UNIVERSITY OF SOUTH CAROLINA

DEPARTMENT OF PHYSICS AND ASTRONOMY

COLLOQUIUM

Speaker:

Joseph E. Johnson, PhD
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Title:

A New Kind of Number
Encompassing Logical And Numerical Uncertainty With Applications to Computer Systems and Quantum Theory

Abstract:

In every domain of science and engineering (and certainly in astronomy, and financial investments!), we must deal with limited accuracy for numerical values. Even in the case of quantum mechanical measurement of position, we have an absolute limit on accuracy of position as the Compton wave length of the particle. Numerical uncertainties can be treated (a) via the usual cursory rules for significant digits, (b) computationally exhausting Monte Carlo programs, or (c) by advanced but problematic methods using distributions. The basic problem is that numerical error is a probability distribution, and thus is a function and no longer a number. So one is left to deal with an 'arithmetic of these functions,' which neither closes mathematically, nor which has nice properties. Not only are these problems mathematically convoluted, but the representation of uncertainty is not well managed by our computer software. We would like to have algorithms that automatically can exercise the kind of judgment that is similar to human reasoning in the manipulation of sequential approximation, uncertainty, estimation, utility, and value.

I propose a new kind of number, which is a smooth generalization of the reals and complex numbers in the same way that they are generalizations of the integers and rationals. We achieve this by (a) first generalizing the underlying elements of logic (1 & 0) to continuous probabilities, then (b) generalizing Boolean logic (AND, OR, NOT..) in a smooth way. Then we are able to build a smooth generalization of traditional arithmetic upon this logic. A number of interesting results will be shown but the most important new result is that these new numbers (and they are of the same order of infinity as the reals and are isomorphic to them!) can represent any probability distribution with any degree of accuracy. Thus one interesting conjecture is that they could be used to represent the wave function in quantum theory by a 'number' rather than a vector in Hilbert space. A very real application is for a rebuild of computer logic that allows for the automatic creation and annihilation of processes based upon the IF { } THEN {} ELSE {} which can now spawn branches each with its own probability of truth.

Technically: It is suggested that we generalize the traditional information bits of 1 and 0 to a Markov Lie Group representation, (x_1, x_0), which objects are to represent the probability to be '1' or 'T' and '0' or 'F' respectively and thus $x_1 + x_0 = 1$. We call these objects 'bit vectors' or 'bittors' in analogy with the spinor representations of SL(2,C). We then define a type of product of these objects, $z_i = ca_i^{jk} x_j y_k$ (summed over repeated indices i,j,k = 1,0) and where a represents the 16 different logical operations AND, OR, XOR, etc. This system is a smooth generalization of Boolean logic. We also define a sum operation, $z_i = a_m xm_i$ where a_m is any representation of the Markov Group. This operation can be interpreted as a weighted average of probabilities. Numbers (generalizing the binary numbers 110.10..) are defined as outer products of these Markov representations and can represent numerical uncertainty and probability distributions.

Jones Physical Science Center
Rogers Room
PSC 409

Thursday, March 20, 2008
3:30 pm

Refreshments at 3:15 pm **Everyone Invited**

Hosted by:
Sanjib Mishra