BARYON FORM FACTORS AT LARGE MOMENTUM TRANSFERS

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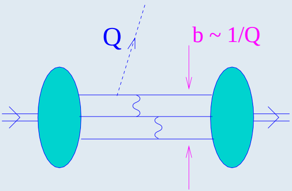
How to transfer a large momentum to a weekly bound system?

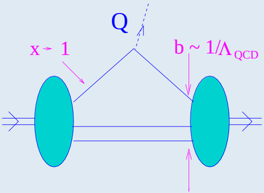
Heuristic picture:

- quarks can acquire large transverse momenta when they exchange gluons
- "hard" gluon exchanges can be separated from "soft" nonperturbative wave functions
- hard gluons can only be exchanged at small transverse separations



Hard rescattering:

Small b
Average $0 < x < 1$



Soft (Feynman):

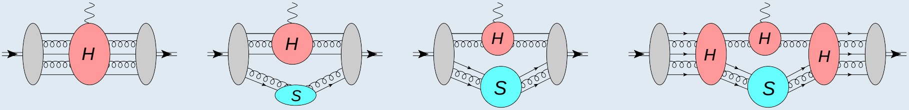
Average b
Large $x \to 1$

In practice three-quark states indeed seem to dominate, however

- "Squeesing" to small transverse separations occurs very slowly
- Helicity selections rules do not work. Orbital angular momentum?
- $\Rightarrow$ More complicated nonperturbative input needed

How to transfer a large momentum to a weekly bound system?

Can be formalized separating hard (H) and soft (S) momentum flow regions



A) B) C) DM)

Expect at $Q^2 \to \infty$:

$$F_1(Q^2) \sim \frac{\alpha_s^2(Q^2)}{\pi^2} \frac{1}{Q^4}\,, \qquad F_2(Q^2) \sim \frac{1}{Q^6}$$

- **A)**: $1/Q^4$, factorizable in terms of distribution amplitudes, only $F_1(Q^2)$
- **B)**: $1/Q^6(?)$, nonfactorizable, involves large transverse distances
- **C)**: $1/Q^8(?)$, nonfactorizable, involves large transverse distances
- **DM)** Duncan–Mueller: $1/Q^4$, nonfactorizable, contributes at NNLO

Main problem: leading term suppressed by $(\alpha_s/\pi)^2 \sim 0.01$

Light-cone wave functions vs. Distribution Amplitudes

- **Nucleon light-cone wave function** — Brodsky, Lepage

$$|P\uparrow\rangle^{\ell_z=0} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}}\,\psi^{L=0}(x_i,\vec{k}_i)\times$$
$$\times\Big\{\Big|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\uparrow(x_3,\vec{k}_3)\Big\rangle - \Big|u^\uparrow(x_1,\vec{k}_1)d^\downarrow(x_2,\vec{k}_2)u^\uparrow(x_3,\vec{k}_3)\Big\rangle\Big\}$$

- **Leading-twist-three distribution amplitude**

Brodsky, Lepage, Peskin, Chernyak, Zhitnitsky

$$\Phi_3(x_1,x_2,x_3;\mu) = 2\int^{\mu}[d^2\vec{k}]\,\psi^{L=0}(x_1,x_2,x_3;\vec{k}_1,\vec{k}_2,\vec{k}_3)$$

can be studied using the OPE

$$\Phi_3(x_i;\mu) = 120 f_N x_1x_2x_3\Big\{1 + c_{10}(x_1-2x_2+x_3)L^{\frac{8}{3\beta_0}}$$
$$+ c_{11}(x_1-x_3)L^{\frac{20}{9\beta_0}} + c_{20}\left[1+7(x_2-2x_1x_3-2x_2^2)\right]L^{\frac{14}{3\beta_0}}$$
$$+ c_{21}(1-4x_2)(x_1-x_3)L^{\frac{40}{9\beta_0}} + c_{22}\left[3-9x_2+8x_2^2-12x_1x_3\right]L^{\frac{32}{9\beta_0}} + \ldots\Big\}$$

- $f_N(\mu_0)$: **wave function at the origin**
- $c_{nk}(\mu_0)$: **shape parameters**

$L \equiv \alpha_s(\mu)/\alpha_s(\mu_0)$

Braun, Manashov, Rohwild

Wave functions vs. Distribution amplitudes (II)

