Progress on Formulating Bethe-Salpeter Kernels for Studying Hadron Excitations

Sixue Qin

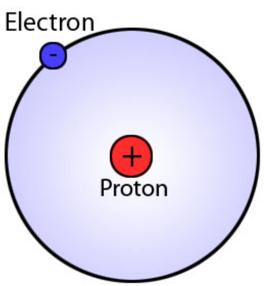
Argonne National Laboratory

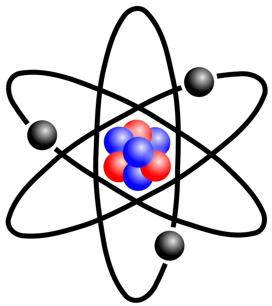
2015-10-14@Trento

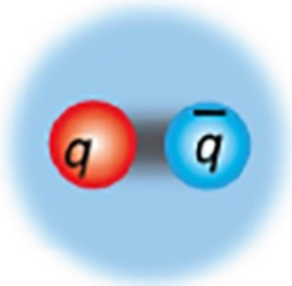
Fundamental Forces versus **Bound States**

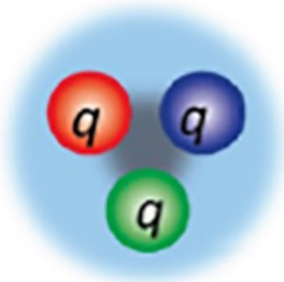
QED

QCD



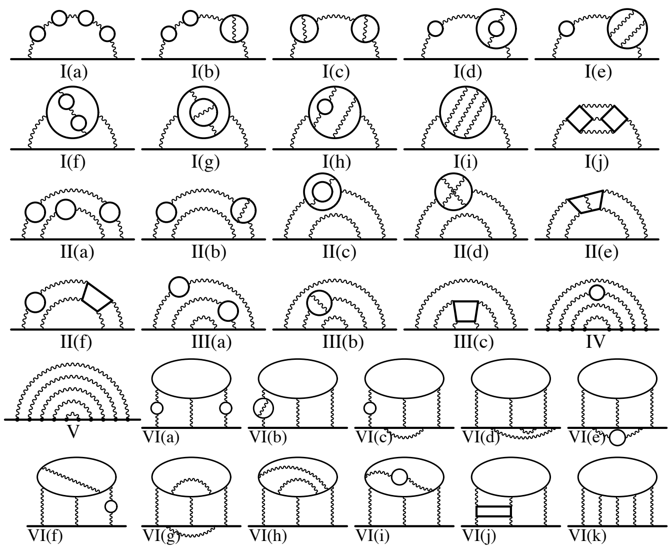






Fundamental Forces versus Bound States

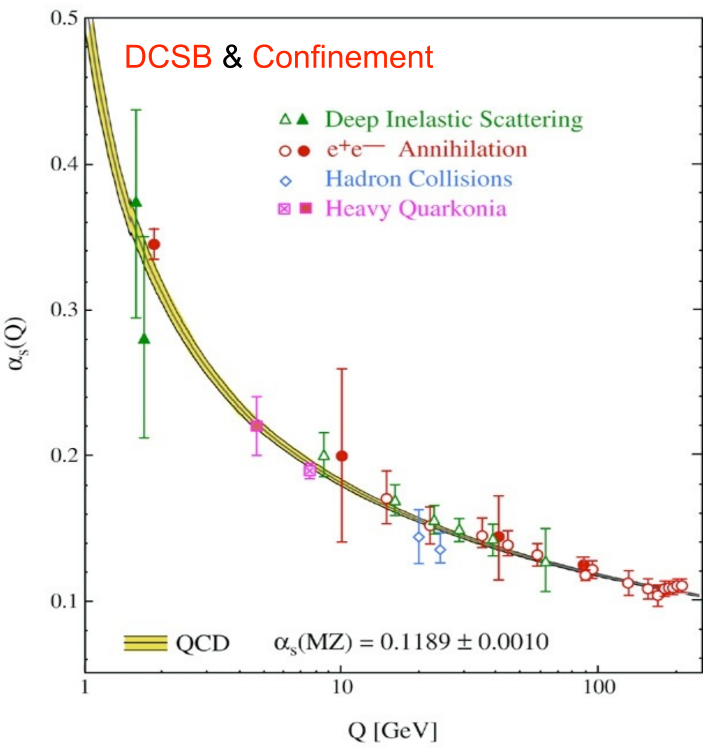
Perturbative



$$\alpha^{-1} = 137.035\ 999\ 174\ (35)$$

QED fine-structure constant

Non-perturbative



QCD running coupling constant

Dyson-Schwinger Equations: Equation of motion of Green functions

Classical Mechanics | Quantum Field Theory

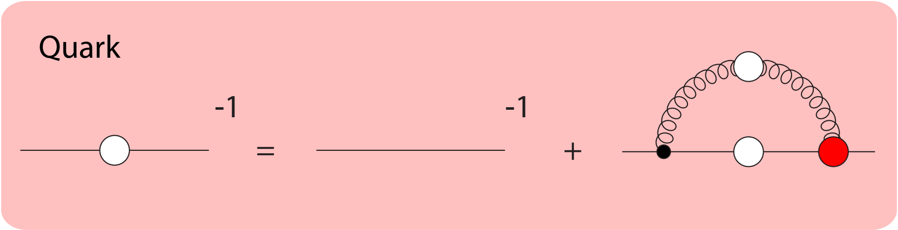
Principle of Least Action

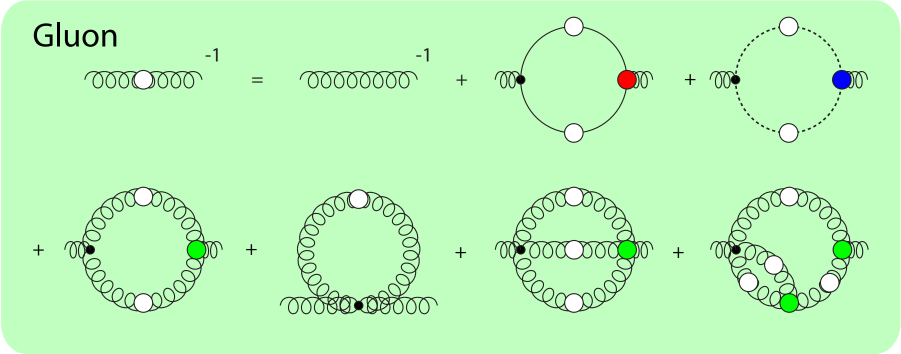
$$\frac{\delta S[q]}{\delta q} = 0$$

$$\left\langle \frac{\delta S[\phi(x)]}{\delta \phi(x)} \right\rangle = 0$$

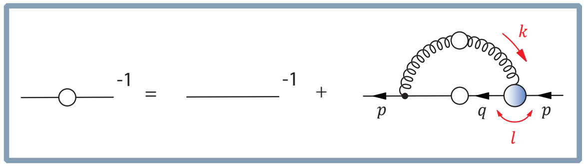
Equations of Motion (EoM)

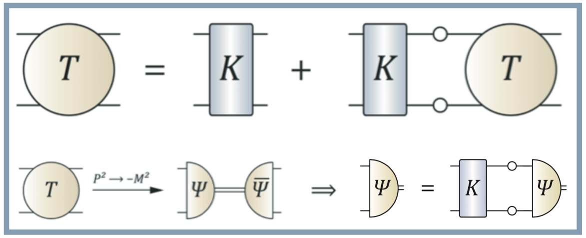
Euler-Lagrange Equation | Dyson-Schwinger Equation

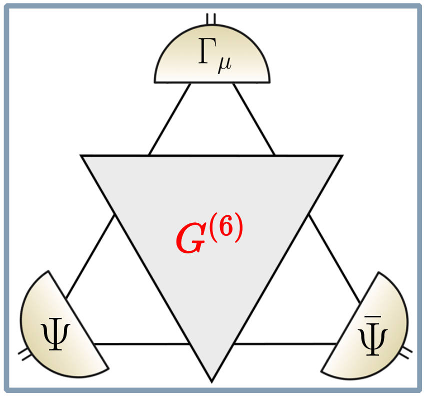




Dyson-Schwinger Equations: Equations for meson properties







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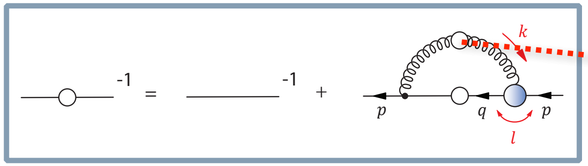
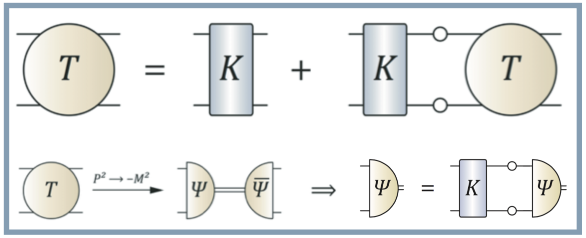
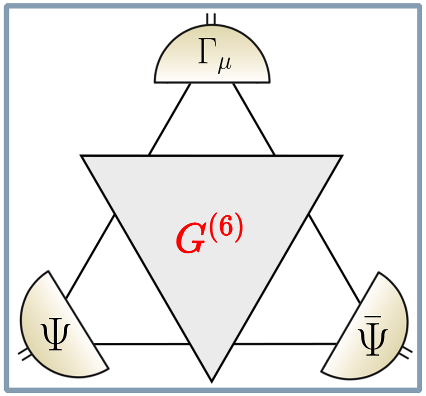


