Coupled channel dynamics in Λ and Σ production

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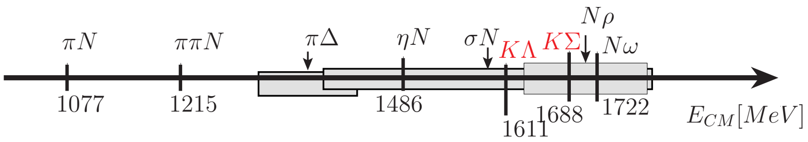
The model

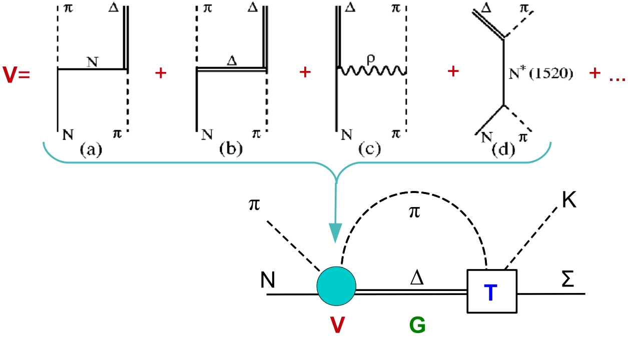


Athens-Bonn-Jülich-Washington collaboration

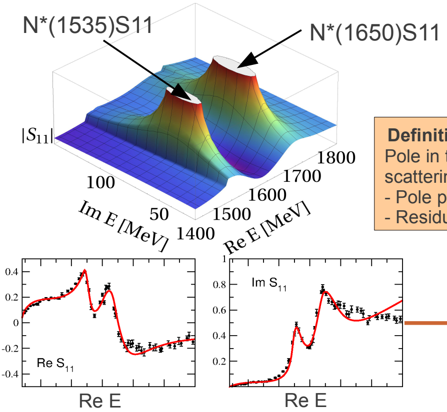
- Dynamical coupled-channels model for elastic & inelastic meson-baryon scattering up to and beyond 2 GeV
- Lagrangian of Wess & Zumino + additional channels(SU3)
- 12 Nucleon and 10 Delta Resonances with total spin up to $J = 9/2$
- Simultaneous fit to πN, ηN, $K\Sigma$, and $K\Lambda$
- Used as input for a gauge invariant analysis of pion photoproduction
- respects unitarity(2-body) and analyticity; Resonance poles and residues

The scattering equation

$$T^I_{\mu\nu}(\vec{k}',\lambda',\vec{k},\lambda) = V^I_{\mu\nu}(\vec{k}',\lambda',\vec{k},\lambda) + \sum_{\gamma,\lambda''}\int d^3q\, V^I_{\mu\gamma}(\vec{k}',\lambda',\vec{q},\lambda'')\frac{1}{Z-E_\gamma(q)+i\epsilon}T^I_{\gamma\nu}(\vec{q},\lambda'',\vec{k},\lambda)$$



Analyticity



Definition of a resonance:
Pole in the complex plane of the scattering energy E (≡ z ≡ W);
- Pole position (“mass & width”)
- Residues (“branching ratios”)

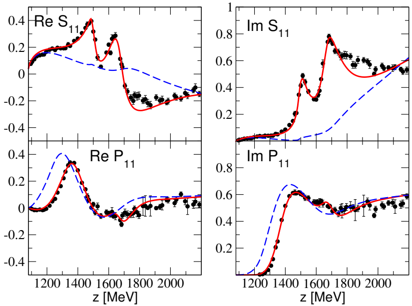
Extractions of resonance parameters in terms of poles and residues

Not every bump is a resonance and not every resonance is a bump.

Moorhouse 1960ties

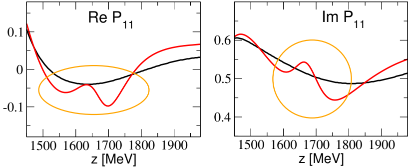
$\pi N \to \pi N$: Partial wave amplitudes I=1/2 (preliminary)

- $S_{11}(1535)$ and $S_{11}(1650)$



- dynamical Roper $P_{11}(1440)$
- genuine $P_{11}(1710)$

Detail P_{11}:



- Inclusion of $P_{11}(1710)$ necessary to improve $K\Lambda$
- Input in the fit: energy-dependent solution (black line)
- But: Our solution matches single-energy solution (data points)

- Coupled-channels essential
- Signal for a $N^*(17XX)P_{11}$
- Single-energy solutions are not data
  $\Rightarrow$ Fit to πN observables required

$\pi N \to \pi N$: Partial wave amplitudes I=1/2 (preliminary)

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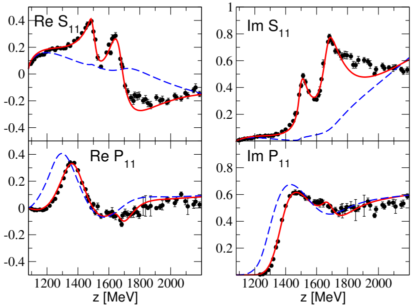

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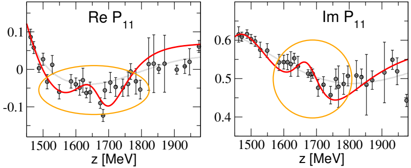

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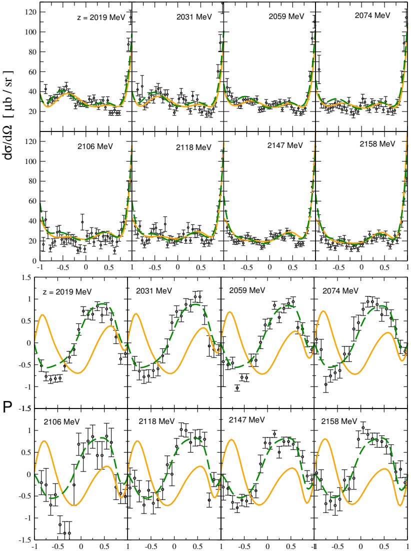
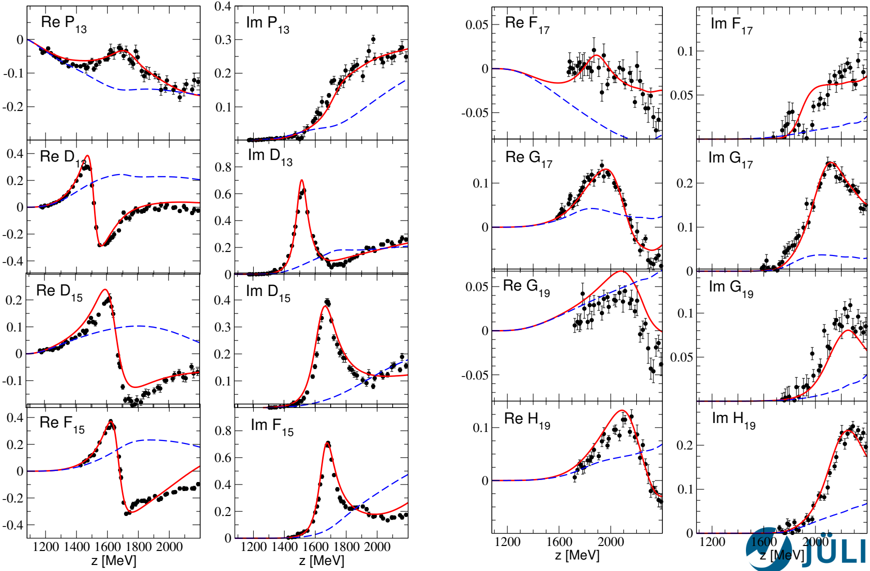
Importance of polarization measurements: The $P_{33}(1920)$

