Amplitude analyses at JPAC and their prospects for γNN^*

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ECT*, Trento, October 12-16, 2015

PRiSMA

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

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CLAS collaboration
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Viktor Mokeev (JLab)
HASPECT
Marco Battaglieri (Genova)
Derek Glazier (Glasgow)
…

GlueX collaboration
Matthew Shepherd (IU)
Justin Stevens (JLab)
…

COMPASS collaboration
Mikhail Mikhasenko (Bonn)
Fabian Krinner (TUM)
Boris Grube (TUM)
…

Joint Physics Analysis Center (JPAC)

Projects

$$\eta, \omega, \phi \to 3\pi$$
$$\omega, \phi \to \pi\gamma^*$$
$$J/\psi\,(\psi') \to 3\pi$$

$$\pi N \to \pi N$$
$$\pi N \to \eta N$$
$$KN \to KN$$
$$\gamma N \to \pi N$$

$$\gamma p \to K^+K^- p$$
$$\gamma p \to \pi^0 \eta p$$
$$\pi^- p \to \pi^- \eta p$$

$$XYZ, etc.$$

Formalisms

Regge Theory
Dispersive Relations
& Unitarity
Dual Models
Isobar Models
…

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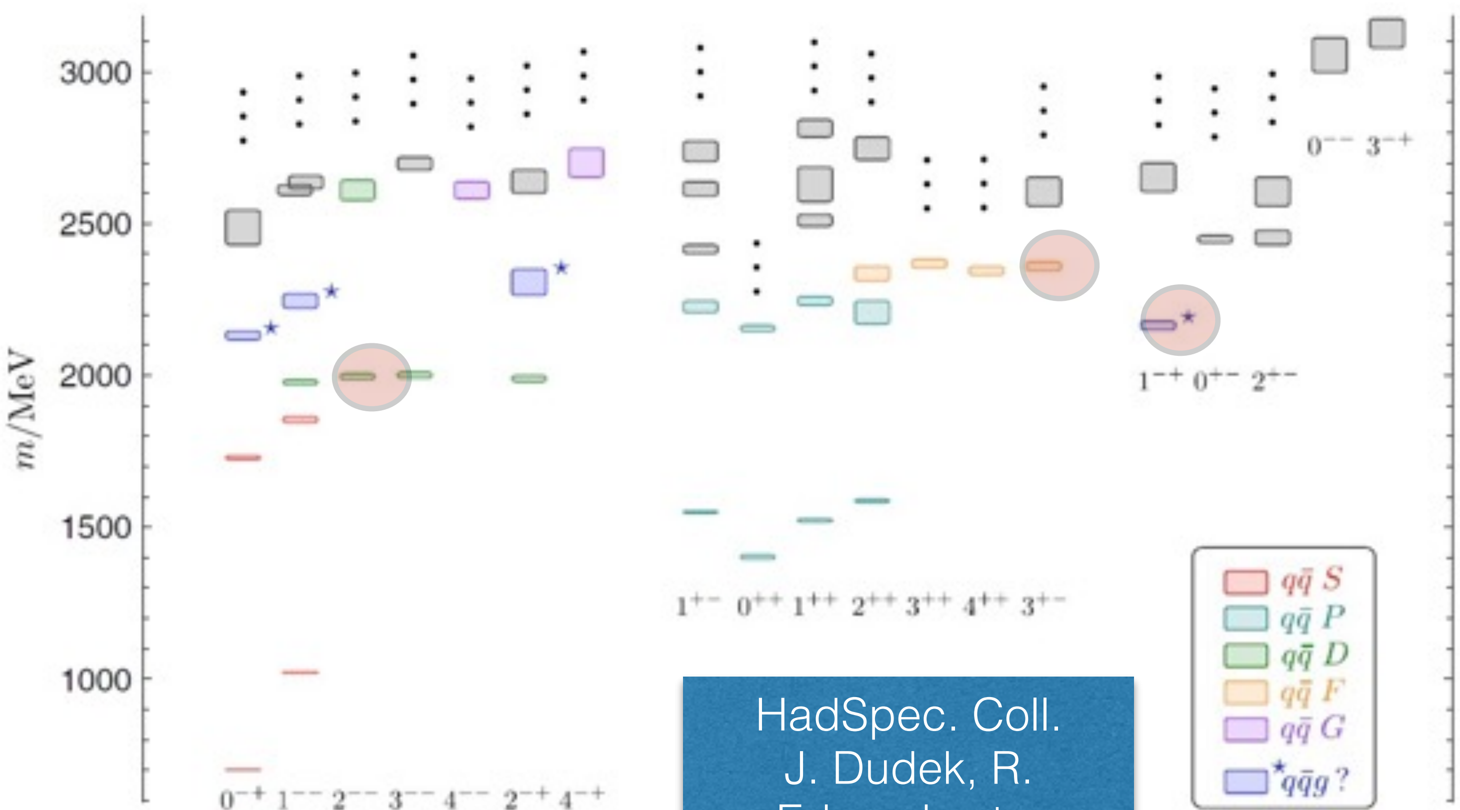
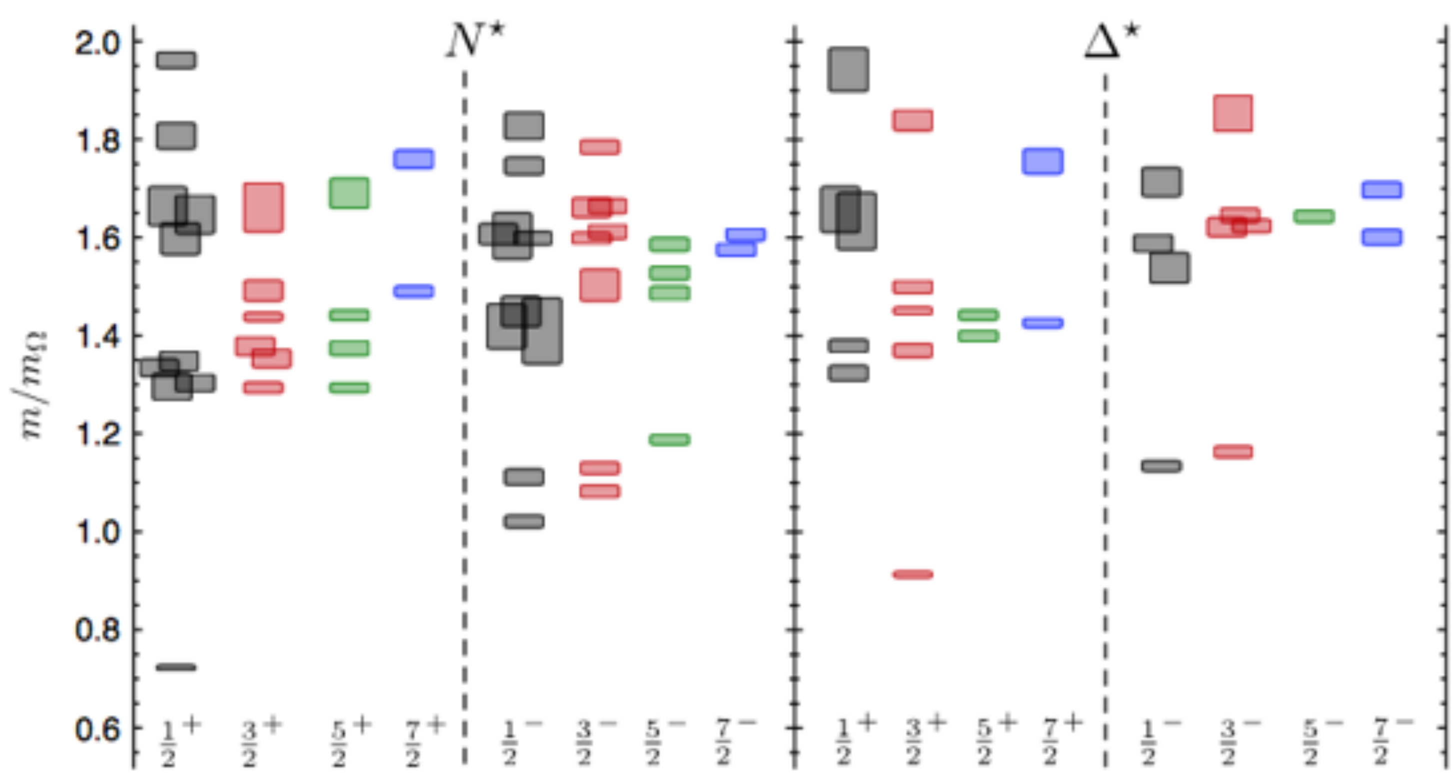
- First principle constraints
- Current projects
- Prospects for γNN*
- Summary

Hadron Spectroscopy



HadSpec. Coll.
J. Dudek, R.
Edwards etc.