Gluon propagator





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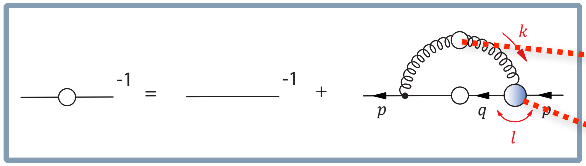
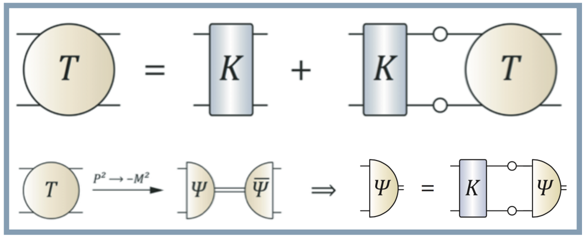
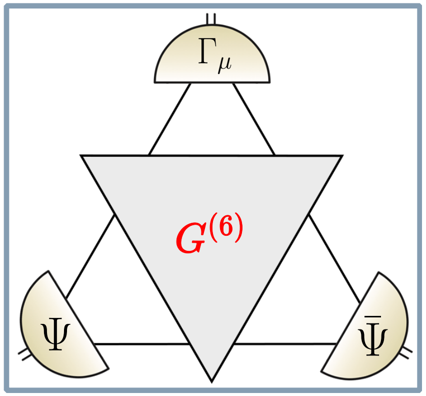


Gluon propagator

Quark-gluon vertex





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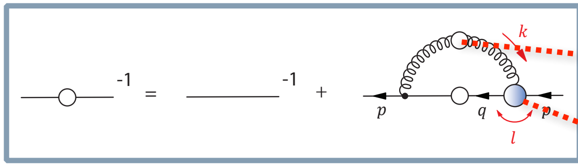
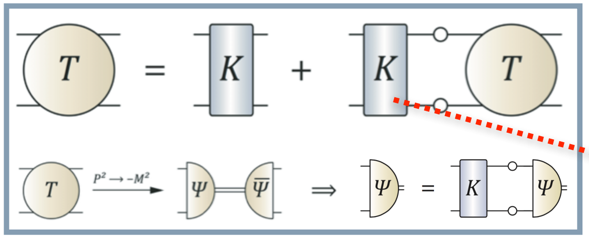
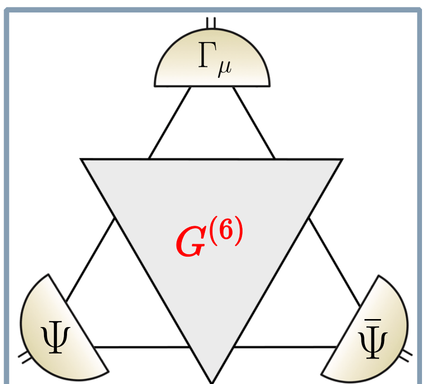




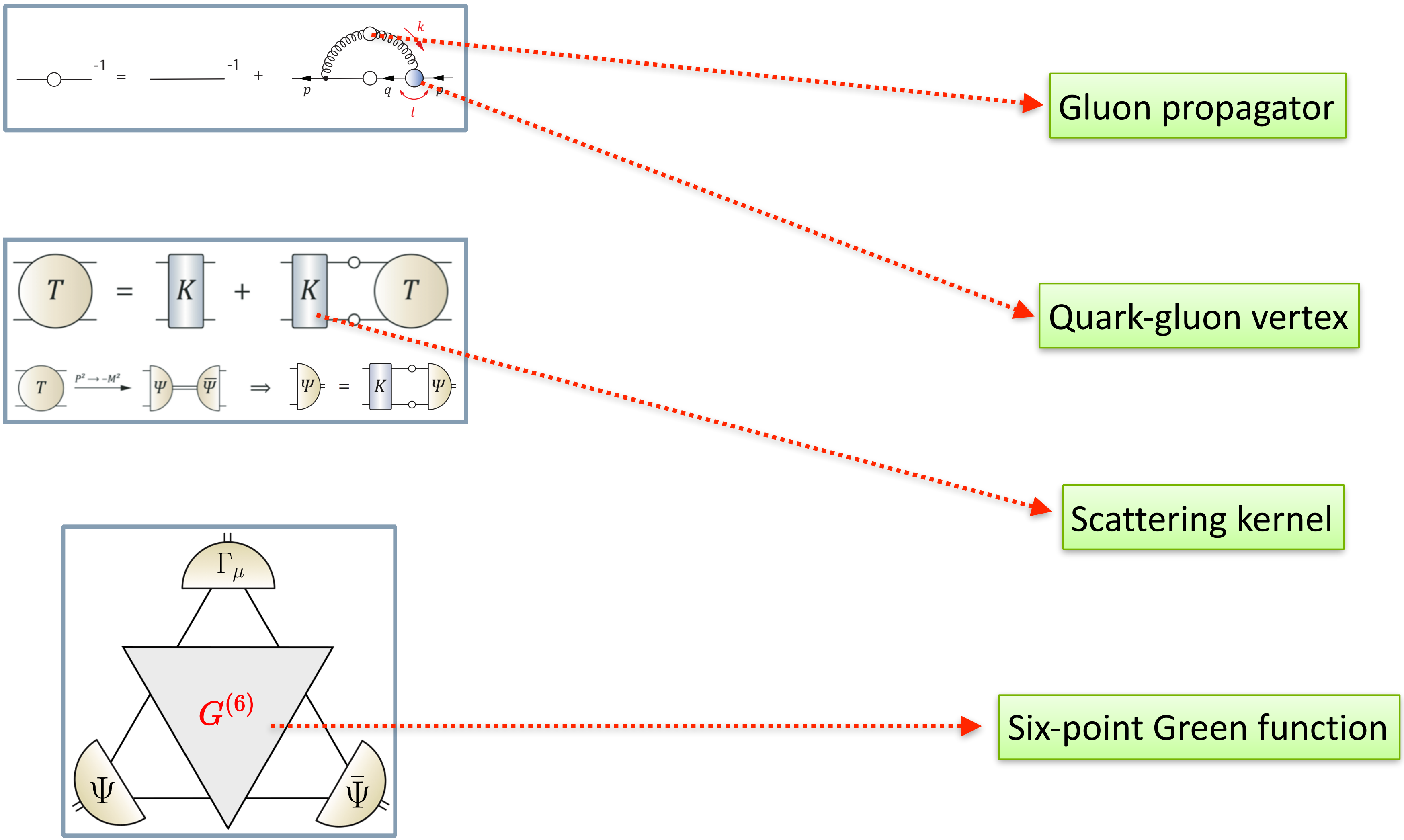


Gluon propagator

Quark-gluon vertex

Scattering kernel

Dyson-Schwinger Equations: Equations for meson properties



Dyson-Schwinger Equations: The simplest approximation

I. Gluon propagator

II. Quark-gluon vertex

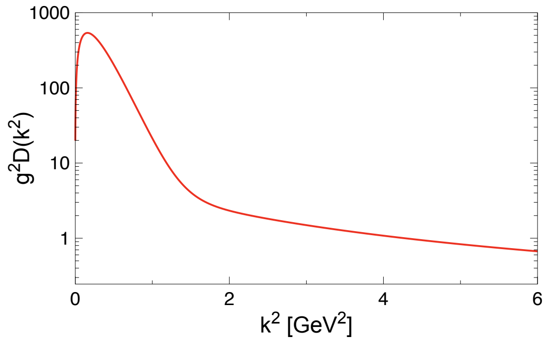
III. Scattering kernel

IV. Six-point Green function

Dyson-Schwinger Equations: The simplest approximation

I. Gluon propagator

Maris-Tandy model



II. Quark-gluon vertex

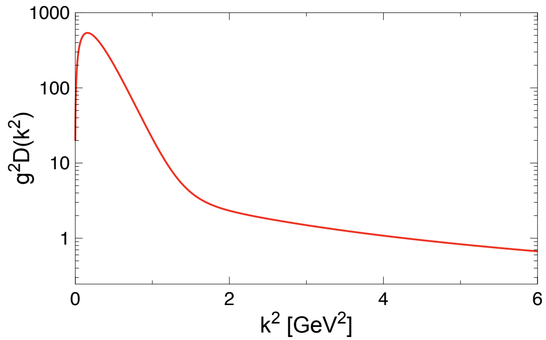
III. Scattering kernel

IV. Six-point Green function

Dyson-Schwinger Equations: The simplest approximation

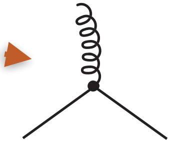
I. Gluon propagator

Maris-Tandy model



II. Quark-gluon vertex

rainbow approximation



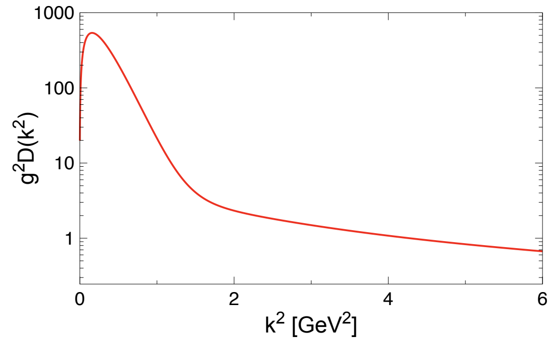
III. Scattering kernel

IV. Six-point Green function

Dyson-Schwinger Equations: The simplest approximation

I. Gluon propagator

Maris-Tandy model



II. Quark-gluon vertex

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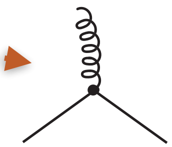
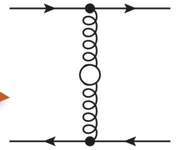


III. Scattering kernel

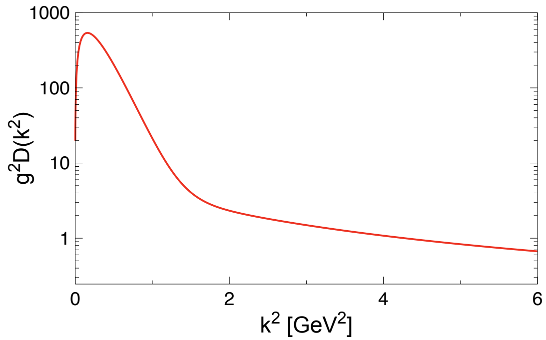
ladder approximation



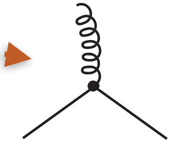
IV. Six-point Green function

Dyson-Schwinger Equations: The simplest approximation

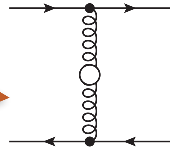
I. Gluon propagator — Maris-Tandy model



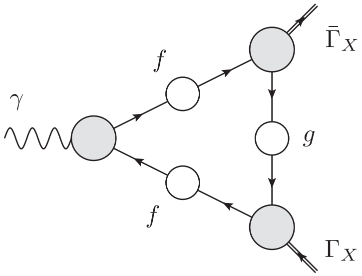
II. Quark-gluon vertex — rainbow approximation