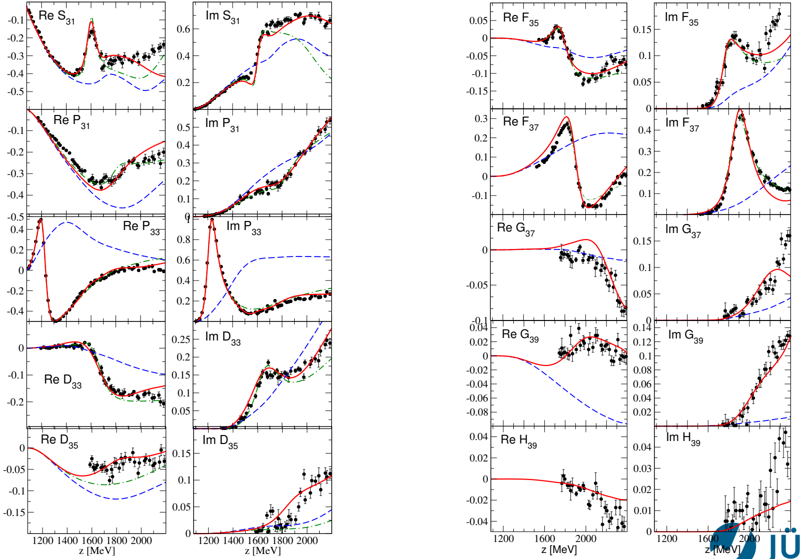


- Orange: Only $d\sigma/d\Omega$ in fit: no $P_{33}(1920)$ necessary
- Green: $d\sigma/d\Omega$ + P in fit: Need for $P_{33}(1920)$!

$\pi N \to \pi N$: Partial wave amplitudes I=1/2 (preliminary)

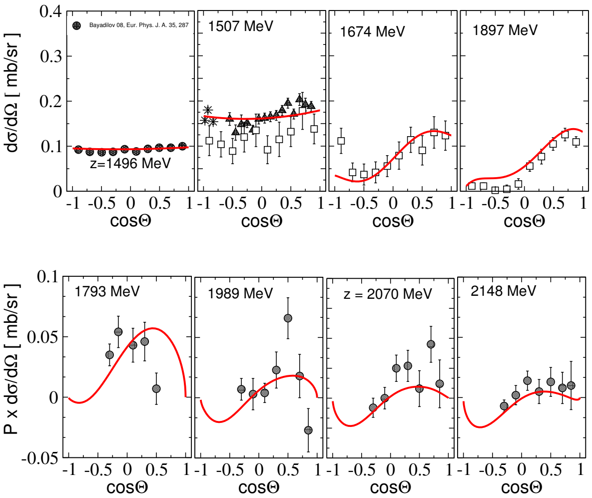


$\pi N \to \pi N$: Partial wave amplitudes I=3/2 (preliminary)



$\pi^- p \to \eta N$: Cross section and Polarization (preliminary)

Selected results



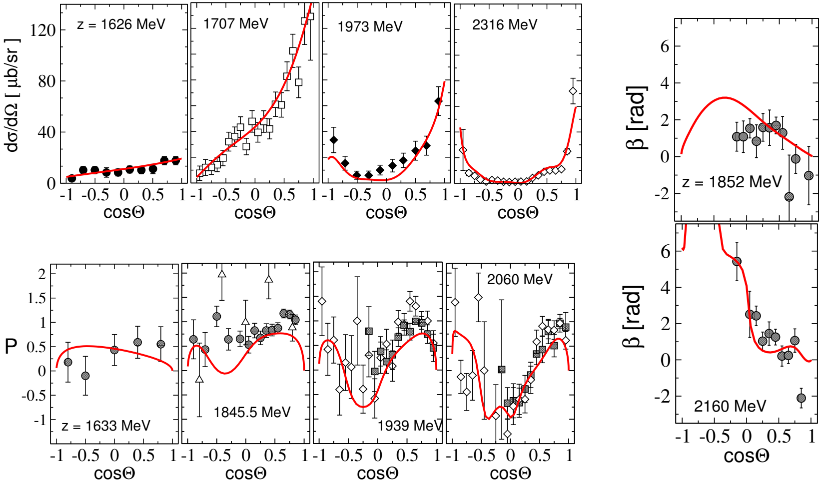
Difficult data situation

- Inconsistencies among different data sets
- Data at higher energies questionable
- Polarization data questionable

Full results ηN

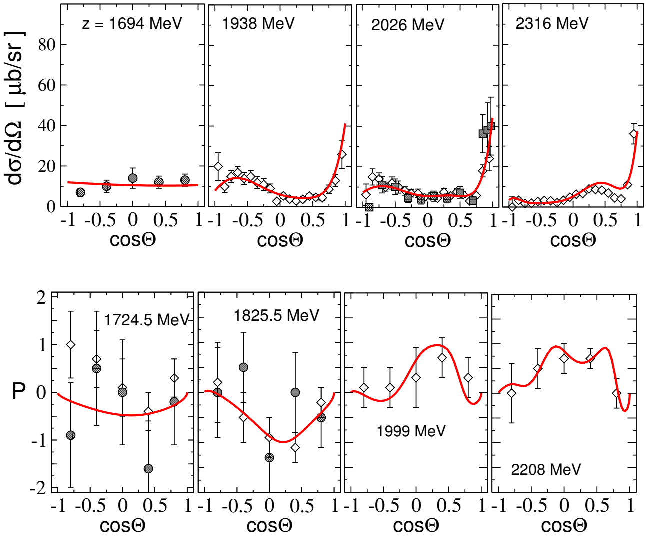
$\pi^- p \to K^0\Lambda$: $d\sigma/d\Omega$, Polarization & Spinrotation angle (preliminary)

Seclected results



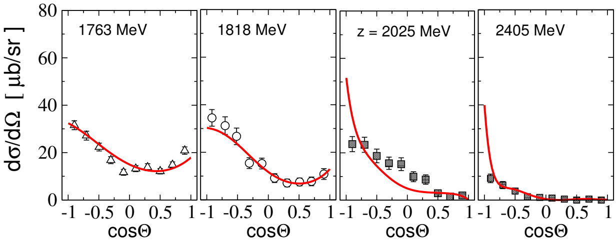
Full results $K\Lambda$

$\pi^- p \to K^0\Sigma^0$: $d\sigma/d\Omega$ & Polarization (preliminary)



Full results $K^0\Sigma^0$

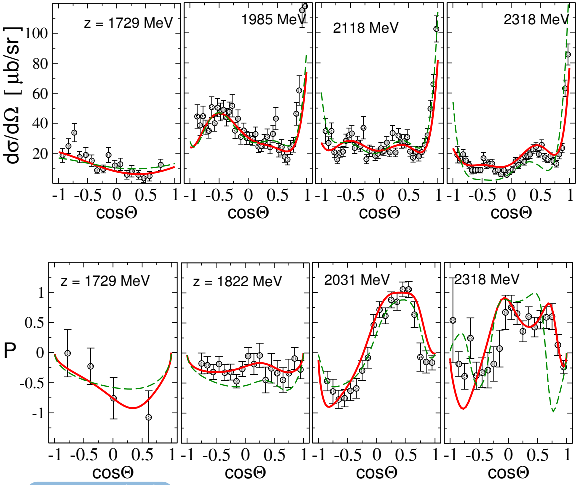
$\pi^- p \to K^+\Sigma^-$: $d\sigma/d\Omega$ (preliminary)

