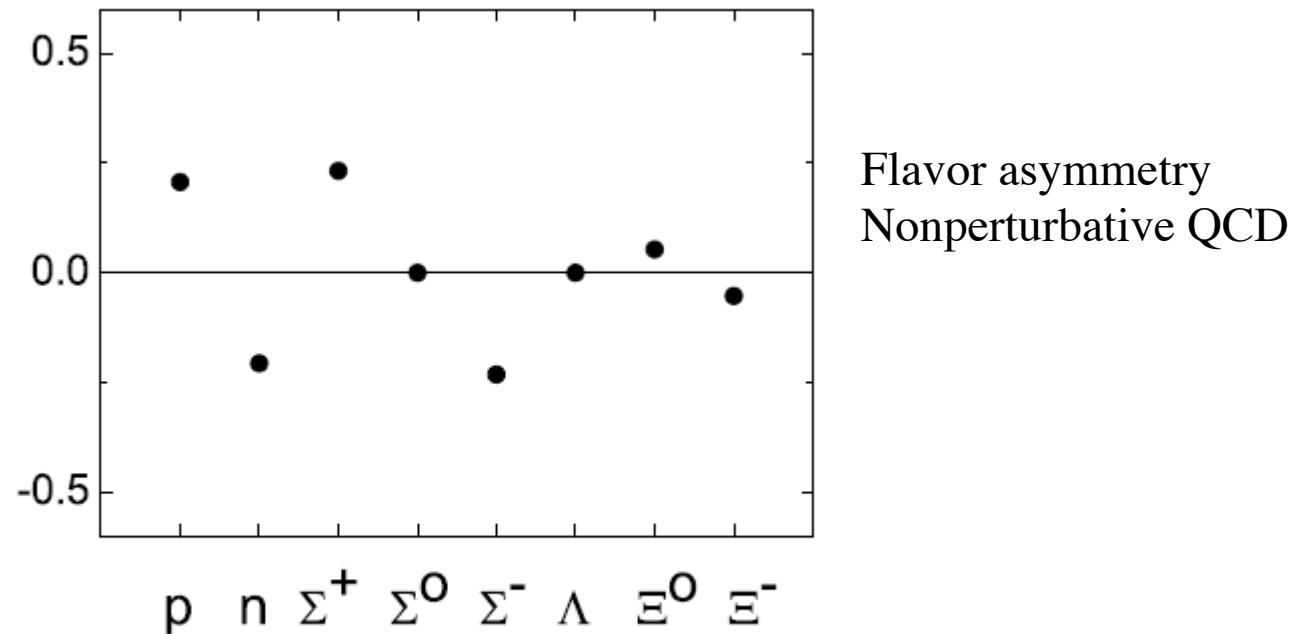


Figure 1. Flavor asymmetry of octet baryons

Pauli blocking (Field & Feynman, 1977) **too small**

Pion dressing of the nucleon (Thomas et al., 1983)

Meson cloud models

Flavor asymmetries of octet baryons

Santopinto, Bijker, PRC 82,062202(R) (2010)

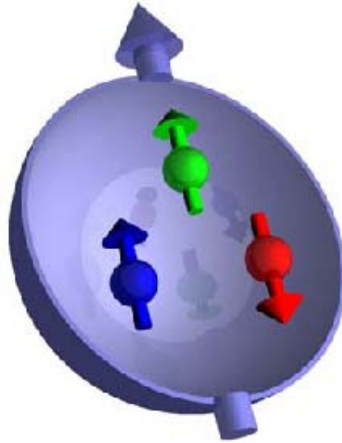
TABLE III. Relative flavor asymmetries of octet baryons.

Model	$\mathcal{A}(\Sigma^+)/\mathcal{A}(p)$	$\mathcal{A}(\Xi^0)/\mathcal{A}(p)$	Ref.
Unquenched CQM	0.833	-0.005	present
Chiral QM	2	1	Eichen
Balance model	3.083	2.075	Y.-J Zhang
Octet couplings	0.353	-0.647	Alberg