- **Contributions of orbital angular momentum** — Ji, Ma, Yuan, '03

$$|P\uparrow\rangle^{\ell_z=1} = \int \frac{[dx][d^2\vec{k}]}{12\sqrt{x_1x_2x_3}} \left[k_1^+\psi_1^{L=1}(x_i,\vec{k}_i) + k_2^+\psi_2^{L=1}(x_i,\vec{k}_i)\right] \times$$
$$\times\left\{\left|u^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)d^\downarrow(x_3,\vec{k}_3)\right\rangle - \left|d^\uparrow(x_1,\vec{k}_1)u^\downarrow(x_2,\vec{k}_2)u^\downarrow(x_3,\vec{k}_3)\right\rangle\right\}$$

are related to higher-twist-four distribution amplitudes — Belitsky, Ji, Yuan, '03

$$\Phi_4(x_2,x_1,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_3}\, k_3^-\left[k_1^+\psi_1^{L=1} + k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$

$$\Psi_4(x_1,x_2,x_3;\mu) = 2\int^\mu \frac{[d^2\vec{k}]}{m_N x_2}\, k_2^-\left[k_1^+\psi_1^{L=1} + k_2^+\psi_2^{L=1}\right](x_i,\vec{k}_i)$$

$$k^\pm = k_x \pm i k_y$$

and, again, can be studied using OPE — Braun, Fries, Mahnke, Stein '00

$$\Phi_4(x_i;\mu) = 12\lambda_1 x_1x_2 + 12 f_N x_1x_2\left[1+\frac{2}{3}(1-5x_3)\right] + \ldots$$

$$\Psi_4(x_i;\mu) = 12\lambda_1 x_1x_3 + 12 f_N x_1x_3\left[1+\frac{2}{3}(1-5x_2)\right] + \ldots$$

- **to this accuracy only one new nonperturbative constant $\lambda_1(\mu)$**

Wave functions vs. Distribution amplitudes (III)

- **New:**
  Wandzura-Wilczek–type relations for spin-1/2 baryons

Braun, Manashov, Rohrwild, '09

Let

$$\Phi_3(x_i,\mu) = 120 x_1 x_2 x_3 \sum_{n=0}^{\infty}\sum_{k=0}^{N} c_{nk}^{N}(\mu)\mathcal{P}_{nk}(x_i)$$

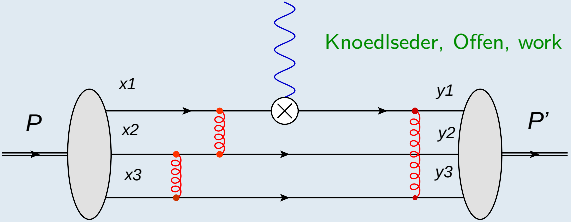
$$\int [dx]\, x_1 x_2 x_3 \mathcal{P}_{nk}(x_i)\mathcal{P}_{n'k'} = \mathcal{N}_{nk}\delta_{nn'}\delta_{kk'}\,, \qquad c_{nk}(\mu) = c_{nk}(\mu_0)\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\gamma_{nk}/\beta_0}$$

Then

$$\Phi_4^{WW}(x_i) = -\sum_{n,k}\frac{240\, c_{nk}}{(n+2)(n+3)}\left(n+2-\frac{\partial}{\partial x_3}\right) x_1 x_2 x_3 \mathcal{P}_{nk}(x_i)$$

$$\Psi_4^{WW}(x_i) = -\sum_{n,k}\frac{240\, c_{nk}}{(n+2)(n+3)}\left(n+2-\frac{\partial}{\partial x_2}\right) x_1 x_2 x_3 \mathcal{P}_{nk}(x_i)$$

Region A): towards NLO accuracy



Knoedlseder, Offen, work in progress

Challenging calculation:

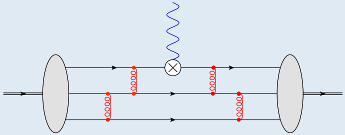
- up to 4000 Feynman diagrams (though most of them redundant)
- up to seven-point integrals (though planar kinematics)
- up to tensor rank four

Interesting because of

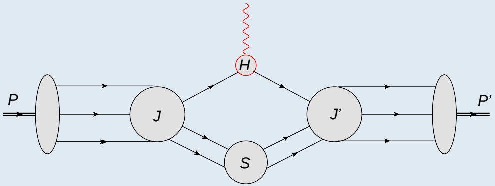
♡ a new color structure compared to LO

♡ imaginary part (timelike form factors)

Region DM): 30 years later...