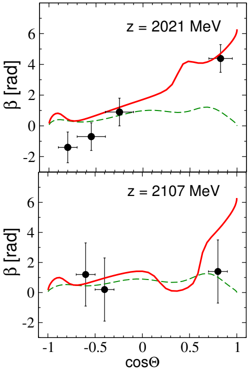


No polarization data

Full results $K^+\Sigma^-$

$\pi^+ p \rightarrow K^+\Sigma^+$ (preliminary)
Cross section, polarization and spinrotation parameter



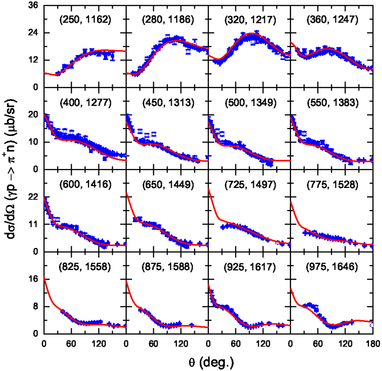


Green dashed line: Jülich model solution from NPA 851, 58 (2011)

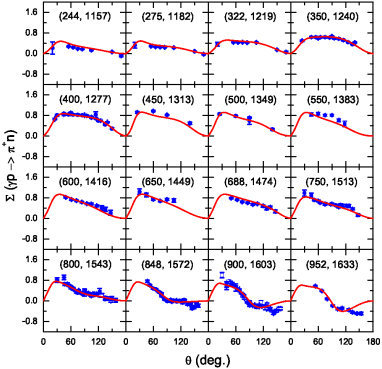
Full results $K^+\Sigma^+$

Photoproduction: $d\sigma/d\Omega$ and Σ_γ for $\gamma p \to \pi^+ n$

F. Huang, M. Döring, K. Nakayama *et al.*, Phys. Rev. C85 (2012) 054003



Differential cross section for $\gamma p \to \pi^+ n$



Photon spin asymmetry for $\gamma p \to \pi^+ n$

Data: CNS Data analysis center [CBELSA/TAPS, JLAB, MAMI,...]

Resonance content: I $= 1/2$ (preliminary)

	Re(z_0), -2Im(z_0)	$\lvert r \rvert, \theta$	$\Gamma_{\pi N}/\Gamma_{\text{tot}}$	$(\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{\eta N})/\Gamma_{\text{tot}}$	$(\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{K\Lambda})/\Gamma_{\text{tot}}$	$(\Gamma^{1/2}_{\pi N}\Gamma^{1}_{K}$
$N(1440)P_{11}$	1343	69.5	48.7			
$1/2^+$ **** [a]	266	-113				
$N(1520)D_{13}$	1523	37.8	75.78			
$3/2^-$ ****	100	-357.1				
$N(1535)S_{11}$	1496	13	38.13	43.9		
$1/2^-$ ****	66	-40				
$N(1650)S_{11}$	1677	30.2	44.95	15.69	18.82	
$1/2^-$ ****	134	36				
$N(1710)P_{11}$	1678	8.8	15.25	22.93	21.20	
$1/2^+$ ***	116	-321.2				
$N(1675)D_{15}$	1643	33.4	36.63	2.85	1.95	
$5/2^-$ ****	180	-32.7				
$N(1680)F_{15}$	1664	40.6	65.44	1.59	0.1	
$5/2^+$ ****	124	-20.7				
$N(1720)P_{13}$	1742	16.7	12.8	4.91	4.11	2.
$3/2^+$ ****	258	-32.6				
$N(1990)F_{17}$	1851	4	3.02	0.46	1.32	0.
$7/2^+$ **	260	-99.8				
$N(2190)G_{17}$	2049	31.8	17.65	0.32	1.27	0.
$7/2^-$ ****	356	-23.3				
$N(2220)H_{19}$	2146	38	15.82	0.06	1.79	0.
$9/2^+$ ****	470	-65.7				
$N(2250)G_{19}$	2117	15.6	6.27	0.13	1.36	0.
$9/2^-$ ****	488	-63.4				

Resonance content: I = 3/2 (preliminary)

	Re(z_0), -2Im(z_0)	$\lvert r\rvert, \theta$	$\Gamma_{\pi N}/\Gamma_{\text{tot}}$	$(\Gamma_{\pi N}^{1/2}\Gamma_{K\Sigma}^{1/2})/\Gamma_{\text{tot}}$
$\Delta(1232)P_{33}$	1216	51	100	
$3/2^+$ ****	96	-39.3		
$\Delta(1620)S_{31}$	1598	16	41.63	
$1/2^-$ ****	76	-106		
$\Delta(1700)D_{33}$	1636	32.1	16.72	
$3/2^-$ ****	370	-25.7		
$\Delta(1905)F_{35}$	1757	10.6	10.65	0.2
$5/2^+$ ****	198	-56.8		
$\Delta(1910)P_{31}$	1797	47.8	24.62	2.32
$1/2^+$ ****	378	-125.4		
$\Delta(1920)P_{33}$	1848	28	6.24	15.28
$3/2^+$ ***	818	-351.6		
$\Delta(1930)D_{35}$	1767	16.9	7.18	2.93
$5/2^-$ ***	452	-100.7		
$\Delta(1950)F_{37}$	1892	55.2	50.74	3.89
$7/2^+$ ****	216	-20.5		
$\Delta(2200X)G_{37}$	2133	16.8	7.56	0.56
$7/2^-$ *	438	-64		
$\Delta(2400)G_{39}$	1933	16.5	5.76	0.89
$9/2^-$ **	552	-116.7		

Summary and outlook

- Lagrangian based, field theoretical description of meson-baryon interaction
- Unitarity and analyticity are ensured; branch points in the complex plane included

  $\rightarrow$ precise, model independent determination of resonance parameters
- Coupled channel formalism links different reactions in one combined description:
  - $\pi N \rightarrow \pi N$, $\pi^+ p \rightarrow K^+\Sigma^+$,..., $\gamma N \rightarrow \pi N$,....
- Extension to kaon photoproduction

Matching with lattice

The lattice results depend on a finite volume V and a pion mass $M_{pion}^{lattice} >= M_{pion}^{exp}$.
The dynamical coupled channel approach is based on a Lagrangian and therefore can vary V and M_{pion}.

May we hope to interpolate between experiment and the state-of-the-art lattice?

Matching with quark dynamics

Question:

Do the towers of excited baryons predicted by quark models merge into a continuum after coupling to meson-baryon dynamics is considered?