$$\Sigma^\pm p \rightarrow \ell^+ \ell^- + X \text{ (e.g., at CERN).}$$

3. Proton Spin Crisis

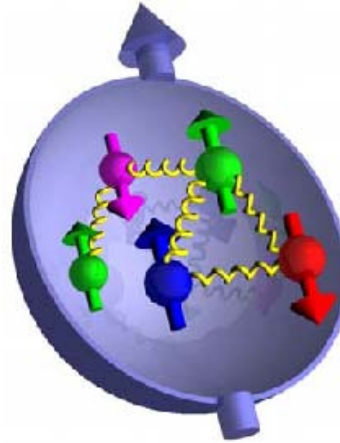
1980's



Naive parton model
3 valence quarks

$$\frac{1}{2} = \frac{1}{2}(\Delta u + \Delta d)$$

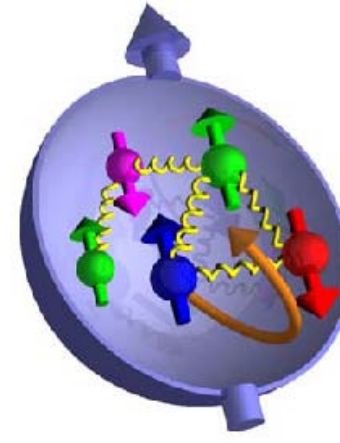
1990's



QCD: contributions from
sea quarks and gluons

$$\frac{1}{2} = \frac{1}{2} \underbrace{(\Delta u + \Delta d + \Delta s)}_{\Delta \Sigma} + \Delta G + \Delta L$$

2000's

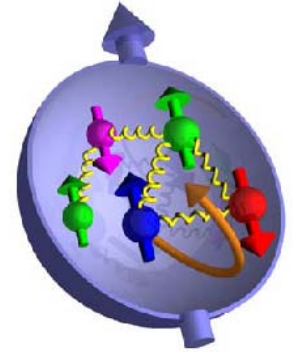


.. and orbital angular
momentum

$$\left. \begin{array}{l} \Delta u = 0.842 \\ \Delta d = -0.427 \\ \Delta s = -0.085 \end{array} \right\} \Delta \Sigma = 0.330 \pm 0.039$$

HERMES, PRD 75, 012007 (2007)
COMPASS, PLB 647, 8 (2007)

Proton Spin



- COMPASS@CERN: Gluon contribution is small (sign undetermined)
- Unquenched quark model

Ageev et al., PLB 633, 25 (2006)
Platchkov, NPA 790, 58 (2007)

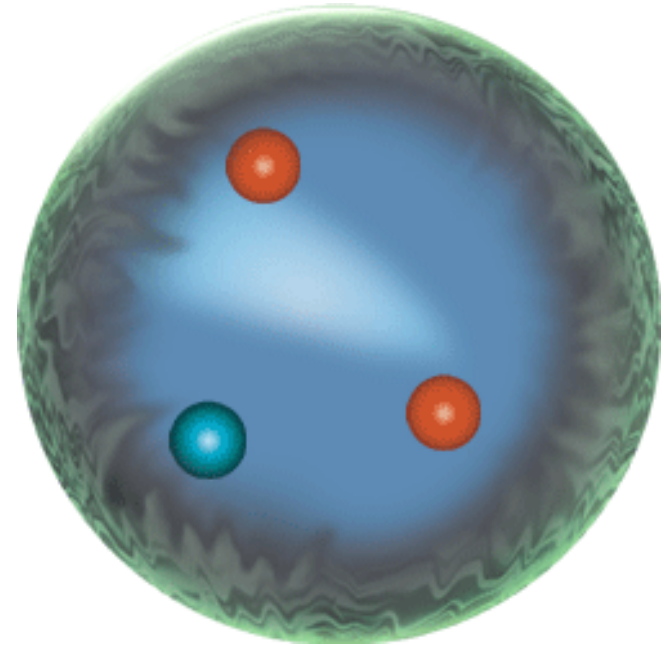
		CQM	Unquenched QM		
			Valence	Sea	Total
p	$\Delta\Sigma$	1	0.378	0.298	0.676
	$2\Delta L$	0	0.000	0.324	0.324
	$2\Delta J$	1	0.378	0.622	1.000

- More than half of the proton spin from the sea!
- Orbital angular momentum

Suggested by Myhrer & Thomas, 2008, but not explicitly calculated

4. Strangeness in the Proton

- The strange (anti)quarks come uniquely from the sea: there is no contamination from up or down valence quarks
- The strangeness distribution is a very sensitive probe of the nucleon's properties
- Flavor content of form factors
- New data from Parity Violating Electron Scattering experiments: SAMPLE, HAPPEX, PVA4 and G0 Collaborations



“There is no excellent beauty that hath not some strangeness in the proportion”
(Francis Bacon, 1561-1626)

