The Discovery of the Wave Nature of the Electron

de Broglie's Theory of Matter Waves

Luis Suarez
University of South Carolina
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Louis de Broglie

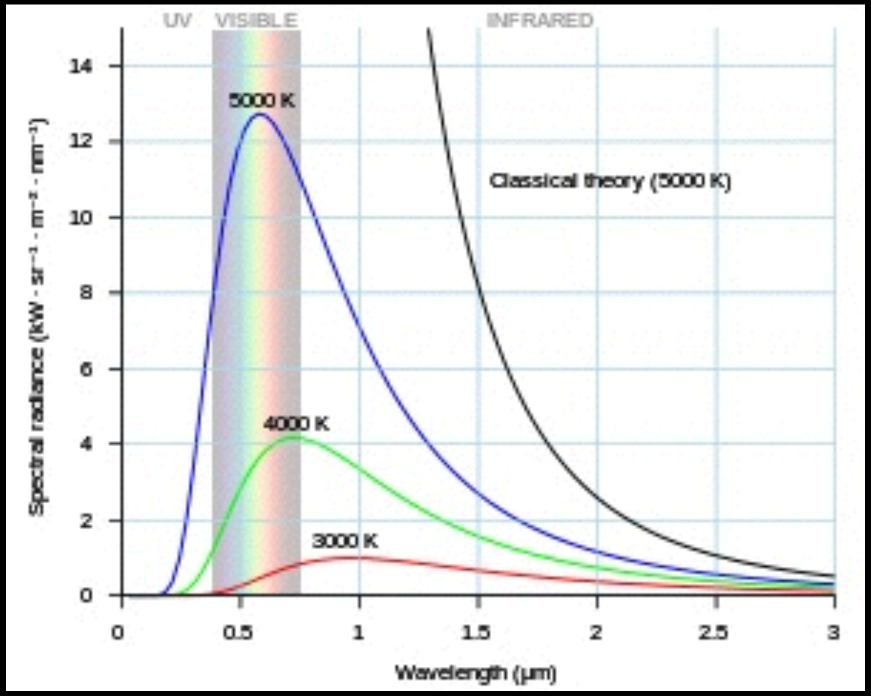
1892 - 1987

A brief history of light

- Is it a stream of particles or a wave?
- Newton thought it was "corpuscular."
- 19th century science seemed to confirm that light is fundamentally wave-like, while matter is fundamentally particle-like.

A brief history of light

- In 1899, Max Planck revived the idea that light was granular. He postulated that radiation is emitted as discrete packets of energy, with energy given by $E = hf$.



Source: Wikipedia

A brief history of light

- This idea was further supported by the photoelectric effect.
- Experiment showed that the kinetic energy of electrons dislodged from an irradiated material increases linearly with the frequency of radiation, and is *independent of intensity*.
- In 1905, Einstein proposed that radiation comes in the form of discrete packets - “photons” - with energy $E = hf$.
- So it seemed light is both particle-like and wave-like.