Why not a pragmatic approach?
Quark model $\rightarrow M_{N^*}$
couplings $N^* \rightarrow N\pi, N\eta, \Lambda K, \Sigma K.$
Match:

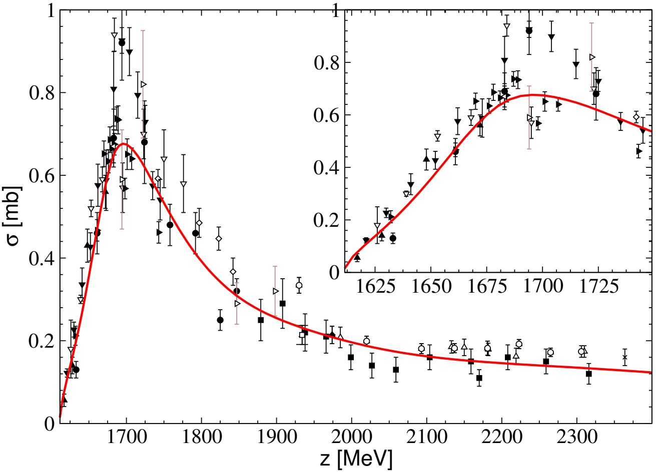
$$\langle N^* \mid H_{quark} \mid N\pi\rangle = \langle\psi_{N^*} \mid -\mathcal{L}_{int} \mid \psi_N \pi\rangle$$

for $q_{on-shell} = \frac{\sqrt{M_{N^*}}}{2}(1-(\frac{m_N+m_\pi}{M_{N^*}})^2)^{\frac{1}{2}}(1-(\frac{m_N-m_\pi}{M_{N^*}})^2)^{\frac{1}{2}}.$

Use as input in time-ordered perturbation theory!

This might be the first step for a generalization to electroproduction.
Analogy: quasielastic bump in nuclear physics
$=$ superposition of nuclear modes coupled to continuum.

$\pi^- p \to K^0 \Lambda$: Total cross section (preliminary)



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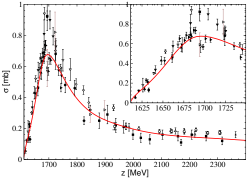
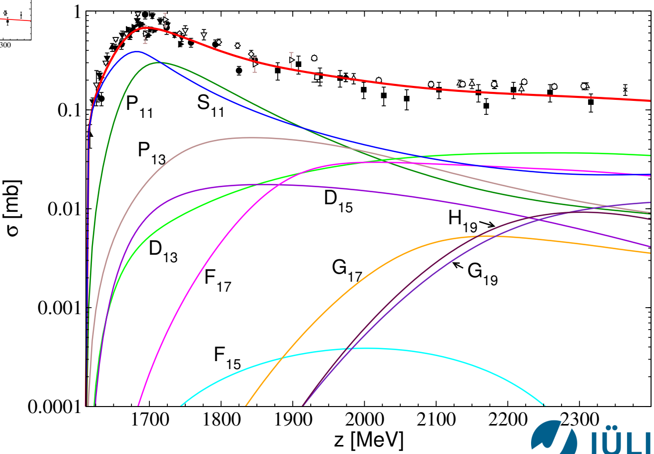
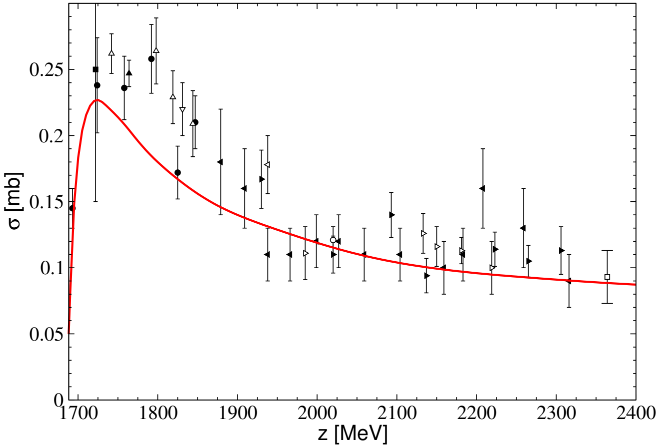


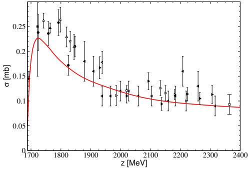
Partial wave content:



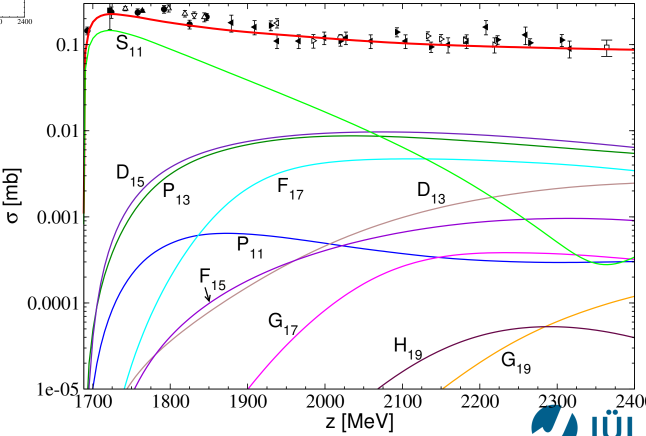
$\pi^- p \to K^0 \Sigma^0$: Total cross section (preliminary)



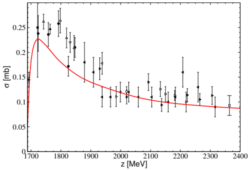
$\pi^- p \to K^0\Sigma^0$: Total cross section (preliminary)



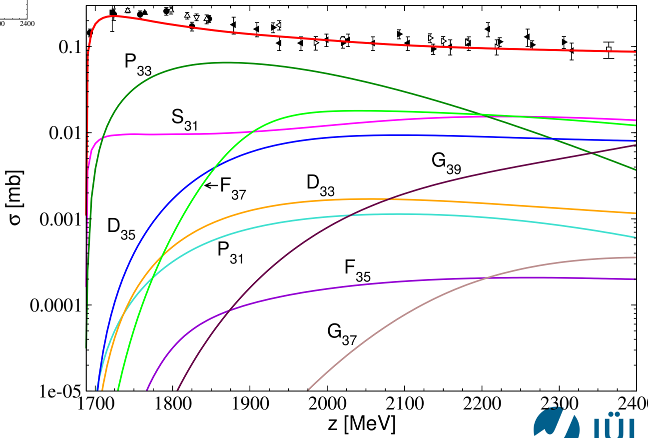
Partial wave content $I = 1/2$:



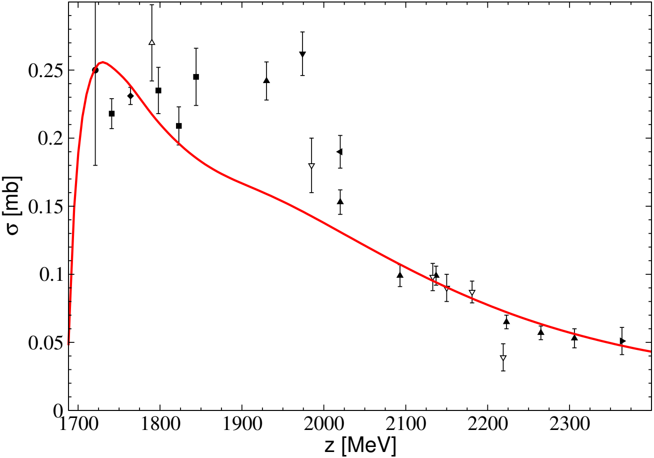
$\pi^- p \to K^0\Sigma^0$: Total cross section (preliminary)



Partial wave content $I = 3/2$:



$\pi^- p \to K^+\Sigma^-$: Total cross section (preliminary)