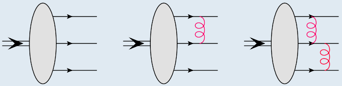
Kivel, Vanderhaeghen, '11-'12



lowest order $\sim \alpha_s^4$

All-order structure

Kivel 1202.4944:



$$\Phi(x_i,\mu) = \Phi_0(x_i) + \alpha_s \Phi_1(x_i) + \alpha_s^2 \Phi_2(x_i)$$

$$\begin{aligned}
\Phi_1(x_i) &\sim \ln\mu \cdot \mathbb{V}_1 \otimes \Phi_0(x_i) + \Phi_{10}(x_i) \\
\Phi_2(x_i) &\sim \ln^2\mu \cdot \mathbb{V}_1 \otimes \mathbb{V}_1 \otimes \Phi_0(x_i) \\
&\quad + \ln\mu \cdot \mathbb{V}_2 \otimes \Phi_0(x_i) + \Phi_{20}(x_i)
\end{aligned}$$

$\Phi_{20}(x_i)\Big|_{x_3\to 1} \sim (1-x_3)^1$ vs. $\sim (1-x_3)^2$ from evolution

Mixing of hard and soft spectator scattering starting NNLO

B,C): Light-Cone Sum Rules

complicated because involves large transverse distances (all twists)

- **Full transverse momentum dependence in the wave functions**
- **All orbital angular momenta**

What can be done:

- **Hope in Sudakov suppression of large transverse distances, k_T factorization**

  unfortunately seems to be too weak

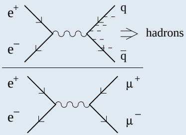
- **Models for complete baryon wave functions quark models, AdS/QCD, . . .**

  strong model dependence

- **Calculate contributions of large transverse distances in terms of DA using dispersion relations and duality LCSR**

  work in progress

From distribution amplitides to form factors: Duality

$R(s = E_{\rm cm}^2) =$ 

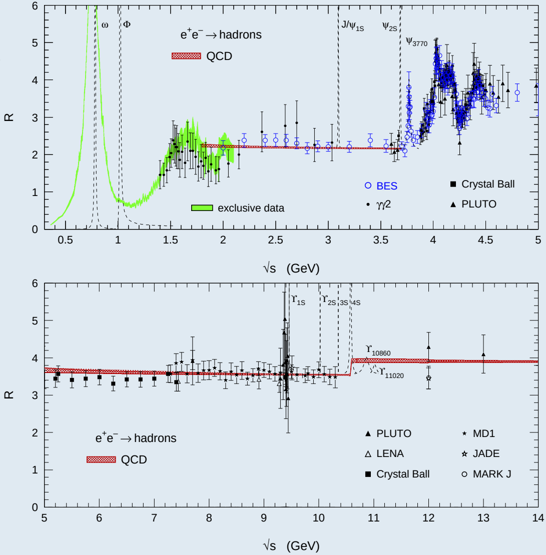
observe

$$R^{\rm QCD}(s) \neq R^{\rm exp}(s)$$

in the resonance region $\sqrt{s} < 1.5 GeV$, but

$$\int_0^{s_0} ds\, R^{\rm QCD}(s) = \int_0^{s_0} ds\, R^{\rm exp}(s)$$

s_0 is called interval of duality



Davier et al., Eur.Phys.J.C27:497-521,2003

a consequence of two major principles:

- unitarity $\leftarrow$ probability interpretation of wave functions

$$R(s) \quad = \quad \frac{1}{\pi}\mathrm{Im}\,\Pi(s=q^2)$$

where

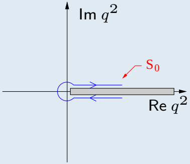
$$i\int d^4x\, e^{iqx}\langle 0|\,T\{j_\mu^{\rm em}(x)j_\nu^{\rm em}(0)\}|0\rangle \;=\; (q_\mu q_\nu - g_{\mu\nu}q^2)\Pi(q^2)$$