III. Scattering kernel — ladder approximation

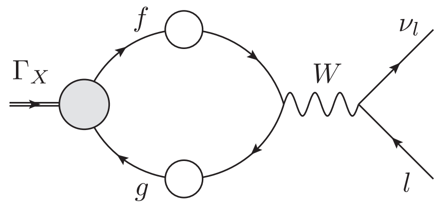


IV. Six-point Green function — impulsion approximation

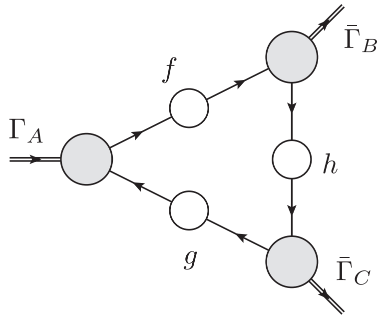


Rainbow-Ladder truncation: T = 0

✦ Leptonic decay



✦ Strong decay

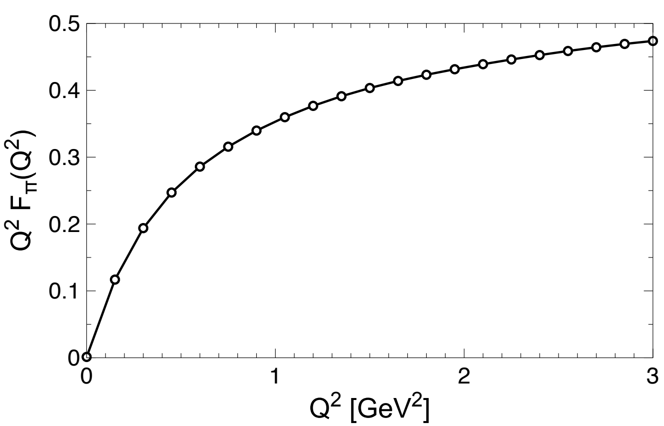


✦ PDF, GPD, TMD, and etc.

✦ Meson spectroscopy Qin et. al., PRC 85, 035202 (2012)

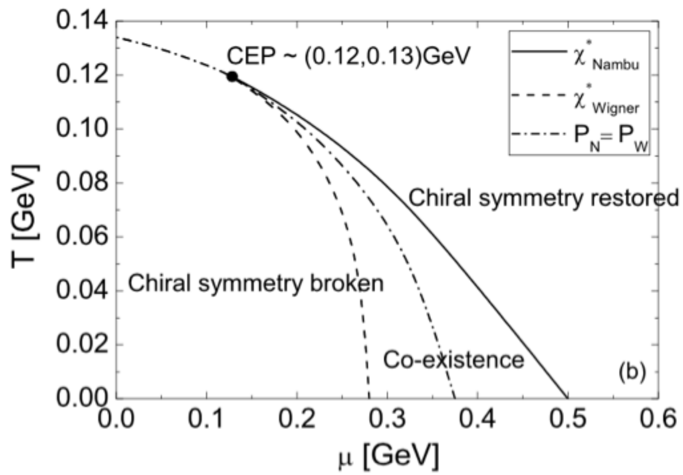
$(D\omega)^{1/3}$	0.72	0.8	0.8	0.8	0.8	-
ω	0.4	0.4	0.5	0.6	0.7	-
$m^{\zeta}_{u,d}$	0.0037	0.0034	0.0034	0.0034	0.0034	-
m^{ζ}_{s}	0.084	0.082	0.082	0.082	0.082	-
$A(0)$	1.58	2.07	1.70	1.38	1.16	-
$M(0)$	0.50	0.62	0.52	0.42	0.29	-
M_π	0.138*	0.139*	0.134	0.136	0.139	0.138
f_π	0.093*	0.094*	0.093	0.090	0.081	0.092
$\rho_\pi^{1/2}$	0.48	0.49	0.49	0.49	0.48	-
M_K	0.496*	0.496*	0.495	0.497	0.503	0.496
f_K	0.11	0.11	0.11	0.11	0.10	0.113
$\rho_K^{1/2}$	0.54	0.55	0.55	0.55	0.55	-
M_ρ	0.74	0.76	0.74	0.72	0.67	0.777
f_ρ	0.15	0.14	0.15	0.14	0.12	0.153
M_ϕ	1.07	1.09	1.08	1.07	1.05	1.020
f_ϕ	0.18	0.19	0.19	0.19	0.18	0.168
M_σ	0.67	0.67	0.65	0.59	0.46	-
$\rho_\sigma^{1/2}$	0.52	0.53	0.53	0.51	0.48	-

✦ Form factor



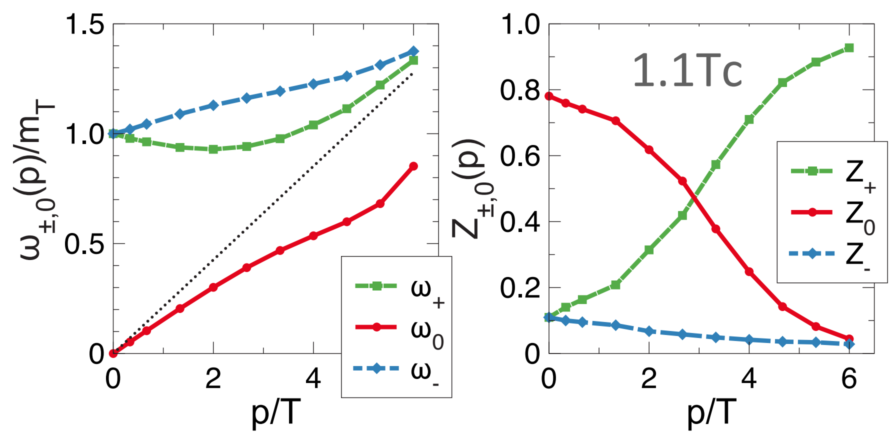
Rainbow-Ladder truncation: T > 0

✦ QCD phase diagram



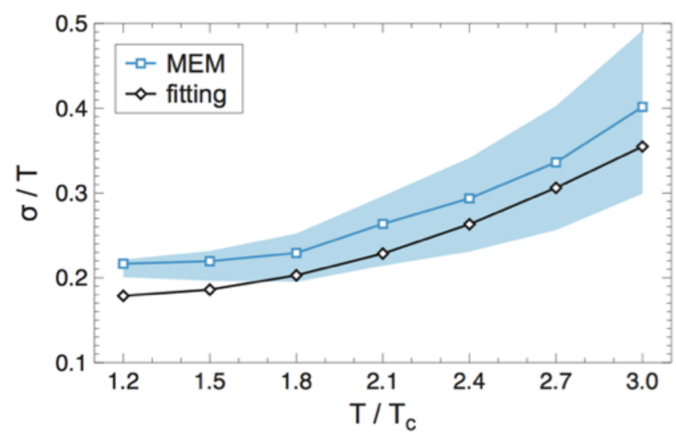
Qin et. al., PRL 106, 172301 (2011)

✦ sQGP collective excitations



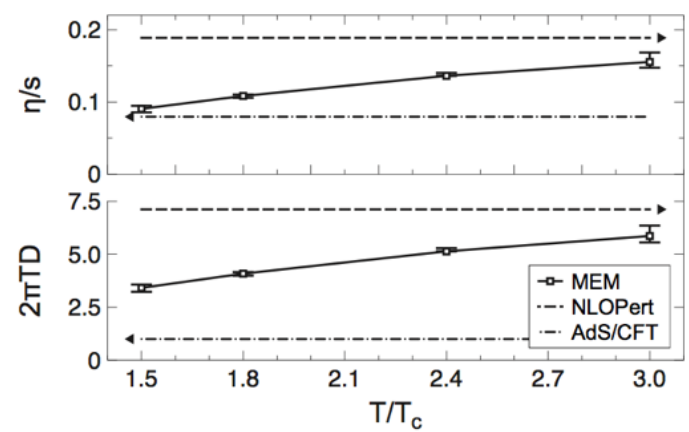
Qin et. al., PRD 84, 014017 (2011)

✦ QGP electrical conductivity



Qin, PLB 742, 358 (2015)

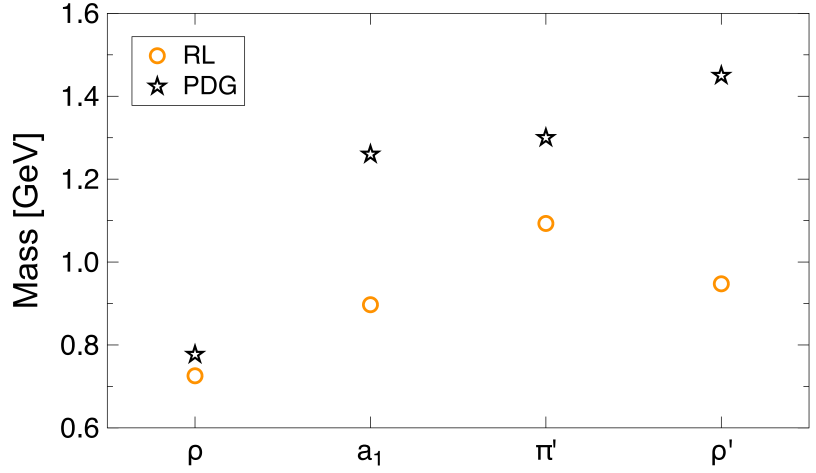
✦ QGP viscosity



Qin et. al., PLB 734, 157 (2014)