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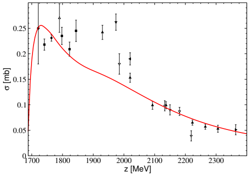

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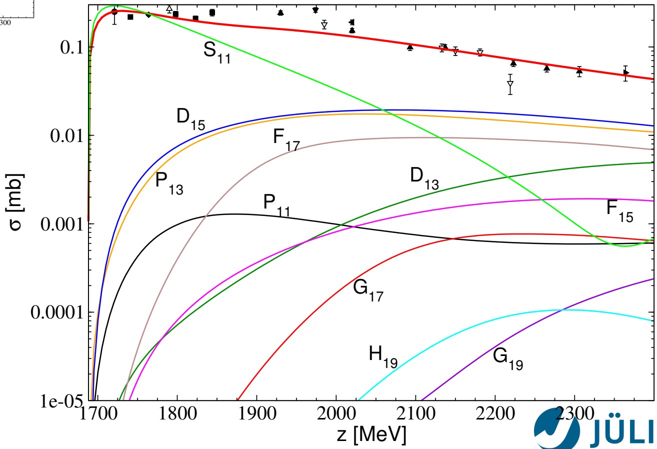

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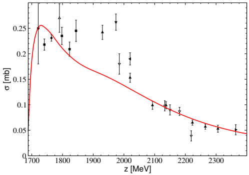
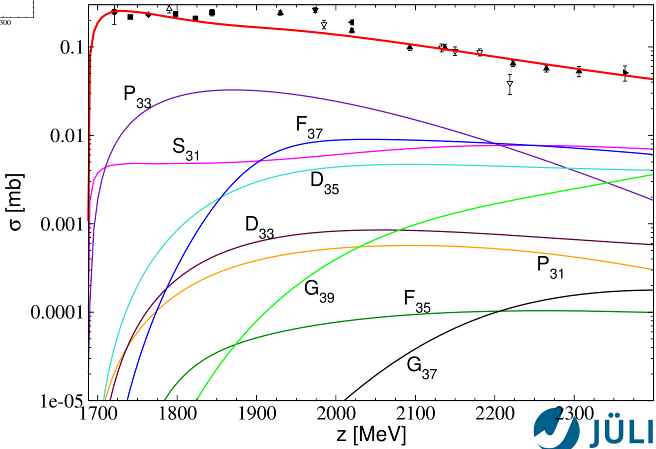
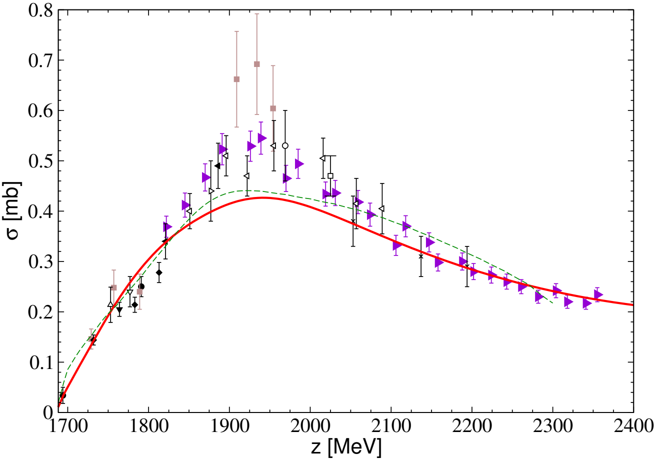


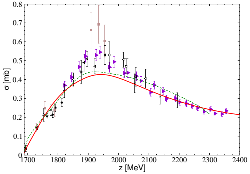
Partial wave content $I = 3/2$:



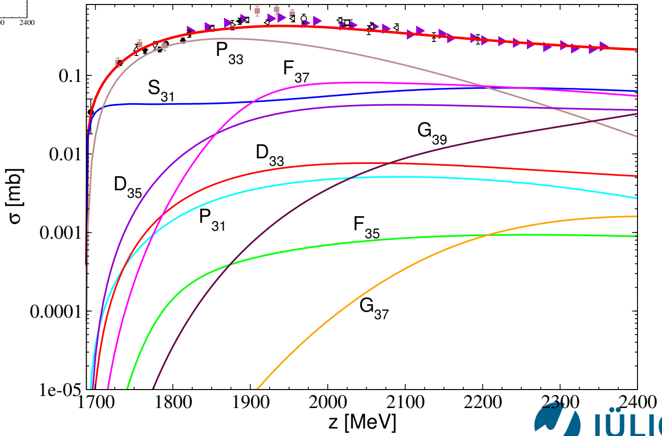
$\pi^+ p \to K^+ \Sigma^+$: Total cross section (preliminary)



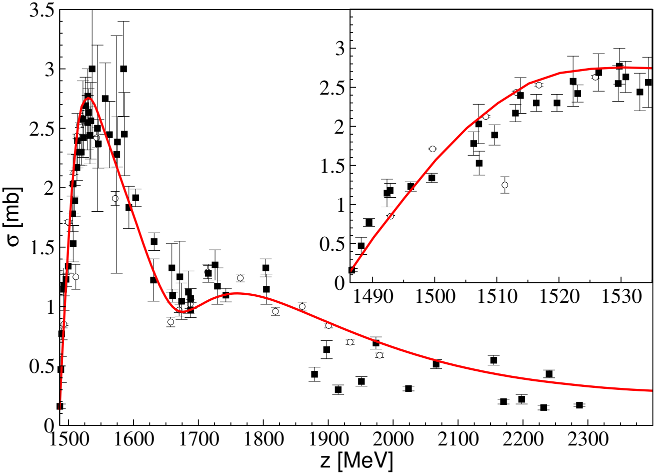
$\pi^+ p \to K^+\Sigma^+$: Total cross section (preliminary)



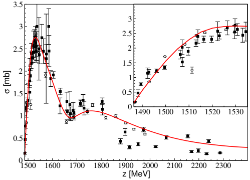
Partial wave content:



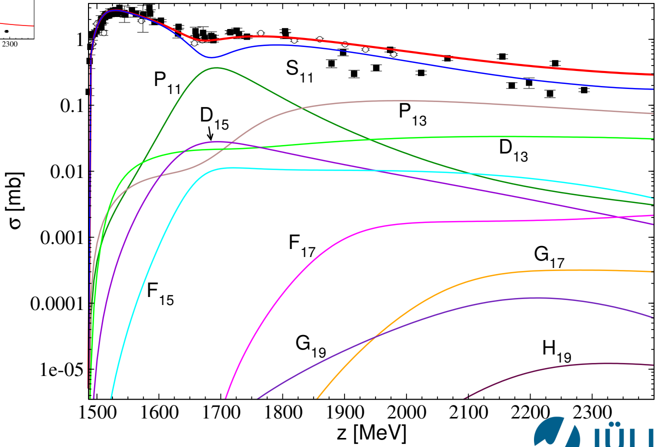
$\pi^- p \to \eta N$: Total cross section (preliminary)



$\pi^- p \to \eta N$: Total cross section (preliminary)



Partial wave content:



Photoproduction: Coupled channels and gauge invariance

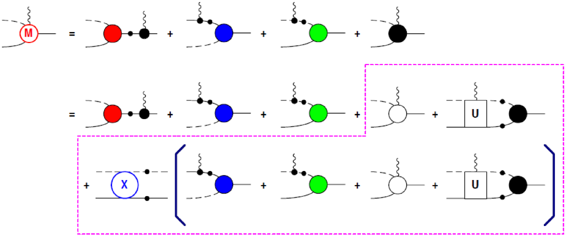
Haberzettl, PRC56 (1997), Haberzettl, Nakayama, Krewald, PRC74 (2006)