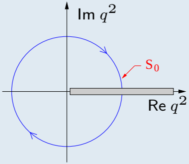
- analyticity $\leftarrow$ causality

$$R(s) \;=\; \frac{1}{2\pi i}[\Pi(q^2+i\epsilon)-\Pi(q^2-i\epsilon)]$$

$$\int_0^{s_0} ds\, R(s) \;=\; \frac{1}{2\pi i}\oint dq^2\; R(q^2)$$

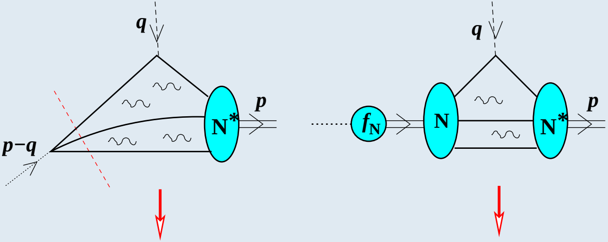
$$\simeq\; \frac{1}{2i}\oint dq^2\; R^{\rm pQCD}(q^2)$$





because the region of $q^2 \sim \Lambda^2_{\rm QCD}$ is avoided

- a development of this idea ⇒ Light-Cone Sum Rules:



$$\frac{1}{\pi}\int_0^{s_0} ds\ \mathrm{Im}\ T(p,q) \quad \overset{duality}{=} \quad f_N\, F_{N\to N^*}(Q^2)$$

- $T(p,q)$ is calculated in terms of N^* distribution amplitudes
  Balitsky, Braun, Kolesnichenko, Nucl.Phys.B312:509-550,1989
  Braun, Halperin, Phys.Lett.B328:457-465,1994

- Leading term is a Feynman (soft) contribution; hard terms can be added systematically and without double counting

Light-Cone Sum Rules vs. QCD Sum Rules (SVZ)

- Different expansion parameter: twist (LCSR) vs. dimension (QCDSR)
- Different nonperturbative input: hadron DAs (LCSR) vs. condensates (QCDSR
- Different goals: calculation of form factors in terms of WFs at small separations (LCSR)
  calculation of form factors "from first principles" (QCDSR)
- Crucial advantage: LCSR are consistent with power counting at large Q^2

$$F_{QCDSR}(Q^2) \simeq \left\{\frac{1}{Q^8}+\frac{\alpha_s}{Q^6}+\frac{\alpha_s^2}{Q^4}\right\} + \langle\alpha_s G^2\rangle\left\{\frac{1}{Q^4}+\frac{\alpha_s}{Q^2}\right\} + \alpha_s\langle\bar q q\rangle^2\left\{\frac{1}{Q^2}+\frac{\alpha_s}{Q^0}\right\} + \ldots$$

$$F_{LCSR}(Q^2) \simeq \left\{\frac{1}{Q^8}+\frac{\alpha_s}{Q^6}+\frac{\alpha_s^2}{Q^4}\right\}\otimes[\text{twist-3}] + \frac{1}{Q^6}\otimes\left\{\text{twist-4} + \text{twist-5} + \ldots\right\}$$

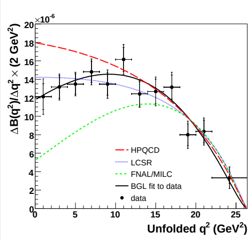
- Higher accuracy: Dispersion relation in one (LCSR) vs. two (QCDSR) variables

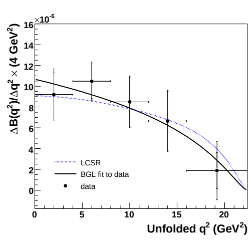
- State of the art: Weak B–decays, $B \to (\pi, \rho, K^*)\ell\bar{\nu}_\ell$ etc.

  - ...
  - Ball, Braun; Phys. Rev. D **58**, 094016 (1998) [392 citations]
  - Khodjamirian *et al.*; Phys. Rev. D **62** (2000) 114002 [174 citations]
  - Ball, Zwicky; Phys. Rev. D **71**, 014015 (2005) [391 citations]
  - Ball, Zwicky; Phys. Rev. D **71**, 014029 (2005) [297 citations]
  - Khodjamirian, Mannel, Offen, Wang; Phys. Rev. D **83**, 094031 (2011)

Recent Highlights: $B \to \pi \ell \nu$, $B \to \eta \ell \nu$, $B \to \omega \ell \nu$

J. P. Lees *et.al* [The BABAR Collaboration], arXiv:1208.1252





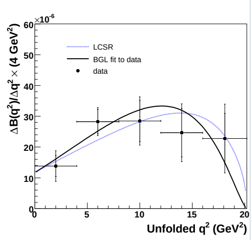


Table: Values of $|V_{ub}|$ derived from the combined $B \to \pi \ell^+ \nu$ and $B^+ \to \omega \ell^+ \nu$ decays. The three uncertainties on $|V_{ub}|$ are statistical, systematic and theoretical, respectively.