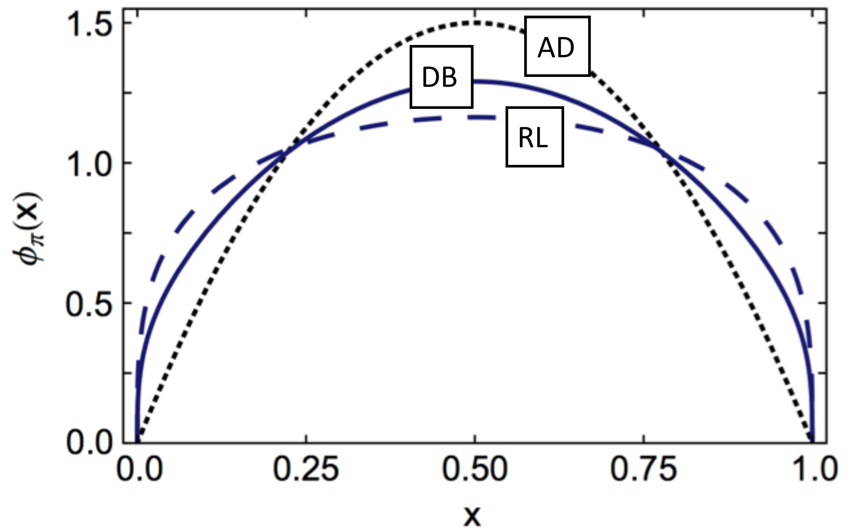
Rainbow-Ladder truncation: Drawbacks

- Rho-a1 mass splitting: too small



- Radial excitation states: wrong ordering and wrong magnitudes

- Parton distribution function: too broad



- Pion form factor: rho monopole form, and etc.

Chang et. al., PRL 110, 132001 (2013)

Is there a **systematic** way to truncate the DSEs in order to approach the full QCD?

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I. Gluon propagator

II. Quark-gluon vertex

III. Scattering kernel

IV. Six-point Green function

I. Gluon propagator: Dynamically massive gluon

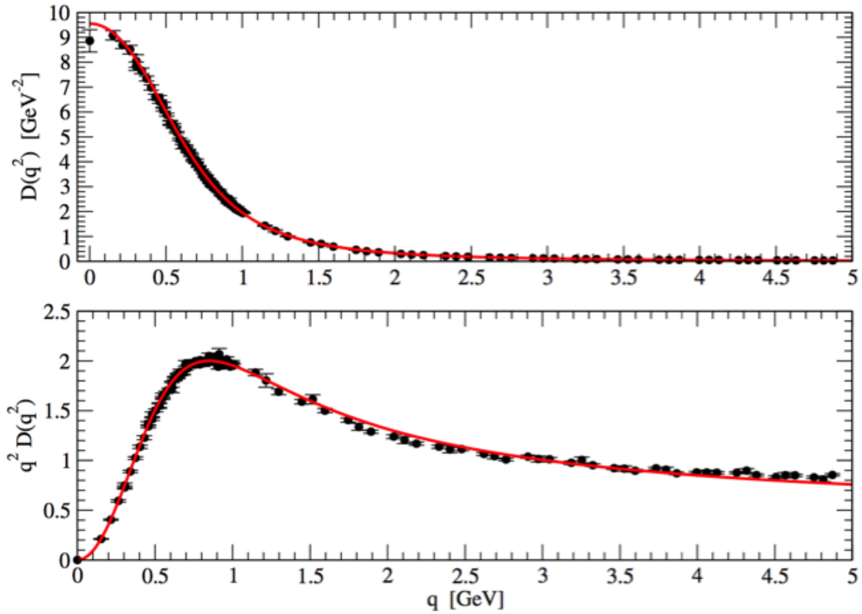
- In Landau gauge (a fixed point of the renormalization group):

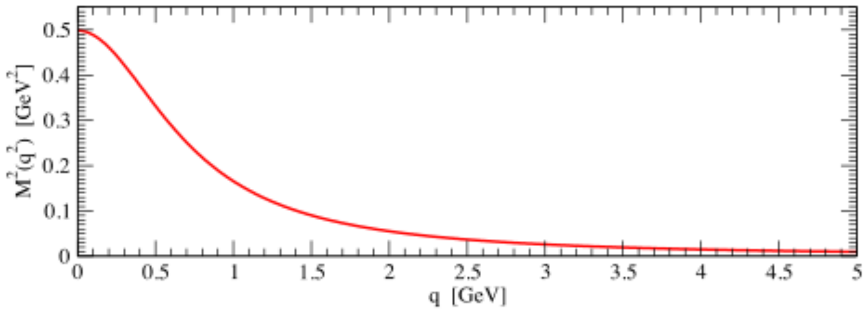
$$g^2 D_{\mu\nu}(k) = \mathcal{G}(k^2)(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2})$$

- Modeling the dress function: ***gluon mass scale*** + ***effective running coupling constant***

$$\mathcal{G}(k^2) \approx \frac{4\pi\alpha_{RL}(k^2)}{k^2 + m_g^2(k^2)},$$

$$m_g^2(k^2) = \frac{M_g^4}{M_g^2 + k^2},$$





O. Oliveira et. al., J.Phys. G38, 045003 (2011)

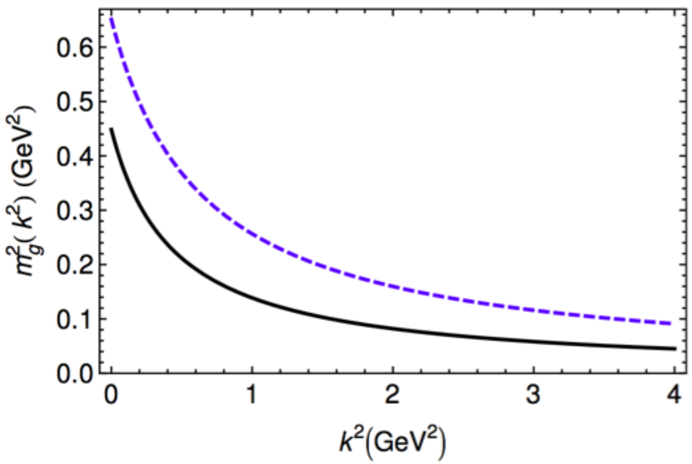
I. Gluon propagator: Dynamically massive gluon

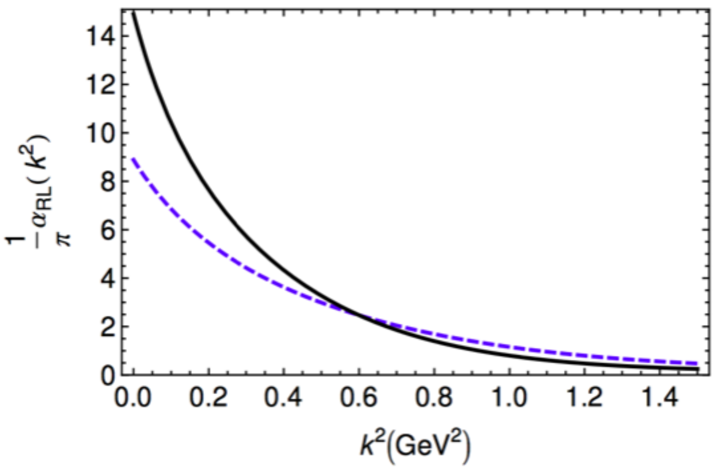
Model the gluon propagator as two parts: Infrared + Ultraviolet. The former is an expansion of delta function; The latter is a form of one-loop perturbative calculation.

$$\delta^4(k) \overset{\omega \sim 0}{\approx} \frac{1}{\pi^2}\frac{1}{\omega^4} e^{-k^2/\omega^2} \qquad \mathcal{G}(s) = \frac{8\pi^2}{\omega^4} D e^{-s/\omega^2} + \frac{8\pi^2 \gamma_m \mathcal{F}(s)}{\ln\left[\tau + \left(1 + s/\Lambda_{\mathrm{QCD}}^2\right)^2\right]}$$

Qin et. al., PRC 84, 042202R (2011)

- The gluon mass scale is *typical values of lattice QCD* in our parameter range: *Mg* in [0.6, 0.8] GeV.
- The gluon mass scale is inversely proportional to the confinement length.





ω = 0.5 GeV (solid curve) and ω = 0.6 GeV (dashed curve)

II. Quark-gluon vertex: (Abelian) Ward-Green-Takahashi Identities

- **Gauge symmetry (vector current conservation):** vector WGTI

$$\psi(x) \to \psi(x) + ig\alpha(x)\psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi}(x) - ig\alpha(x)\bar{\psi}(x)$$

$$iq_\mu \Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p)$$

- **Chiral symmetry (axial-vector current conservation):** axial-vector WGTI

$$\psi(x) \to \psi(x) + ig\alpha(x)\gamma^5\psi(x),$$
$$\bar{\psi}(x) \to \bar{\psi}(x) + ig\alpha(x)\bar{\psi}(x)\gamma^5,$$

$$q_\mu \Gamma^A_\mu(k,p) = S^{-1}(k) i\gamma_5 + i\gamma_5 S^{-1}(p) - 2im\Gamma_5(k,p)$$

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- **Lorentz symmetry + (axial-)vector current conservation:** transverse WGTIs

$$\delta_T \phi^a(x) = \delta_{\mathrm{Lorentz}}(\delta\phi^a(x)) = -\frac{i}{2}\epsilon^{\mu\nu} S^{(\delta\phi^a)}_{\mu\nu}(\delta\phi^a(x)).$$
$$S^{(\mathrm{spinor})}_{\mu\nu} = \frac{1}{2}\sigma_{\mu\nu}, \qquad (S^{(\mathrm{vector})}_{\mu\nu})^\alpha_\beta = i(\delta^\alpha_\mu g_{\nu\beta} - \delta^\alpha_\nu g_{\mu\beta});$$