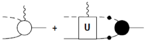
Gauge invariance: Generalized Ward-Takahashi identity (WTI)

$$k_\mu M^\mu = -|F_s\tau\rangle S_{p+k} Q_i S_p^{-1} + S_{p'}^{-1} Q_f S_{p'-k} |F_u\tau\rangle + \Delta^{-1}_{p-p'+k} Q_{pi} \Delta_{p-p'} |F_t\tau\rangle$$

Photoproduction amplitude:

$$M^\mu = \underbrace{M_s^\mu + M_u^\mu + M_t^\mu}_{\text{coupling to external legs}} + \underbrace{M_{int}^\mu}_{\text{coupling inside hadronic vertex}}$$

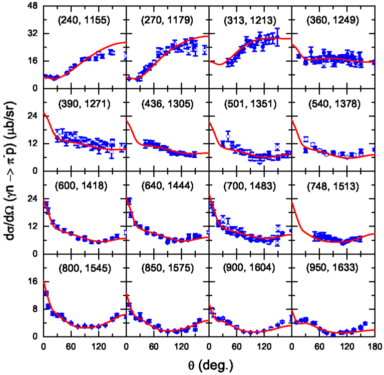


Strategy: Replace 

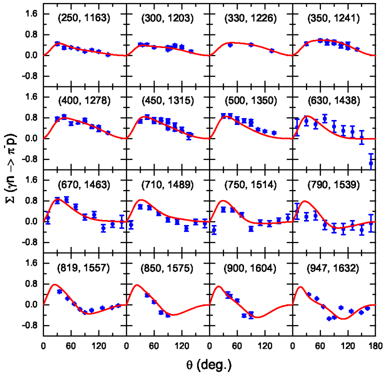
by phenomenological contact term such that the generalized WTI is satisfied

Photoproduction: $d\sigma/d\Omega$ and Σ_γ for $\gamma n \to \pi^- p$

F. Huang, M. Döring, K. Nakayama *et al.*, Phys. Rev. C85 (2012) 054003



Differential cross section for $\gamma n \to \pi^- p$

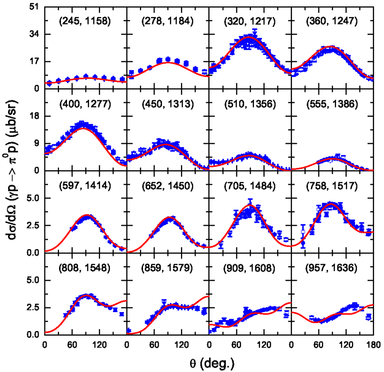


Photon spin asymmetry for $\gamma n \to \pi^- p$

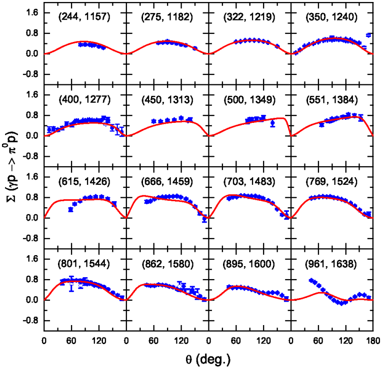
Data: CNS Data analysis center [CBELSA/TAPS, JLAB, MAMI,...]

Photoproduction: $d\sigma/d\Omega$ and Σ_γ for $\gamma p \to \pi^0 p$

F. Huang, M. Döring., K. Nakayama *et al.*, Phys. Rev. C85 (2012) 054003



Differential cross section for $\gamma p \to \pi^0 p$

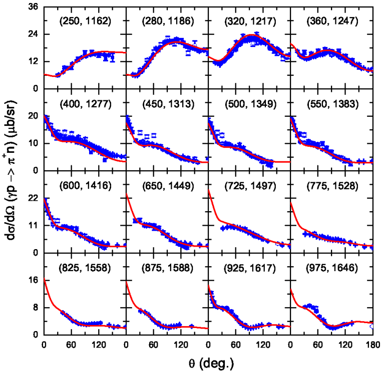


Photon spin asymmetry for $\gamma p \to \pi^0 p$

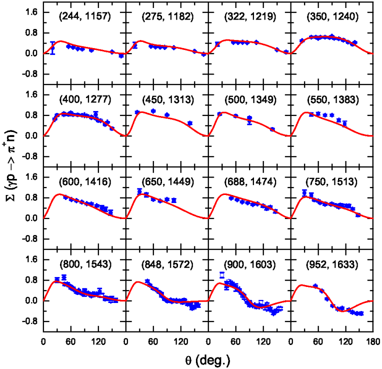
Data: CNS Data analysis center [CBELSA/TAPS, JLAB, MAMI,...]

Photoproduction: $d\sigma/d\Omega$ and Σ_γ for $\gamma p \to \pi^+ n$

F. Huang, M. Döring, K. Nakayama *et al.*, Phys. Rev. C85 (2012) 054003



Differential cross section for $\gamma p \to \pi^+ n$



Photon spin asymmetry for $\gamma p \to \pi^+ n$

Data: CNS Data analysis center [CBELSA/TAPS, JLAB, MAMI,...]

/appendix

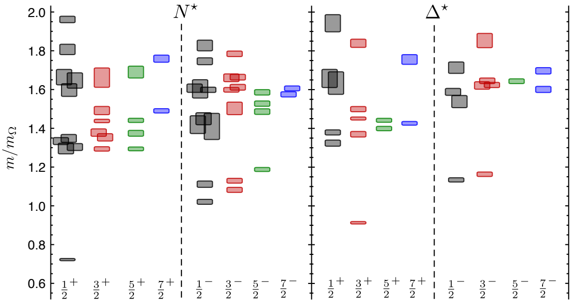
Error estimates for masses: $\Delta(1905)F_{35}$

Table: Error estimates of bare mass m_b and bare coupling f for the $\Delta(1905)F_{35}$ resonance.

m_b [MeV]	πN	ρN	$\pi\Delta$	ΣK
2258^{+44}_{-43}	$0.0500^{+0.0011}_{-0.0012}$	$-1.62^{+1.29}_{-1.61}$	$-1.15^{+0.030}_{-0.022}$	$0.120^{+0.0065}_{-0.0059}$

Motivation: Baryon spectrum, πN scattering

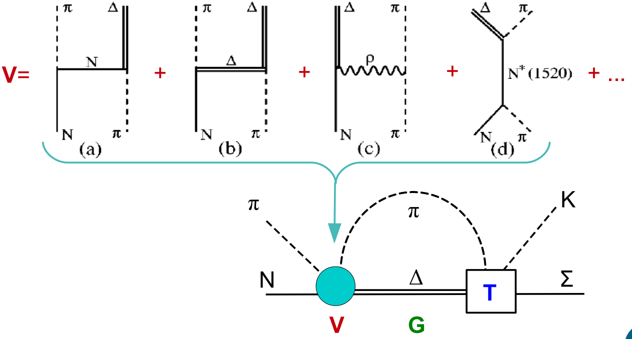
- Higher energies: more states are predicted than seen in elastic πN scattering ("misssing resonance problem")
  $\rightarrow$ coupling to other channels like multi-pion and KY
- Predicted resonances from recent lattice calculations [Edwards *et al.*, Phys.Rev. D84 (2011)]:



$m_\pi = 396 MeV$

The scattering equation

$$T^{I}_{\mu\nu}(\vec{k}',\lambda',\vec{k},\lambda) = V^{I}_{\mu\nu}(\vec{k}',\lambda',\vec{k},\lambda) + \sum_{\gamma,\lambda''}\int d^3q\, V^{I}_{\mu\gamma}(\vec{k}',\lambda',\vec{q},\lambda'')\frac{1}{Z-E_\gamma(q)+i\epsilon}T^{I}_{\gamma\nu}(\vec{q},\lambda'',\vec{k},\lambda)$$



The scattering equation

$$T^{I}_{\mu\nu}(\vec{k}', \lambda', \vec{k}, \lambda) = V^{I}_{\mu\nu}(\vec{k}', \lambda', \vec{k}, \lambda) + \sum_{\gamma, \lambda''} \int d^3q \, V^{I}_{\mu\gamma}(\vec{k}', \lambda', \vec{q}, \lambda'') \frac{1}{Z - E_\gamma(q) + i\epsilon} T^{I}_{\gamma\nu}(\vec{q}, \lambda'', \vec{k}, \lambda)$$

Features:

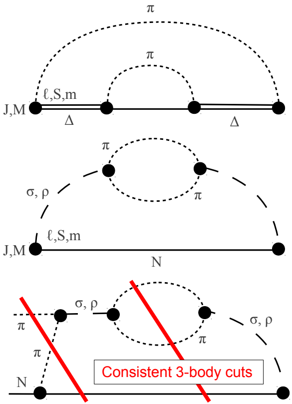
- Coupled channels πN, ηN, $K\Lambda$, $K\Sigma$, $\pi\pi N$ [$\pi\Delta$, σN, ρN]
- Hadron exchange: relevant degrees of freedom in 2nd and 3rd resonance region
- t- and u-channel processes: "background", all channels are linked
- s-channel processes: genuine resonances (only a minimum)
- Channels/reactions linked (SU(3) symmetry in Lagrangian framework)
- No on-shell factorization, full analyticity (dispersive parts)

s-, t- and u-channel exchanges

- s-channel states coupling to πN, ηN, $K\Lambda$, $K\Sigma$, $\pi\Delta$, ρN.
- t- and u-channel exchanges:

	πN	ρN	ηN	πΔ	σN	KΛ	KΣ
πN	N,Δ,$(\pi\pi)_\sigma$, $(\pi\pi)_\rho$	N, Δ, Ct., π, ω, a_1	N, a_0	N, Δ, ρ	N, π	Σ, Σ*, K*	Λ, Σ, Σ*, K*
ρN		N, Δ, Ct., ρ	-	N, π	-	-	-
ηN			N, f_0	-	-	K*, Λ	Σ, Σ*, K*
πΔ				N, Δ, ρ	π	-	-
σN					N, σ	-	-
KΛ						Ξ, Ξ*, f_0, ω, ϕ	Ξ, Ξ*, ρ
KΣ							Ξ, Ξ*, f_0, ω, ϕ, ρ

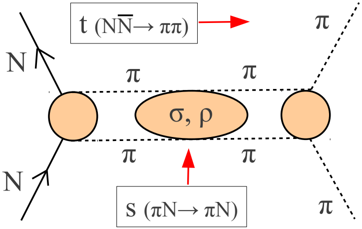
$\pi\pi N$ states: $\pi\Delta$, σN, ρN



- $\pi\pi/\pi N$ subsystems fit the respective phase shifts.
- Towards a consistent inclusion of 3-body cuts.
- Allow for *a-priori* 3-body unitarity per construction [Aaron, Almado, Young, PR 174 (1968) 2022].

Crossing symmetry

at the level of the potential (not the amplitude)



- For $\sigma(600)$ and $\rho(770)$ quantum numbers: πN t-channel interaction from $\bar{N}N \to \pi\pi$ (analytically continued) data.
- Use of crossing symmetry and dispersion techniques

  [Schütz *et al.* PRC 49 (1994) 2671].

Scattering equation: partial wave decomposition

Expand $T(\vec{k}', \lambda', \vec{k}, \lambda)$ in terms of the eigenstates of the total angular momentum J:

$$\langle \lambda' \vec{k}' | T | \lambda \vec{k} \rangle = \frac{1}{4\pi} \sum_J (2J+1) D^J_{\lambda\lambda'}(\Omega_{k'k}, 0)^* \langle \lambda' \vec{k}' | T^J | \lambda \vec{k} \rangle$$

Scattering equation:

$$\frac{1}{4\pi} \sum_J (2J+1) D^J_{\lambda\lambda'}(\Omega_{k'k}, 0)^* \langle \lambda_3 \lambda_4 \vec{k}' | T^J | \lambda_1 \lambda_2 \vec{k} \rangle = \langle \lambda_3 \lambda_4 \vec{k}' | V^{J'} | \lambda_1 \lambda_2 \vec{k} \rangle +$$

$$+ \frac{1}{4\pi} \sum_{\gamma_1, \gamma_2} \sum_{J', J''} \int dq d\Omega_{qk} q^2 (2J'+1)(2J''+1) D^{J'}_{\gamma\lambda'}(\Omega_{k'q}, 0)^* D^{J''}_{\lambda\gamma}(\Omega_{qk}, 0)^*$$

$$\times \langle \lambda_3 \lambda_4 \vec{k}' | V^{J'} | \gamma_1 \gamma_2 \vec{q} \rangle G(q) \langle \gamma_1 \gamma_2 \vec{q} | T^{J''} | \lambda_1 \lambda_1 \vec{k} \rangle$$

$$\int d\Omega_{qk} D^{J'}_{\gamma\lambda'}(\Omega_{k'q}, 0)^* D^{J''}_{\lambda\gamma}(\Omega_{qk}, 0)^* = D^{J'}_{\lambda\lambda'}(\Omega_{k'k}, 0)^* \frac{4\pi}{2J'+1} \delta_{J'J''}$$

Scattering equation: partial wave decomposition

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$$\times \langle \lambda_3 \lambda_4 \vec{k}' | V^{J'} | \gamma_1 \gamma_2 \vec{q} \rangle G(q) \langle \gamma_1 \gamma_2 \vec{q} | T^{J''} | \lambda_1 \lambda_1 \vec{k} \rangle$$

$$\int d\Omega_{qk} D^{J'}_{\gamma\lambda'}(\Omega_{k'q}, 0)^* D^{J''}_{\lambda\gamma}(\Omega_{qk}, 0)^* = D^{J'}_{\lambda\lambda'}(\Omega_{k'k}, 0)^* \frac{4\pi}{2J'+1} \delta_{J'J''}$$