	q^2 (GeV^2)	$\Delta\mathcal{B}$ (10^{-4})	$\Delta\zeta$ (ps^{-1})	$\lvert V_{ub}\rvert$ (10^{-3})	χ^2/ndf
$B \to \pi \ell^+ \nu$					
HPQCD	$16 - 26.4$	$0.37 \pm 0.02 \pm 0.02$	2.02 ± 0.55	$3.47 \pm 0.10 \pm 0.08^{+0.60}_{-0.39}$	2.7/4
FNAL	$16 - 26.4$	$0.37 \pm 0.02 \pm 0.02$	$2.21^{+0.47}_{-0.42}$	$3.31 \pm 0.09 \pm 0.07^{+0.37}_{-0.30}$	3.9/4
LCSR	$0 - 12$	$0.83 \pm 0.03 \pm 0.04$	$4.59^{+1.00}_{-0.85}$	$3.46 \pm 0.06 \pm 0.08^{+0.37}_{-0.32}$	8.0/6
LCSR2	0			$3.34 \pm 0.10 \pm 0.05^{+0.29}_{-0.26}$	
$B^+ \to \omega \ell^+ \nu$					
LCSR3	$0 - 20.2$	$1.19 \pm 0.16 \pm 0.09$	14.2 ± 3.3	$3.20 \pm 0.21 \pm 0.12^{+0.45}_{-0.32}$	2.24/5

- Schematic structure of a LCSR for baryon form factors

Braun, Lenz, Mahnke, Stein, Phys. Rev. D **65**, *074011 (2002)*
Braun, Lenz, Wittmann, Phys. Rev. D **73**, *094019 (2006)*

$$F(Q^2) \simeq \underbrace{\frac{1}{Q^6}\left\{F_{tw=3} + F_{tw=4} + \frac{\Lambda^2}{s_0}F_{tw=5} + \ldots\right\}}_{\text{soft+hard}} + \underbrace{\frac{1}{Q^4}\left(\frac{\alpha_s(Q)}{\pi}\right)^2 F^{tw=3}_{pQCD}}_{\text{hard (pQCD)}}$$

where

$$F_{tw=3} = f_{tw=3}(Q^2, s_0) \otimes \Phi_3(\mu_F = s_0)\,, \qquad F_{tw=4} = f_{tw=4}(Q^2, s_0) \otimes \Phi_4(\mu_F = s_0)\,,$$

- for each twist obtain an expansion

$$f_{tw=n} = f^{(0)}_{tw=n} + \left(\frac{\alpha_s(s_0)}{\pi}\right) f^{(1)}_{tw=n} + \left(\frac{\alpha_s(s_0)}{\pi}\right)^2 f^{(2)}_{tw=n} + \ldots$$

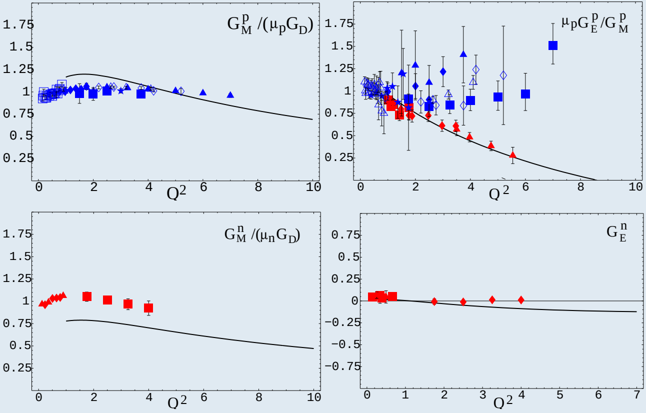
$$f^{(1)}_{tw=n} = c_2 \ln^2(Q^2/s_0) + c_1 \ln(Q^2/s_0) + c_0\,, \qquad \text{etc.}$$

- hierarchy of twists based on $s_0 \gg \Lambda_{QCD}$, hence $\alpha_s(s_0) \ll 1$

Light-Cone Sum Rules (LO): Nucleon Electromagnetic Formfactors

proton

neutron



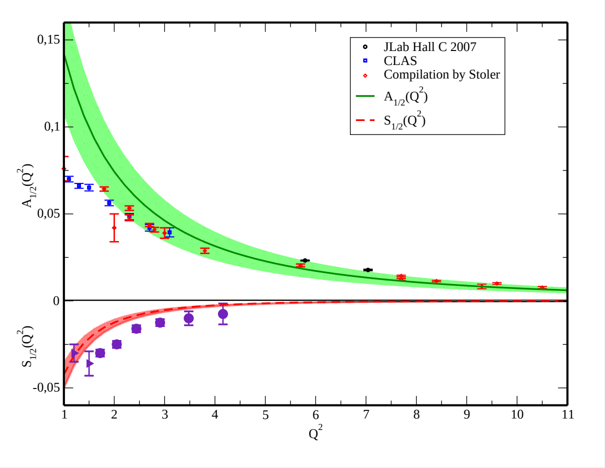
Braun, Lenz, Wittmann; PRD73:094019,2006