He, PRD, 80, 016004 (2009)

$$\begin{aligned} q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = {} & S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) \\ & + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) \\ & + A^V_{\mu\nu}(k,p), \end{aligned}$$
$$\begin{aligned} q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) = {} & S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) \\ & + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) \\ & + V^A_{\mu\nu}(k,p), \qquad \sigma^5_{\mu\nu} = \gamma_5\sigma_{\mu\nu} \end{aligned}$$

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He, PRD, 80, 016004 (2009)

$$q_\mu \Gamma_\nu(k,p) - q_\nu \Gamma_\mu(k,p) = S^{-1}(p)\sigma_{\mu\nu} + \sigma_{\mu\nu} S^{-1}(k) + 2im\Gamma_{\mu\nu}(k,p) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma^A_\rho(k,p) + A^V_{\mu\nu}(k,p),$$

$$q_\mu \Gamma^A_\nu(k,p) - q_\nu \Gamma^A_\mu(k,p) = S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu} S^{-1}(k) + t_\lambda \varepsilon_{\lambda\mu\nu\rho}\Gamma_\rho(k,p) + V^A_{\mu\nu}(k,p), \qquad \sigma^5_{\mu\nu} = \gamma_5\sigma_{\mu\nu}$$

The longitudinal and transverse WGTIs express the vertex divergences and curls, respectively.

$$\nabla \cdot \Phi \qquad \nabla \times \Phi$$

II. Quark-gluon vertex: Solution of WGTIs

Define two projection tensors and contract them with the transverse WGTIs,

$$T^1_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta} t_\alpha q_\beta \mathbf{I}_D, \qquad T^2_{\mu\nu} = \frac{1}{2}\varepsilon_{\alpha\mu\nu\beta}\gamma_\alpha q_\beta.$$

one can decouple the WGTIs and obtain a group of equations for the vector vertex:

$$q_\mu i\Gamma_\mu(k,p) = S^{-1}(k) - S^{-1}(p),$$

$$\begin{aligned} q\cdot tt\cdot\Gamma(k,p) &= T^1_{\mu\nu}\left[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)\right] \\ &\quad + t^2 q\cdot\Gamma(k,p) + T^1_{\mu\nu}V^A_{\mu\nu}(k,p), \\ q\cdot t\gamma\cdot\Gamma(k,p) &= T^2_{\mu\nu}\left[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)\right] \\ &\quad + \gamma\cdot tq\cdot\Gamma(k,p) + T^2_{\mu\nu}V^A_{\mu\nu}(k,p). \end{aligned}$$

Qin et. al., PLB 722, 384 (2013)

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$$q\cdot t\gamma\cdot\Gamma(k,p) = T^2_{\mu\nu}\left[S^{-1}(p)\sigma^5_{\mu\nu} - \sigma^5_{\mu\nu}S^{-1}(k)\right] + \gamma\cdot tq\cdot\Gamma(k,p) + T^2_{\mu\nu}V^A_{\mu\nu}(k,p).$$

They are a group of full-determinant linear equations.
Thus, a unique solution for the vector vertex is exposed:

$$\Gamma^{\rm Full}_\mu(k,p) = \Gamma^{\rm BC}_\mu(k,p) + \Gamma^{\rm T}_\mu(k,p) + \Gamma^{\rm FP}_\mu(k,p).$$

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$$\Gamma^\mathrm{Full}_\mu(k,p) = \Gamma^\mathrm{BC}_\mu(k,p) + \Gamma^\mathrm{T}_\mu(k,p) + \Gamma^\mathrm{FP}_\mu(k,p).$$

❖ The quark propagator contributes to the longitudinal and transverse parts. The DCSB-related terms are highlighted.

$$\Gamma^\mathrm{BC}_\mu(k,p) = \gamma_\mu\Sigma_A + t_\mu \not{t}\frac{\Delta_A}{2} - it_\mu\Delta_B,$$

$$\Gamma^\mathrm{T}_\mu(k,p) = -\sigma_{\mu\nu}q_\nu\Delta_B + \gamma^T_\mu q^2\frac{\Delta_A}{2} - \left(\gamma^T_\mu[\not{q},\not{t}] - 2t^T_\mu\not{q}\right)\frac{\Delta_A}{4}.$$

$$S(p) = \frac{1}{i\gamma\cdot p\,A(p^2) + B(p^2)}$$

$$\Sigma_\phi(x,y) = \frac{1}{2}[\phi(x)+\phi(y)],$$

$$\Delta_\phi(x,y) = \frac{\phi(x)-\phi(y)}{x-y}.$$

$$X^T_\mu = X_\mu - \frac{q\cdot X q_\mu}{q^2}$$

❖ The unknown high-order terms only contribute to the transverse part, i.e., the longitudinal part has been completely determined by the quark propagator.

Qin et. al., PLB 722, 384 (2013)

III. Scattering kernel: Elements of quark gap equation

The Bethe-Salpeter equation and the quark gap equation are written as

$$\Gamma^H_{\alpha\beta}(k,P) = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta}[S(q_+)\Gamma^H(q,P)S(q_-)]_{\alpha'\beta'},$$

$$S^{-1}(k) = S_0^{-1}(k) + \int_q D_{\mu\nu}(k-q)\gamma_\mu S(q)\Gamma_\nu(q,k),$$

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$$P_\mu\Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k_+)i\gamma_5 + i\gamma_5 S^{-1}(k_-),$$

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$$iP_\mu\Gamma_\mu(k,P) = S^{-1}(k_+) - S^{-1}(k_-).$$

The kernel satisfies the following WGTIs: quark propagator + quark-gluon vertex

$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+) - S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+) - S(q_-)\Gamma_\nu(q_-,k_-)],$$

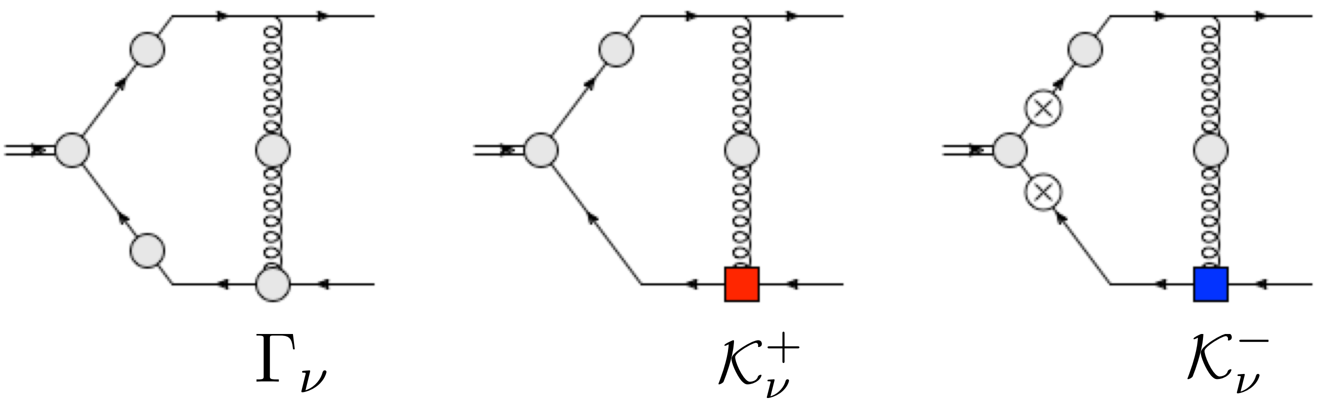
$$\int_q \mathcal{K}_{\alpha\alpha',\beta'\beta}\{S(q_+)[S^{-1}(q_+)\gamma_5 + \gamma_5 S^{-1}(q_-)]S(q_-)\}_{\alpha'\beta'} = \int_q D_{\mu\nu}(k-q)\gamma_\mu[S(q_+)\Gamma_\nu(q_+,k_+)\gamma_5 - \gamma_5 S(q_-)\Gamma_\nu(q_-,k_-)].$$

III. Scattering kernel: Elements of quark gap equation

Assuming the scattering kernel has the following structure:

$$\begin{aligned}\mathcal{K}_{\alpha\alpha',\beta'\beta}(q_\pm,k_\pm)[S(q_+)\ \bigcirc\ S(q_-)]_{\alpha'\beta'} = & -D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\ \bigcirc\ S(q_-)\Gamma_\nu(q_-,k_-)\\ & +D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\ \bigcirc\ \mathcal{K}^+_\nu(q_\pm,k_\pm)\\ & +D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\ \gamma_5 \bigcirc \gamma_5\ \mathcal{K}^-_\nu(q_\pm,k_\pm),\end{aligned}$$

which has three terms including two unknown objects.



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$$+D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\bigcirc \mathcal{K}^+_\nu(q_\pm,k_\pm)$$
$$+D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\,\gamma_5\bigcirc\gamma_5\,\mathcal{K}^-_\nu(q_\pm,k_\pm),$$

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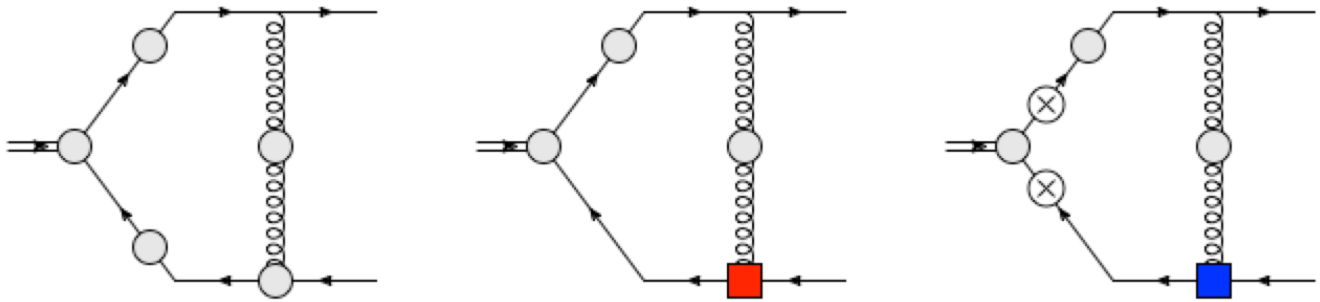