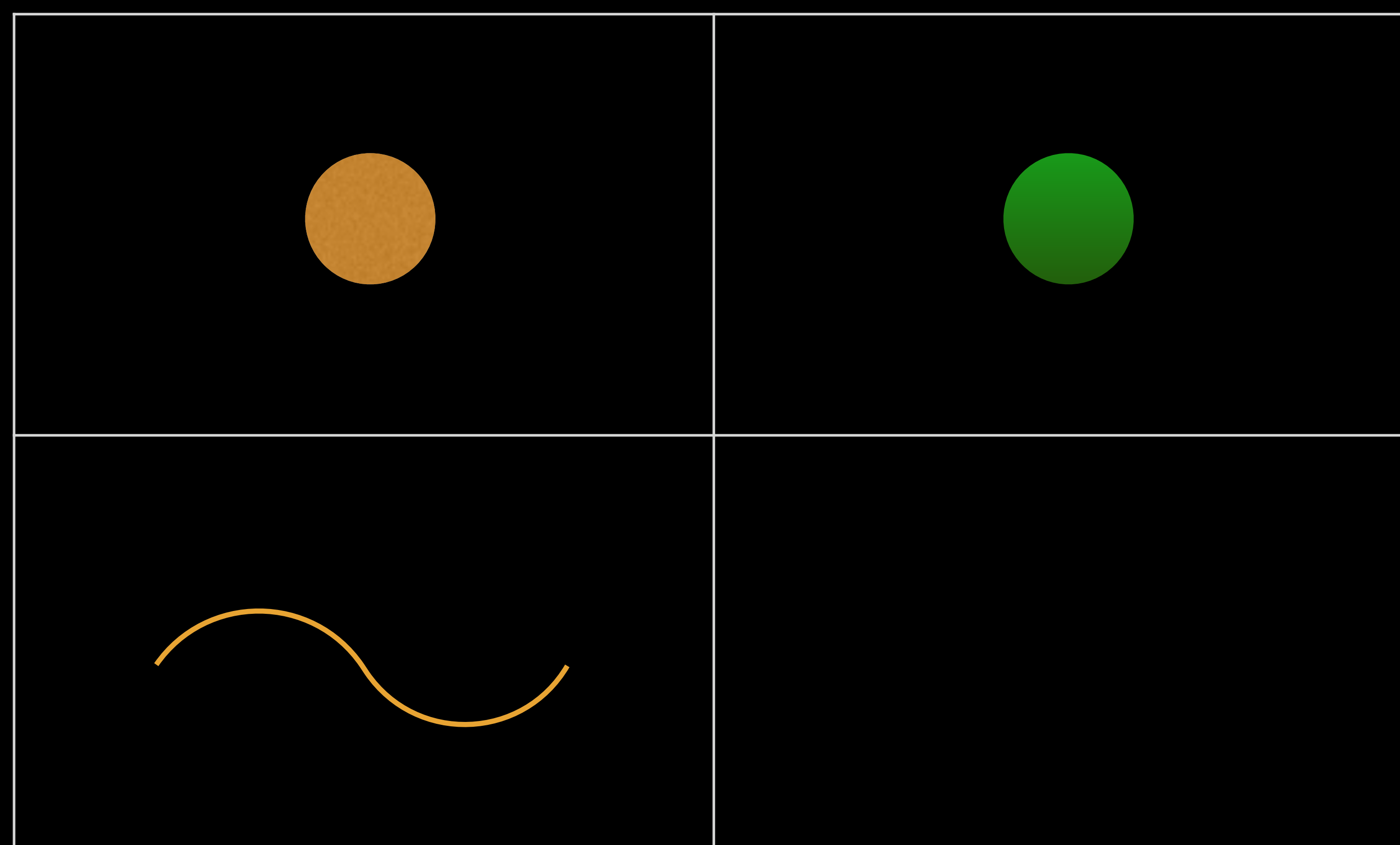
A brief history of light

- In 1913, Bohr introduced his model of the hydrogen atom.
- The model simultaneously explained why the light emitted by hydrogen has very sharp emission lines and why electrons don't continuously lose energy while orbiting the nucleus.
- It is based on Planck's theory of quantized radiation. Bohr stated that electrons could gain/lose energy only by absorbing/emitting photons of specific energy - given by $E = hf$.

“The necessity of assuming for light two contradictory theories - that of waves and that of corpuscles - and the inability to understand why, among the infinity of motions which an electron ought to be able to have in the atom according to classical concepts, only certain ones were possible: such were the enigmas confronting physicists at the time I resumed my studies of theoretical physics.”