- **Nucleon DAs fitted to the** G_E^p/G_M^p **ratio**

$\gamma^* N \to N^*(1535)$: helicity amplitudes

- **A pilot project:** Braun *et al.* Phys.Rev.Lett.103:072001,2009

Electroproduction of $N^*(1535)$ with lattice-constrained N^* distribution amplitides



CLAS data: *I.G. Aznauryan et al., Phys.Rev.C80:055203,2009*

Other applications:

- nucleon axial form factor
  Braun, Lenz, Wittmann; PRD73:094019 (2006)
- nucleon tensor form factor
  Aliev, Azizi, Savci; PRD84, 076005 (2011)
- $\gamma^* N \to \Delta$
  Braun, Lenz, Peters, Radyushkin; PRD73, 034020 (2006)
- Threshold pion production $\gamma^* N \to \pi N$
  Braun, Ivanov, Peters; PRD77, 034016 (2008)
- $\Lambda_b \to p\ell\bar{\nu}_\ell$, $\Lambda_b \to \Lambda\gamma$ ...
  Huang, Wang; PRD69, 094003 (2004)
  Wang, Shen, Lu; PRD80, 074012 (2009)
  Khodjamirian, Klein, Mannel, Wang; JHEP1109, 106 (2011)
- $\Xi_{b,c} \to \Xi(\Sigma)\ell^+\ell^-$, $\Xi'_{b,c} \to \Xi(\Sigma)\ell^+\ell^-$,
  Azizi, Sarac, Sundu; EPJA48, 2 (2012)

Towards baryon LCSRs with NLO corrections

Passek-Kumericki, Peters, Phys.Rev.D78:033009,2008

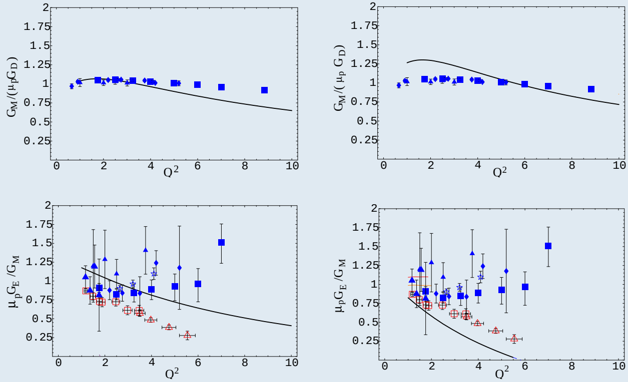
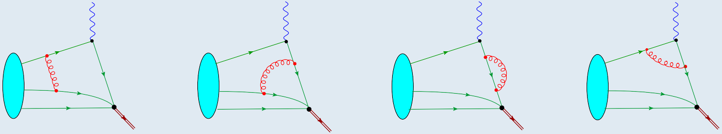


Figure: LCSR results for the electromagnetic proton form factors for a realistic model of nucleon distribution amplitudes. Left panel: Leading order (LO); right panel: next-to-leading order (NLO) for twist-three contributions. Figure adapted from [PassekKumericki:2008sj].

NLO LCSRs for baryon form factors

- **A large project:**
  - A consistent renormalization scheme for three-quark operators — completed
    Krankl, Manashov; PLB703, 519 (2011)
  - Light-cone expansion for three-quark operators for generic coordinates — completed
  - NLO coefficient functions including twist-three and twist-four DAs — 70% completed



  - nucleon mass corrections for twist-four — completed
  - nucleon mass corrections for twist-five (partially known)
- **publication on electromagnetic form factors planned to the end of 2012**
- **a Mathematica code for NLO corrections will be made available at a later time**

Summary

- QCD treatment of form factors is based on the factorization of regions with large momentum flow
- Baryon elastic (and transition) form factors are complicated because several regions contribute significantly
- NLO pQCD calculations on the way (hard scattering)
- Much better understanding of Duncan-Mueller contributions
- NLO LCSR calculations close to completion (soft contributions)
- A large-scale lattice calculation of baryon DAs is on the way