Γ_ν $\quad$ $\mathcal{K}^+_\nu$ $\quad$ $\mathcal{K}^-_\nu$

Ladder-like term

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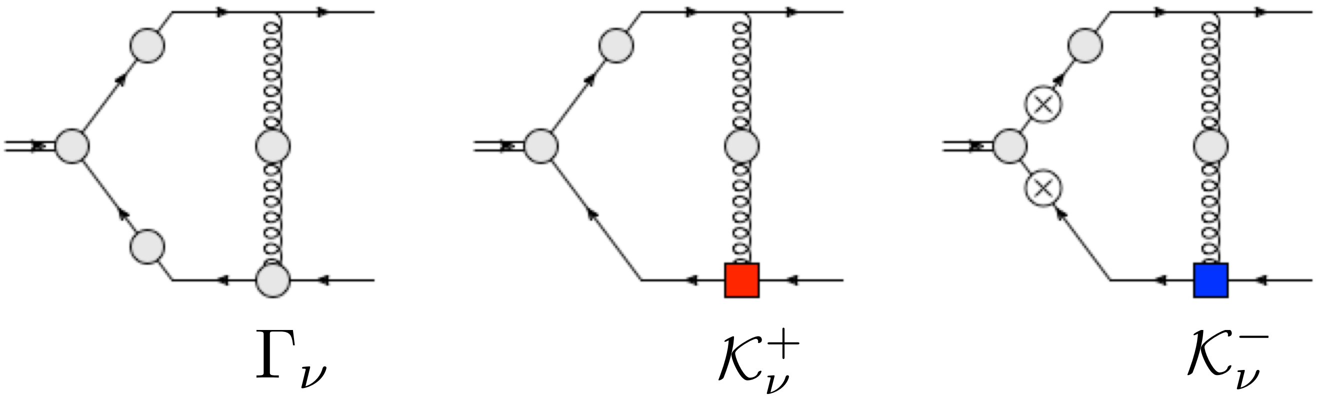

Ladder-like term

Symmetry-rescuing term

III. Scattering kernel: Elements of quark gap equation

Inserting the ansatz for the kernel into its WGTIs, we have

$$\int_q D_{\mu\nu}\gamma_\mu S_+(\Gamma_\nu^+ - \Gamma_\nu^-) = \int_q D_{\mu\nu}\gamma_\mu S_+(S_+^{-1} - S_-^{-1})\mathcal{K}_\nu^+ + \int_q D_{\mu\nu}\gamma_\mu S_+\gamma_5(S_+^{-1} - S_-^{-1})\gamma_5\mathcal{K}_\nu^-,$$

$$\int_q D_{\mu\nu}\gamma_\mu S_+(\Gamma_\nu^+\gamma_5 + \gamma_5\Gamma_\nu^-) = \int_q D_{\mu\nu}\gamma_\mu S_+(S_+^{-1}\gamma_5 + \gamma_5 S_-^{-1})\mathcal{K}_\nu^+ + \int_q D_{\mu\nu}\gamma_\mu S_+(\gamma_5 S_+^{-1} + S_-^{-1}\gamma_5)\mathcal{K}_\nu^-.$$

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Algebraic version of the WGTIs, which the scattering kernel satisfy, are written as

$$\begin{aligned}\Gamma^+_\nu - \Gamma^-_\nu &= (S_+^{-1} - S_-^{-1})\mathcal{K}^+_\nu + \gamma_5(S_+^{-1} - S_-^{-1})\gamma_5\mathcal{K}^-_\nu ,\\ \Gamma^+_\nu\gamma_5 + \gamma_5\Gamma^-_\nu &= (S_+^{-1}\gamma_5 + \gamma_5 S_-^{-1})\mathcal{K}^+_\nu + (\gamma_5 S_+^{-1} + S_-^{-1}\gamma_5)\mathcal{K}^-_\nu .\end{aligned}$$

III. Scattering kernel: Elements of quark gap equation

Inserting the ansatz for the kernel into its WGTIs, we have

$$\int_q D_{\mu\nu}\gamma_\mu S_+ (\Gamma_\nu^+ - \Gamma_\nu^-) = \int_q D_{\mu\nu}\gamma_\mu S_+ (S_+^{-1} - S_-^{-1})\mathcal{K}_\nu^+ + \int_q D_{\mu\nu}\gamma_\mu S_+ \gamma_5 (S_+^{-1} - S_-^{-1})\gamma_5 \mathcal{K}_\nu^-$$

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Eventually, the solution is straightforward:

$$\mathcal{K}_\nu^\pm = (2B_\Sigma A_\Delta)^{-1}[(A_\Delta \mp B_\Delta)\Gamma_\nu^\Sigma \pm B_\Sigma \Gamma_\nu^\Delta].$$

$$S(p) = \frac{1}{i\gamma \cdot p\, A(p^2) + B(p^2)}$$

$$\Gamma_\nu^\Sigma = \Gamma_\nu^+ + \gamma_5 \Gamma_\nu^+ \gamma_5 \qquad \Gamma_\nu^\Delta = \Gamma_\nu^+ - \Gamma_\nu^-$$

$$B_\Sigma = 2B_+ \qquad B_\Delta = B_+ - B_-$$

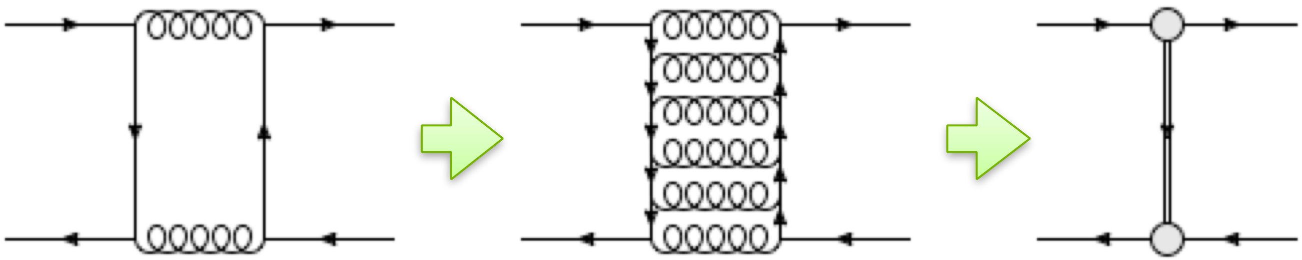
$$A_\Delta = i(\gamma \cdot q_+)A_+ - i(\gamma \cdot q_-)A_-$$

- ✦ The form of scattering kernel is simple.
- ✦ The kernel has no kinetic singularities.
- ✦ All channels share the same kernel.

III. Scattering kernel: Meson cloud

In Quantum Field theory (infinitely many degrees of freedom), high-order Green functions **cannot** completely truncated by low-order ones (unclosed).

For example, meson cloud, e.g., pion cloud, goes into the scattering kernel:



III. Scattering kernel: Meson cloud

The start point is the Bethe-Salpeter equation with meson cloud

$$\Gamma^H_{\alpha\beta}(k,P) = \gamma^H_{\alpha\beta} + \int_q \mathcal{K}(k_\pm, q_\pm)_{\alpha\alpha',\beta'\beta}[S(q_+)\Gamma^H(q,P)S(q_-)]_{\alpha'\beta'}.$$

The color-singlet axial-vector and vector WGTIs (|P| = 0) are written as

$$i\hat{P}_\mu \Gamma_\mu(k,0) = \hat{P}_\mu \frac{\partial S^{-1}(k)}{\partial k_\mu},$$
$$2m\Gamma_5(k,0) = S^{-1}(k)\gamma_5 + \gamma_5 S^{-1}(k),$$

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The Bethe-Salpeter kernel can modify the quark propagator as

$$\left[\hat{P}_\mu \frac{\partial S^{-1}(k)}{\partial k_\mu}\right]_{\alpha\beta} = [i\hat{\not{P}}]_{\alpha\beta} - \int_q \mathcal{K}(k,q)_{\alpha\alpha',\beta'\beta}\left[\hat{P}_\mu \frac{\partial S(q)}{\partial q_\mu}\right]_{\alpha'\beta'},$$

$$\left[S^{-1}(k)\gamma_5 + \gamma_5 S^{-1}(k)\right]_{\alpha\beta} = [2m\gamma_5]_{\alpha\beta} + \int_q \mathcal{K}(k,q)_{\alpha\alpha',\beta'\beta}\left[S(q)\gamma_5 + \gamma_5 S(q)\right]_{\alpha'\beta'},$$

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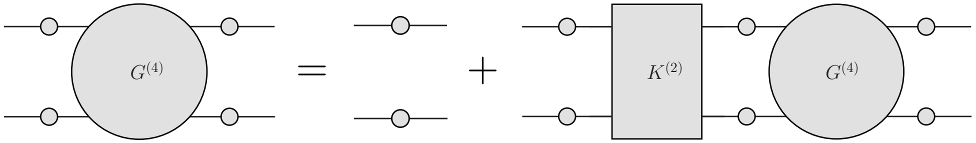
Using the quark dress functions, the new quark gap equation reads

$$\begin{cases} \dfrac{\partial |k| A(k^2)}{\partial |k|} = 1 + \dfrac{1}{4}\displaystyle\int_q \left[k_\mu^{\|}\right]_{\beta\alpha} \mathcal{K}_{\alpha\alpha',\beta'\beta}\left[\frac{\partial S(q)}{\partial q_\mu}\right]_{\alpha'\beta'}, \\ B(k^2) = m + \dfrac{1}{4}\displaystyle\int_q [\gamma_5]_{\beta\alpha}\mathcal{K}_{\alpha\alpha',\beta'\beta}\left[\gamma_5\sigma_B(q^2)\right]_{\alpha'\beta'}, \end{cases}$$