Quark Form Factors

- Charge symmetry $G^{u,p} = G^{d,n} \equiv G^u$
 $G^{d,p} = G^{u,n} \equiv G^d$
 $G^{s,p} = G^{s,n} \equiv G^s$
- Quark form factors

$$\begin{aligned} G^u &= (3 - 4 \sin^2 \Theta_W) G^{\gamma,p} - G^{Z,p} \\ G^d &= (2 - 4 \sin^2 \Theta_W) G^{\gamma,p} + G^{\gamma,n} - G^{Z,p} \\ G^s &= (1 - 4 \sin^2 \Theta_W) G^{\gamma,p} - G^{\gamma,n} - G^{Z,p} \end{aligned}$$

Kaplan & Manohar, NPB 310, 527 (1988)
Musolf et al, Phys. Rep. 239, 1 (1994)

Static Properties

$$G_E(0) = e$$

Electric charge

$$G_M(0) = \mu$$

Magnetic moment

$$\langle r^2 \rangle_E = -6 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0}$$

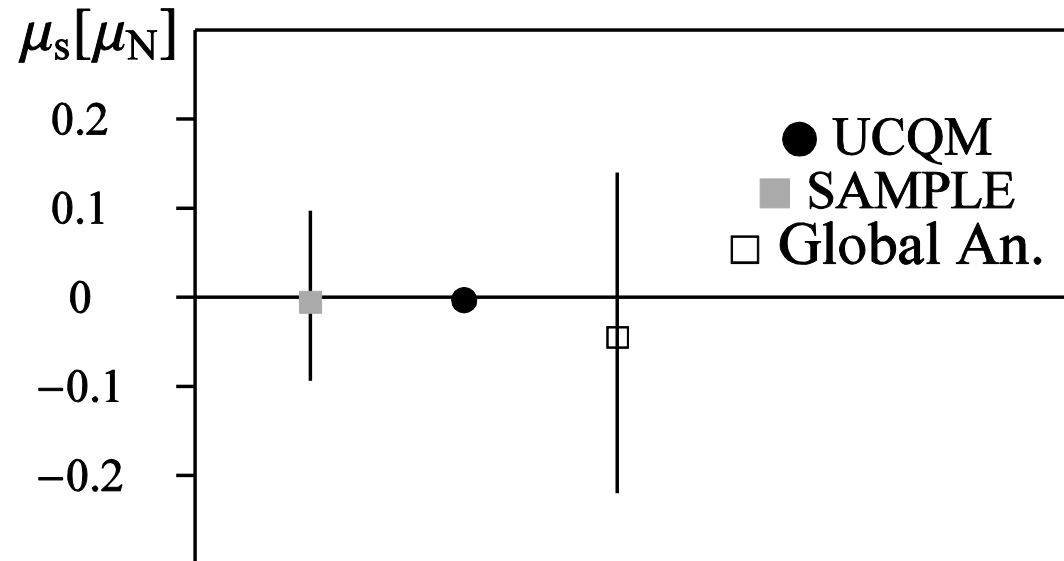
Charge radius

$$\langle r^2 \rangle_M = -\frac{6}{\mu} \left. \frac{dG_M}{dQ^2} \right|_{Q^2=0}$$

Magnetic radius

Strange Magnetic Moment

$$\vec{\mu}_s = \sum_i \mu_{i,s} [2\vec{s}(q_i) + \vec{\ell}(q_i) - 2\vec{s}(\bar{q}_i) - \vec{\ell}(\bar{q}_i)]$$