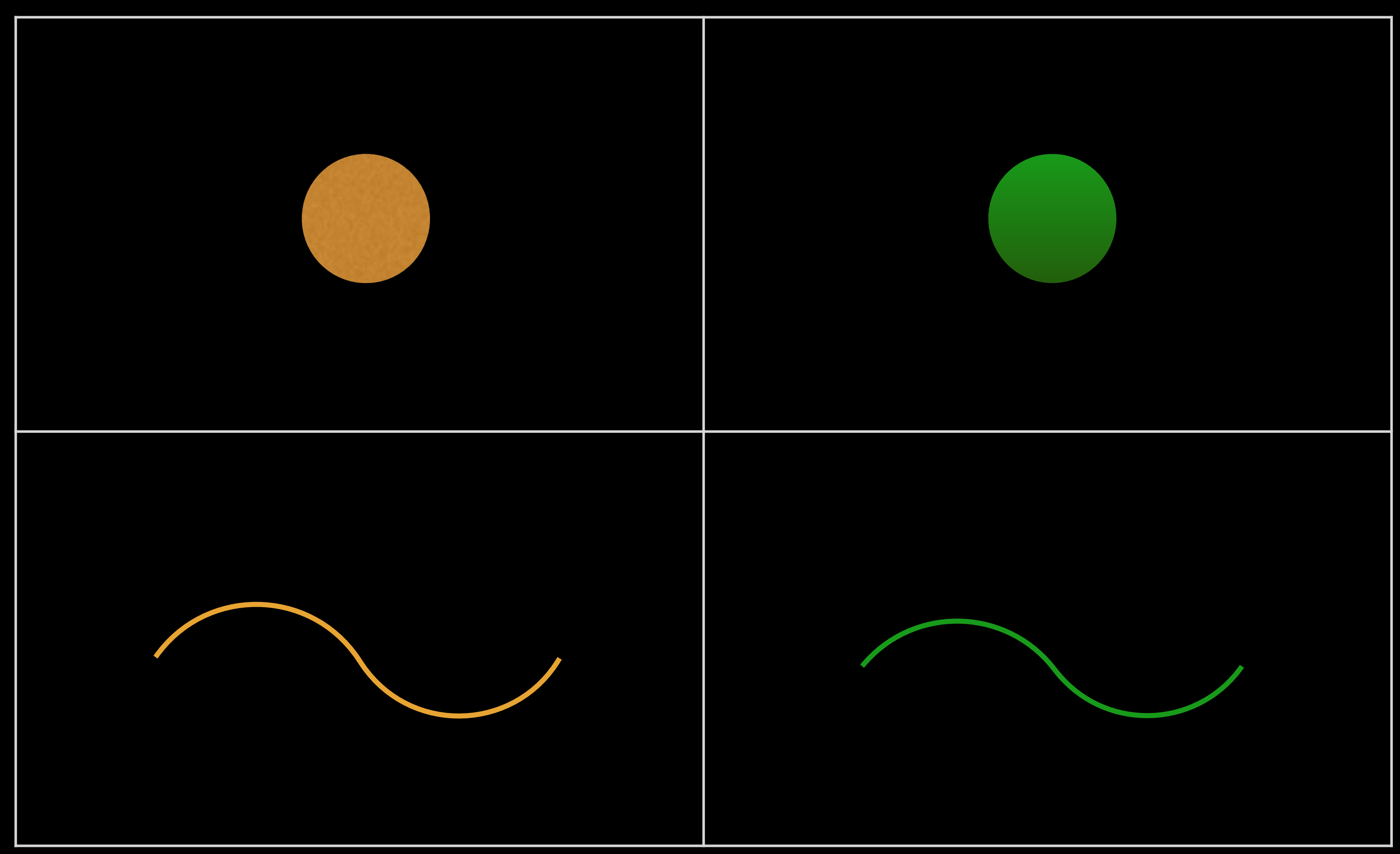
de Broglie's big idea

Light Matter



de Broglie’s big idea

Light Matter



Calculating the wavelength of a free particle

- The plan is to associate a wave with a particle of a given mass, and then to find (if it exists) a mathematical connection between the movement of the particle and the propagation of its associated wave. We will see that the particle's wavelength - the de Broglie wavelength - falls out of the math.

- The simplest case is that of a free particle - i.e. no forces or potentials acting on the particle.

Calculating the wavelength of a free particle: Step 1

In the rest frame of the particle, let the phase of the particle's hypothetical wave be the same at every point and be given by

$$\sin\left(2\pi f_0(t_0 - \tau)\right)$$

So, in its rest frame, the wave we're associating with the particle has infinite wavelength. f_0 is the frequency and t_0 is the intrinsic time of the frame.