IV. Six-point Green function: The norm'n and current conservation

The Dyson-Schwinger equation of the **four-point Green function** is written as



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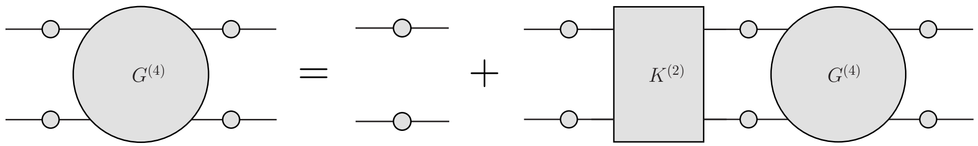
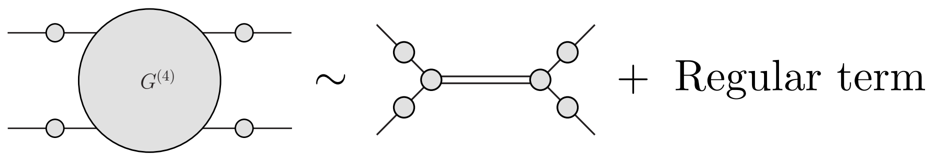


Assuming that there is a **bound state**



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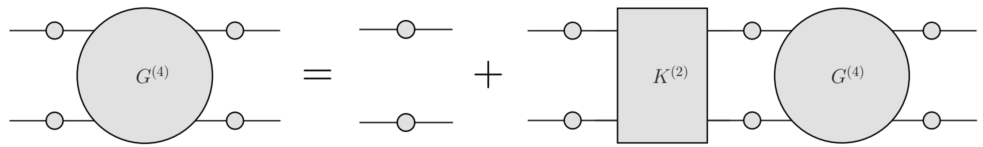

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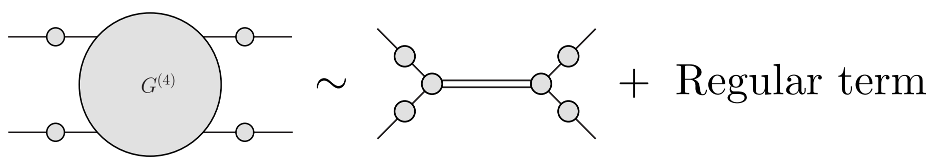
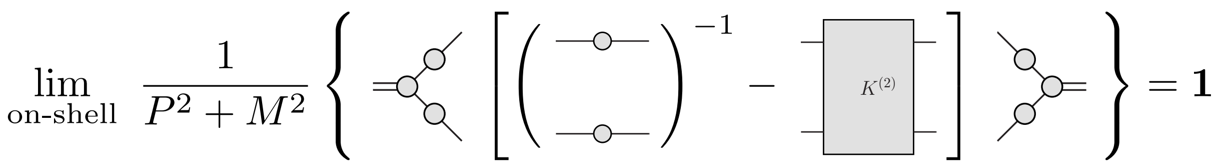
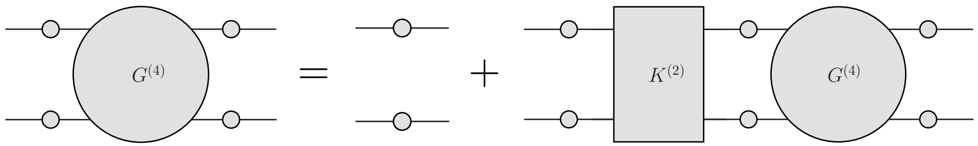


the **wave function** of the bound state has to satisfy the following condition

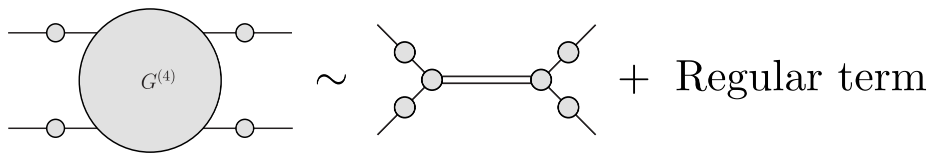


IV. Six-point Green function: The norm'n and current conservation

The Dyson-Schwinger equation of the **four-point Green function** is written as



Assuming that there is a **bound state**



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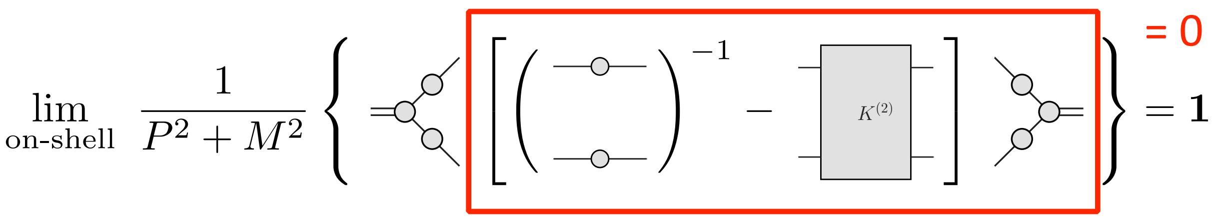

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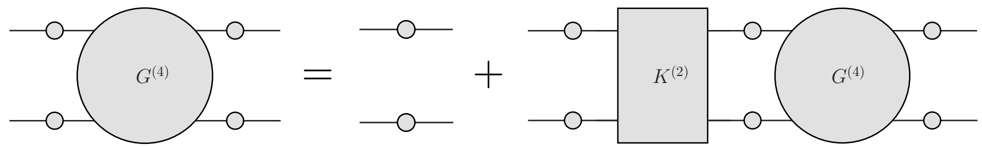

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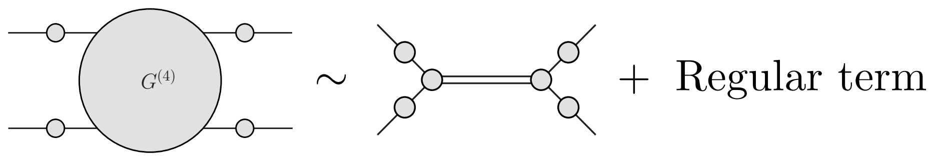

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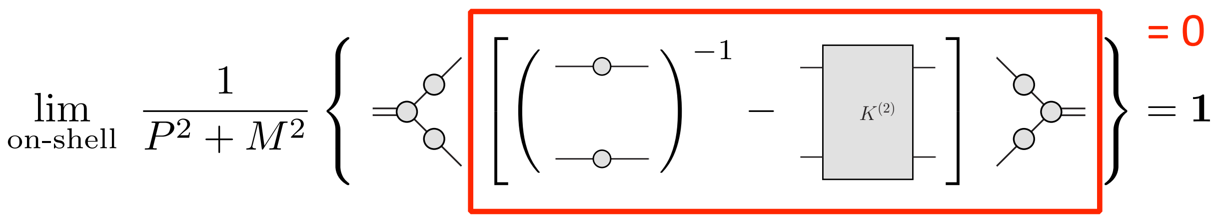
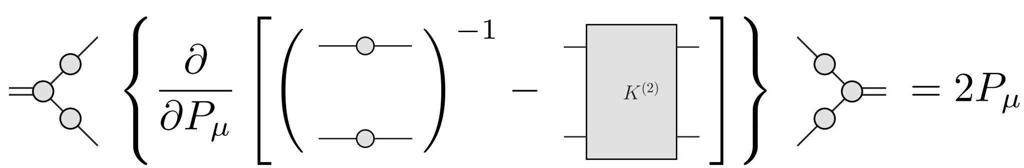
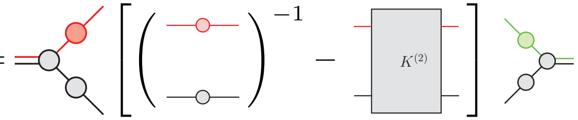


The **differential form** is obtained as

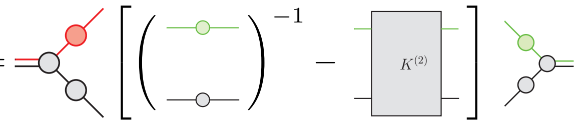


IV. Six-point Green function: The norm'n and current conservation

Introduce a function depending on ***(P, Q)***, i.e., $\mathcal{G}(P,Q) \equiv \mathcal{G}_+(P,Q) - \mathcal{G}_-(P,Q)$

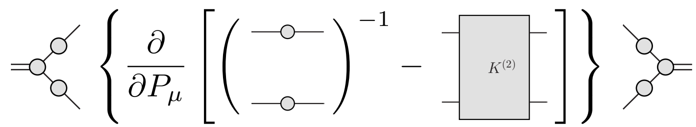
$\mathcal{G}_+(P,Q) =$ 

$q_+ + \frac{Q}{2}$

$\mathcal{G}_-(P,Q) =$ 

$q_+ - \frac{Q}{2}$

Then the function can reproduce the normalization condition as

$\lim_{Q\to 0} \frac{\mathcal{G}(P,Q)}{Q_\mu} =$  $= 2P_\mu$

Inserting the **color-singlet vector Ward identity** into the function,

$$Q_\mu \Gamma_\mu\left(q_+ + \frac{Q}{2}, q_+ - \frac{Q}{2}\right) = S^{-1}\left(q_+ + \frac{Q}{2}\right) - S^{-1}\left(q_+ - \frac{Q}{2}\right) \qquad \mathcal{G}(P,Q) = Q_\mu \Lambda_\mu(P,Q)$$

Eventually, the **form factor** can be defined as $\Lambda_\mu(P,Q) = 2P_\mu F(Q^2)$ with $F(Q^2 = 0) = 1$

IV. Six-point Green function: The norm'n and current conservation

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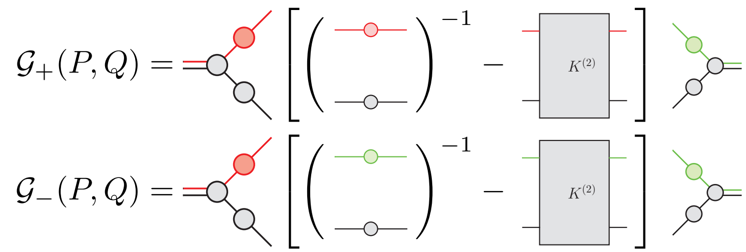