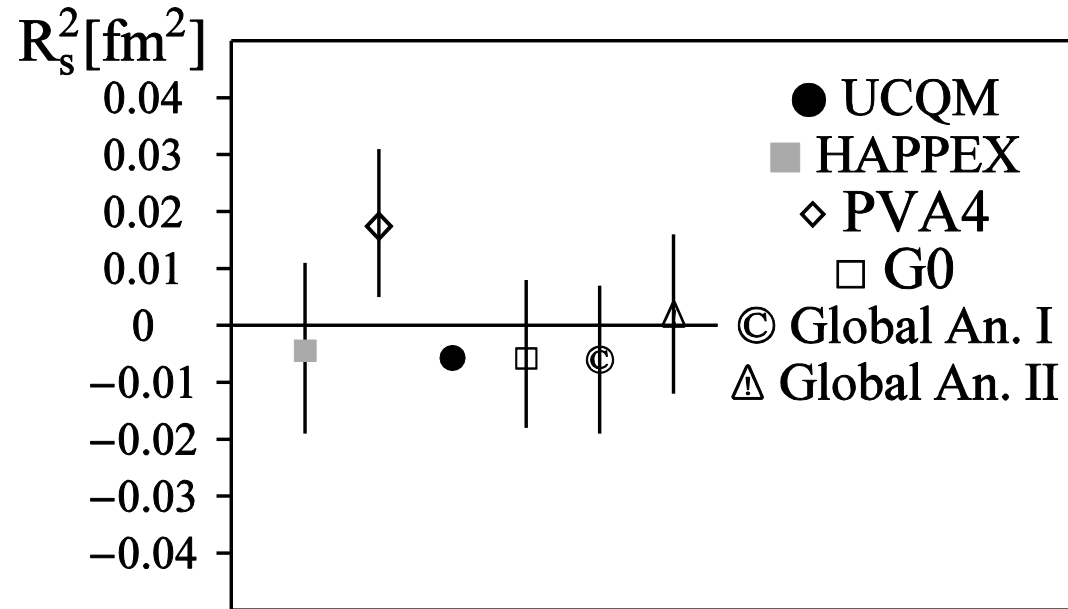


Jacopo Ferretti, Ph.D. Thesis, 2011

Bijker, Ferretti, Santopinto, Phys. Rev. C **85**, 035204 (2012)

Strange Radius

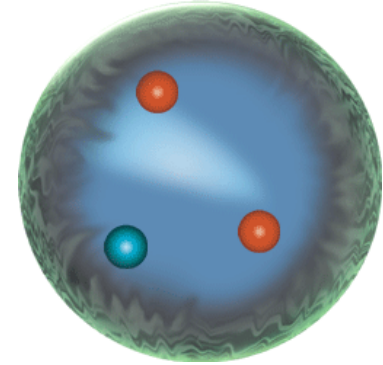
$$R_s^2 = \sum_{i=1}^5 e_{i,s} (\vec{r}_i - \vec{R}_{CM})^2$$



Jacopo Ferretti, Ph.D. Thesis, 2011

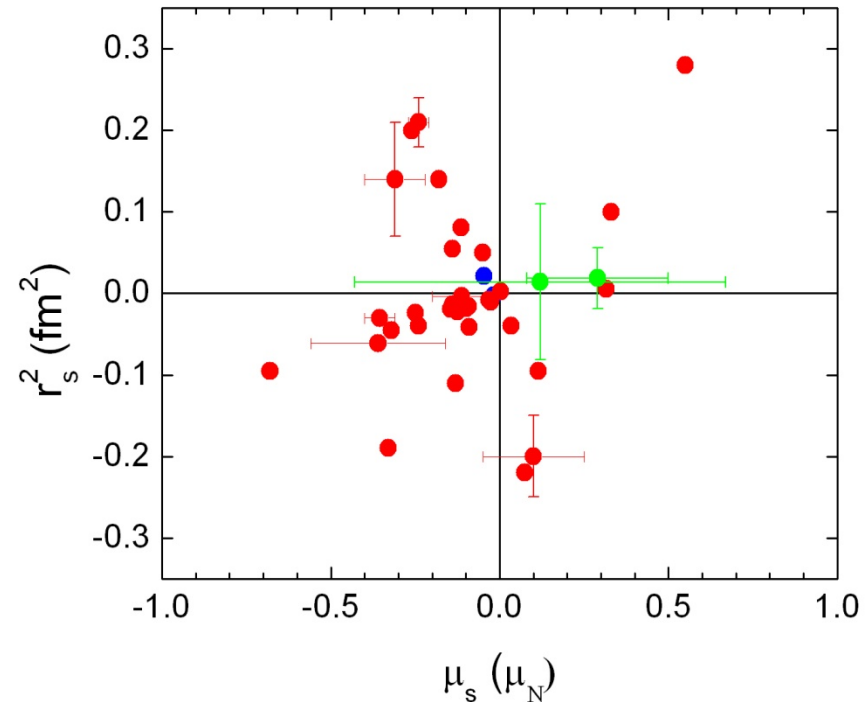
Bijker, Ferretti, Santopinto, Phys. Rev. C **85**, 035204 (2012)

Strange Proton



- Strange radius and magnetic moment of the proton
- Theory
- Lattice QCD
- Global analysis PVES
- Unquenched QM

$$\begin{aligned}\mu_s &= -6 \cdot 10^{-4} (\mu_N) \\ \langle r^2 \rangle_s &= -4 \cdot 10^{-3} (\text{fm}^2)\end{aligned}$$



Jacopo Ferretti, Ph.D. Thesis, 2011