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$$\frac{1}{4\pi} \sum_J (2J+1) D^J_{\lambda\lambda'}(\Omega_{k'k}, 0)^* \langle \lambda_3\lambda_4 \vec{k}' | T^J | \lambda_1\lambda_2 \vec{k} \rangle = \langle \lambda_3\lambda_4 \vec{k}' | V^{J'} | \lambda_1\lambda_2 \vec{k} \rangle +$$
$$+ \frac{1}{4\pi} \sum_{\gamma_1,\gamma_2} \sum_{J', J''} \int dq\, d\Omega_{qk}\, q^2 (2J'+1)(2J''+1) D^{J'}_{\gamma\lambda'}(\Omega_{k'q}, 0)^* D^{J''}_{\lambda\gamma}(\Omega_{qk}, 0)^*$$
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Scattering equation:

$$\langle \lambda_3\lambda_4 \vec{k}' | T^J | \lambda_1\lambda_2 \vec{k} \rangle = \langle \lambda_3\lambda_4 \vec{k}' | V^J | \lambda_1\lambda_2 \vec{k} \rangle + \sum_{\gamma_1,\gamma_2} \int dq q^2 \langle \lambda_3\lambda_4 \vec{k}' | V^J | \gamma_1\gamma_2 \vec{q} \rangle G(q) \langle \gamma_1\gamma_2 \vec{q} | T^J | \lambda_1\lambda_1 \vec{k} \rangle.$$

- 1-dim. integral equation → sizable reduction of numerical effort

Scattering equation in JLS basis

experimental data (partial wave analyses) usually in JLS basis
→ switch from helicity to JLS basis:

$$|JM\lambda_1\lambda_2 k\rangle = \sum_{LS} \langle JMLS|JM\lambda_1\lambda_2\rangle |JMLS\rangle$$

J: total angular momentum
M: z-projection of J
L: orbital angular momentum
S: total spin
λ_i: helicity, $\lambda := \lambda_1 - \lambda_2$

$$\langle JMLS|JM\lambda_1\lambda_2\rangle = \left(\frac{2L+1}{2J+1}\right)^{\frac{1}{2}} \underbrace{\langle L0S\lambda|J\lambda\rangle\langle S_1\lambda_1 S_2\lambda_2|S\lambda\rangle}_{\text{Clebsch-Gordan coefficients}}$$

$$\langle L'S'k'|V^J|LSk\rangle = \sum_{\lambda_1\lambda_2\lambda_3\lambda_4} \langle JML'S'|JM\lambda_3\lambda_4\rangle\langle\lambda_3\lambda_4 k'|V^J|\lambda_1\lambda_2 k\rangle\langle JM\lambda_1\lambda_2|JMLS\rangle$$

Scattering equation in JLS basis:

$$\langle L'S'k'|T^{IJ}_{\mu\nu}|LSk\rangle = \langle L'S'k'|V^{IJ}_{\mu\nu}|LSk\rangle + \sum_{\gamma,L''S''} \int_0^\infty q^2\, dq \langle L'S'k'|V^{IJ}_{\mu\gamma}|L''S''q\rangle\, G(q)\, \langle L''S''q|T^{IJ}_{\gamma\nu}|LSk\rangle$$

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Scattering equation in JLS basis:

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Analytic structure of the scattering amplitude

Analytic properties of the amplitude $\Rightarrow$ important information:

- cuts, poles and zeros on different Riemann sheets determine global behaviour of the amplitude on the physical axis
- parameterization of resonances in a well defined way
- poles and residues: relevant quantities for comparison of different experiments

Jülich model:

- derived within a field theoretical approach
- analyticity is respected

$\Rightarrow$ reliable extraction of resonance properties

Extraction of resonance parameters, pole search on 2nd sheet:

$\rightarrow$ Analytic continuation

Analytic continuation via Contour deformation

...enables access to all Riemann sheets [Nucl.Phys.A829:170-209,2009]

Propagator for a particle with width:

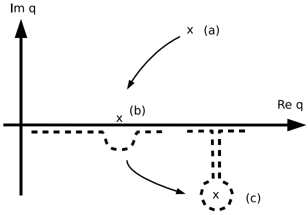
$$G_\sigma(z,k) = \frac{1}{z - \sqrt{k^2 + (m_\sigma^0)^2} - \Pi_\sigma(z', k)}$$

Example: Selfenergy

$$\Pi_\sigma(z) = \int_0^\infty q^2\, dq\, \frac{(v^{\sigma\pi\pi}(q,k))^2}{z - 2\sqrt{q^2 + m_\pi^2} + i\epsilon}$$

- righthand cut along positive real axis, starting at $z_{thresh} = 2m_\pi$
- analytic continuation along the cut to the 2nd sheet:

$$\Pi_\sigma^{(2)} = \Pi_\sigma^{(1)} - 2\mathrm{Im}\Pi_\sigma^{(1)} = \Pi_\sigma^{(1)} + \frac{2\pi i q_{on} E_{on}^{(1)} E_{on}^{(2)}}{z} v^2(q_{on}, k)$$



- case (a), Im $z > 0$: straight integration from $q = 0$ to $q = \infty$.
- case (b), Im $z = 0$: Pole is on real q axis.
- case (c), Im $z < 0$: Deformation gives analytic continuation.
- Special case: Pole at $q = 0$ $\Leftrightarrow$ branch point at $z = m_1 + m_2$ (= threshold).

Analytic continuation via Contour deformation

...enables access to all Riemann sheets [Nucl.Phys.A829:170-209,2009]

Propagator for a particle with width:

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- righthand cut along positive real axis, starting at $z_{thresh} = 2m_\pi$
- analytic continuation along the cut to the 2nd sheet:

$$\begin{aligned}\Pi_\sigma^{(2)} &= \Pi_\sigma^{(1)} - 2\mathrm{Im}\Pi_\sigma^{(1)} \\ &= \Pi_\sigma^{(1)} + \frac{2\pi i q_{on} E_{on}^{(1)} E_{on}^{(2)}}{z} v^2(q_{on}, k)\end{aligned}$$

Scattering equation on the 2nd sheet:

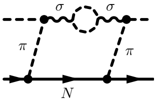
$$\langle q_{cd}| T^{(2)} - V | q_{ab}\rangle = \delta G + \int dq_{mn}\, q_{mn}^2 \frac{\langle q_{cd}|V|q_{mn}\rangle\langle q_{mn}|T^{(2)}|q_{ab}\rangle}{z - E_{mn} + i\epsilon}$$

with

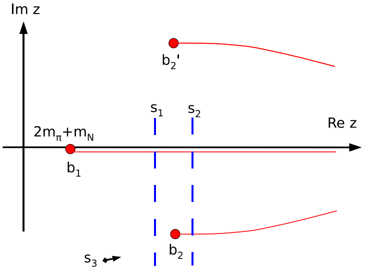
$$\delta G = \frac{2\pi i q_{on} E_{on}^{(1)} E_{on}^{(2)}}{z} \langle q_{cd}|V|q_{mn}^{on}\rangle\langle q_{mn}^{on}|T^{(2)}|q_{ab}\rangle$$

Effective $\pi\pi N$ channels: Analytic structure

new structure, induced by addititonal branch points



$$= \int_0^\infty dk\, k^2 \underbrace{\frac{1}{z-\sqrt{m_N^2+k^2}-\sqrt{(m_\sigma^0)^2+k^2}-\underbrace{\Pi_\sigma(z_\sigma(z,k),k)}_{\to b_1}}}_{\to b_2,\, b_2'}$$



- The cut along Im $z=0$ is induced by the cut of the self energy of the unstable particle.
- The poles of the unstable particle (σ) induce branch points (b_2, b_2') in the σN propagator at

$$z_{b_2} = m_N + z_0,\; z_{b_2'} = m_N + z_0^*$$

3 branch points and 4 sheets for each of the σN, ρN, and $\pi\Delta$ propagators.