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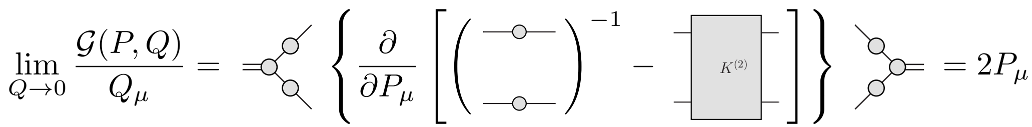

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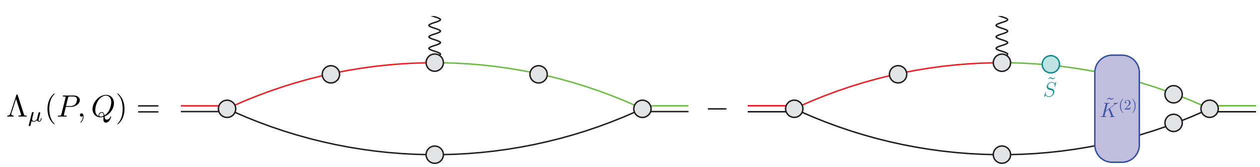


$$\tilde{S}\left(q_+ \pm \frac{Q}{2}\right) = \left[S^{-1}\left(q_+ + \frac{Q}{2}\right) - S^{-1}\left(q_+ - \frac{Q}{2}\right)\right]^{-1}$$

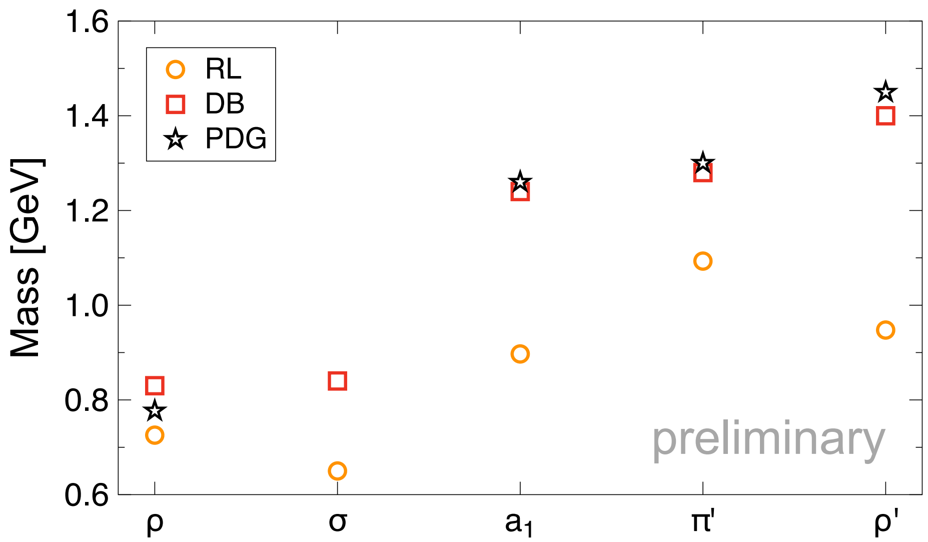
$$\tilde{K}\left(q_+ \pm \frac{Q}{2}, q_-, q'_+ \pm \frac{Q}{2}, q'_-\right) = K\left(q_+ + \frac{Q}{2}, q_-, q'_+ + \frac{Q}{2}, q'_-\right) - K\left(q_+ - \frac{Q}{2}, q_-, q'_+ - \frac{Q}{2}, q'_-\right)$$

Meson spectroscopy: From ground to radial excitation states

Let the quark-gluon vertex include both longitudinal and transverse parts:

$$\Gamma_\mu(p,q) = \Gamma^{\rm BC}_\mu(p,q) + \Gamma^{\rm T}_\mu(p,q) \qquad \Gamma^{\rm T}_\mu(p,q) = \eta\Delta_B\tau^5_\mu + \xi\Delta_B\tau^8_\mu + 4(\eta+\xi)\Delta_A\tau^4_\mu$$

$$\begin{aligned}\tau^4_\mu &= l^{\rm T}_\mu \gamma\cdot k + i\gamma^{\rm T}_\mu \sigma_{\nu\rho} l_\nu k_\rho,\\ \tau^5_\mu &= \sigma_{\mu\nu}k_\nu,\\ \tau^8_\mu &= 3\, l^{\rm T}_\mu \sigma_{\nu\rho} l_\nu k_\rho/(l^{\rm T}\cdot l^{\rm T}).\end{aligned}$$



The correct mass ordering:

$$m_{\rho'} > m_{\pi'} > m_{a_1} > m_\sigma > m_\rho > m_\pi$$

	$-\langle\bar{q}q\rangle_0^{1/3}$	f_π	m_σ	m_ρ	m_{a_1}	$m_{\pi'}$	$m_{\rho'}$
this work	0.220	0.092	0.84	0.83	1.24	1.28	1.40
PDG	-	0.093	0.50	0.78	1.26	1.30	1.45

TABLE I: The fitted spectrum and its comparison with PDG data (Full vertex, $(D\omega)^{1/3} = 0.484$ GeV, $\omega = 0.55$ GeV, $\eta = 0.5$ and $\xi = 1.15$, in the chiral limit where pion is always massless).

Summary

✦ Based on LQCD and WGTIs, a **systematic** and **self-consistent** method to construct **the gluon propagator, the quark-gluon vertex, the scattering kernel,** and **the form factor** beyond the simplest approximation is proposed.

✦ A **demonstration** applying the method to light meson spectroscopy, including **ground** and **radially excited states**, is presented: The new method is powerful.

Outlook

◆ With the **sophisticated** method to solve the DSEs, we can push the DSE approach to a much wider range of applications in **hadron physics**, e.g., baryon in diquark picture.

◆ Hopefully, after more and more successful applications are presented, the DSE approach may provide a **path** to understand **QCD**.

Backups

Sketching scattering kernel: with elements of quark gap equation

Rearranging the scattering kernel as the left- and right-hand forms

$$\begin{aligned}\mathcal{K}_{\alpha\alpha',\beta'\beta}(q_\pm,k_\pm)[S(q_+)\ \bigcirc\ S(q_-)]_{\alpha'\beta'} =& - D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\ \bigcirc\ S(q_-)\Gamma_\nu(q_-,k_-)\\ &+ D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\boxed{\tfrac{1}{2}(\bigcirc + \gamma_5 \bigcirc \gamma_5)}\mathcal{K}^L_\nu(q_\pm,k_\pm)\\ &+ D_{\mu\nu}(k-q)\gamma_\mu S(q_+)\boxed{\tfrac{1}{2}(\bigcirc - \gamma_5 \bigcirc \gamma_5)}\mathcal{K}^R_\nu(q_\pm,k_\pm),\end{aligned}$$

we have the solution as

$$\boxed{\begin{aligned}\mathcal{K}^L_\nu &= B_\Sigma^{-1}\Gamma^\Sigma_\nu,\\ \mathcal{K}^R_\nu &= (B_\Sigma A_\Delta)^{-1}(B_\Sigma\Gamma^\Delta_\nu - B_\Delta\Gamma^\Sigma_\nu).\end{aligned}}$$

For a given Dirac structure, only one of K^L and K^R can survive, e.g.,

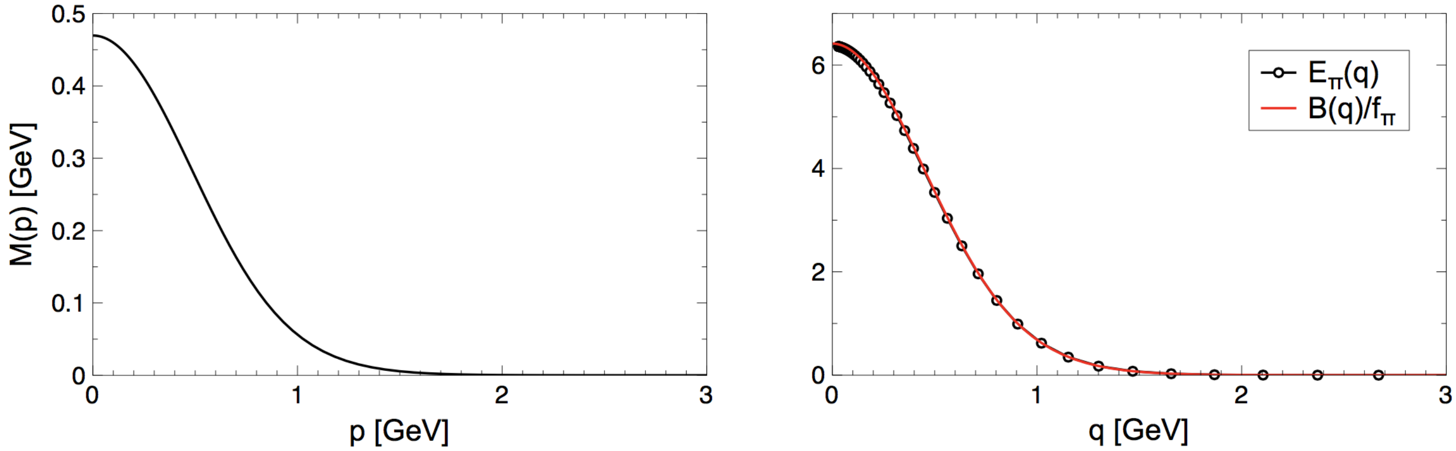
$\bigcirc = \gamma_\mu$	$\gamma_5 \bigcirc \gamma_5 = -\bigcirc$	K^R
$\bigcirc = \gamma_5$	$\gamma_5 \bigcirc \gamma_5 = \bigcirc$	K^L

Meson spectroscopy: From ground to radial excitation states

Let the quark-gluon vertex includes both longitudinal and transverse parts:

$$\Gamma_\mu(p,q) = \Gamma_\mu^{\rm BC}(p,q) + \Gamma_\mu^{\rm T}(p,q)$$

- The longitudinal part is the Ball-Chiu vertex—an exact piece from symmetries.
- The transverse part is the Anomalous Chromomagnetic Moment (ACM) vertex.



To generate the quark mass scale which is comparable to that of LQCD, the coupling strength can be so small that the Rainbow-ladder approximation has NO DCSB at all.