Calculating the wavelength of a free particle: Step 2

Now suppose we are in an inertial frame such that the particle (and its rest frame) are moving along our x-axis with velocity $v = \beta c$. The Lorentz transformation gives

$$t_0 = \frac{t - \frac{\beta x}{c}}{\sqrt{1-\beta^2}}$$

Hence, to us the wave looks like,

$\sin\left(2\pi f(t_0 - \frac{\beta x}{c} - \tau)\right)$, where $f = \frac{f_0}{\sqrt{1-\beta^2}}$ and the phase velocity is $V = \lambda f = \frac{c^2}{v}$

Calculating the wavelength of a free particle: Step 3

"To establish the parallelism of which we have spoken, we must seek to link these parameters to the mechanical parameters, energy and quantity of motion."

This is where Planck's idea - as supported by the ideas of Einstein and Bohr - comes in.

$$E = hf$$

Calculating the wavelength of a free particle: Step 4

Since, according to Einstein, the intrinsic energy of the particle (i.e. in its rest frame) is m_0c^2, we have

$$hf_0 = m_0c^2 \implies f_0 = \frac{m_0c^2}{h}$$

In our frame, the momentum along the x-axis is given by

$$p = \frac{m_0 v}{\sqrt{1-\beta^2}} = \frac{\frac{h}{c^2}\frac{m_0c^2}{h}v}{\sqrt{1-\beta^2}} = \frac{h\frac{f_0}{\sqrt{1-\beta^2}}v}{c^2} = \frac{hfv}{c^2} = \frac{Ev}{c^2}$$

Calculating the wavelength of a free particle: Step 5

Recalling now that the phase velocity is

$$V = \frac{c^2}{v}$$

we have

$$p = \frac{Ev}{c^2} = \frac{hv}{V} = \frac{h}{\lambda} \qquad \Longrightarrow \qquad \lambda = \frac{h}{p}$$

The Davisson-Germer experiment

- It was known that X-rays have a short enough wavelength to be diffracted by crystals: The lattice of atoms in e.g. a salt acts as a diffraction grating.

- An electron accelerated through ~tens of volts will have a momentum which - according to the relation we just derived - gives rise to a wavelength similar to that of an X-ray.

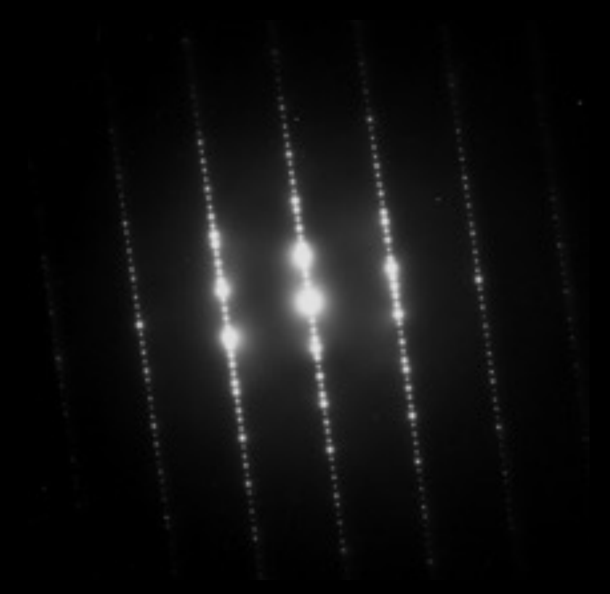
The Davisson-Germer experiment

- Thus, if electrons do indeed have a wave-like nature, it should be possible to obtain a diffraction pattern from a beam of electrons directed through some crystalline material.

The Davisson-Germer experiment

- Thus, if electrons do indeed have a wave-like nature, it should be possible to obtain a diffraction pattern from a beam of electrons directed through some crystalline material.

- This is exactly what Davisson and Germer did in 1927.



In summary…

de Broglie, motivated by aesthetic reasons, successfully united the basic properties of a wave and the dynamics of SR with the fundamental relation $E = hf$ to arrive at his own fundamental relationship

$$\lambda = \frac{h}{p}$$

Sources

- “The wave nature of the electron” : Nobel lecture by Louis de Broglie, given on December 12, 1929.
- Wikipedia

Questions?

Thank you