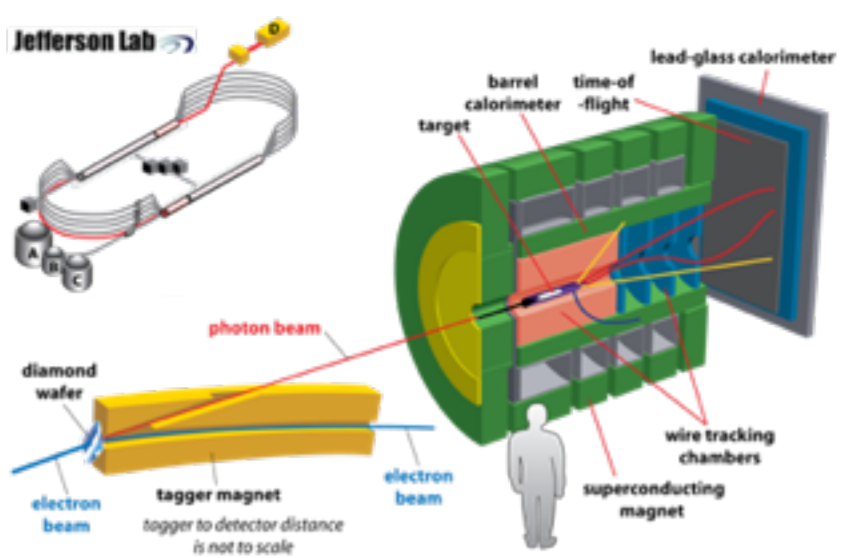
$n^{2s+1}\ell_J$	J^{PC}	$I=1$	$I=1/2$	$I=0$	$I=0$	EXD
1^1S_0	0^{-+}	π	K	η	η'	$R2$
1^3S_1	1^{--}	$\rho(770)$	$K^*(982)$	$\omega(782)$	$\phi(1020)$	$R1$
1^1P_1	1^{+-}	$b_1(1235)$	$K_1(1400)$	$h_1(1170)$	$h_1(1380)$	$R2$
1^3P_0	0^{++}	$a_0(1450)$	$K_0^*(1430)$	$f_0(1370)$	$f_0(1710)$	$R4$
1^3P_1	1^{++}	$a_1(1260)$	$K_1(1270)$	$f_1(1285)$	$f_1(1420)$	$R3$
1^3P_2	2^{++}	$a_2(1320)$	$K_2^{**}(1430)$	$f_2(1270)$	$f_2'(1525)$	$R1$
1^1D_2	2^{-+}	$\pi_2(1670)$	$K_2(1770)$	$\eta_2(1645)$	$\eta_2(1870)$	$R2$
1^3D_1	1^{--}	$\rho(1700)$	$K^*(1680)$	$\omega(1650)$		$R4$
1^3D_2	2^{--}		$K_2^*(1820)$			$R3$
1^3D_3	3^{--}	$\rho_3(1690)$	$K_3^*(1780)$	$\omega_3(1670)$	$\phi_3(1850)$	$R1$
1^1F_3	3^{+-}					$R2$
1^3F_2	2^{++}		$K_2^*(1980)$	$f_2(1910)$	$f_2(2010)$	$R4$
1^3F_3	3^{++}		$K_3(2320)$			$R3$
1^3F_4	4^{++}	$a_4(2040)$	$K_4^{**}(2045)$		$f_4(2050)$	$R1$

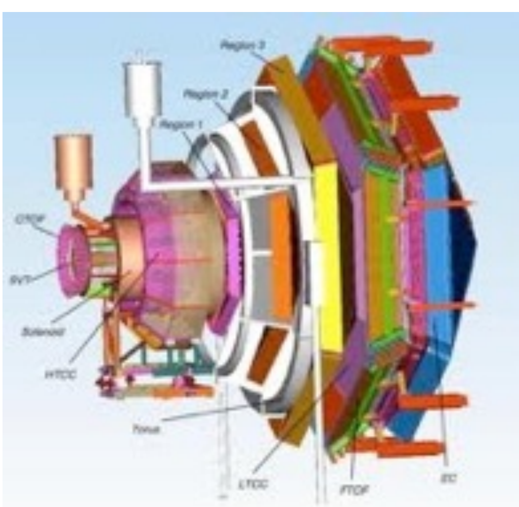


J^P	M_{CQM}	M_{PDG}	Rating	J^P	M_{CQM}	M_{PDG}	Rating
$1/2^-$	1460	1535	****	$1/2^+$	1540	1440	****
$1/2^-$	1535	1650	****	$1/2^+$	1770	1710	***
$1/2^-$	1945	2090	*	$1/2^+$	1880		
$1/2^-$	2030			$1/2^+$	1975		
$1/2^-$	2070			$1/2^+$	2065	2100	*
$1/2^-$	2145			$1/2^+$	2210		
$1/2^-$	2195						
$3/2^-$	1495	1520	****	$3/2^+$	1795	1720	****
$3/2^-$	1625	1700	***	$3/2^+$	1870		
$3/2^-$	1960	2080	**	$3/2^+$	1910		
$3/2^-$	2055			$3/2^+$	1950		
$3/2^-$	2095			$3/2^+$	2030		
$3/2^-$	2165						
$3/2^-$	2180						
$5/2^-$	1630	1675	****	$5/2^+$	1770	1680	****
$5/2^-$	2080			$5/2^+$	1980	2000	**
$5/2^-$	2095	2200	**	$5/2^+$	1995		
$5/2^-$	2180						
$5/2^-$	2235						
$5/2^-$	2260						
$5/2^-$	2295						
$5/2^-$	2305						

PDG & Quark model
Capstic, …

Hadron Spectroscopy



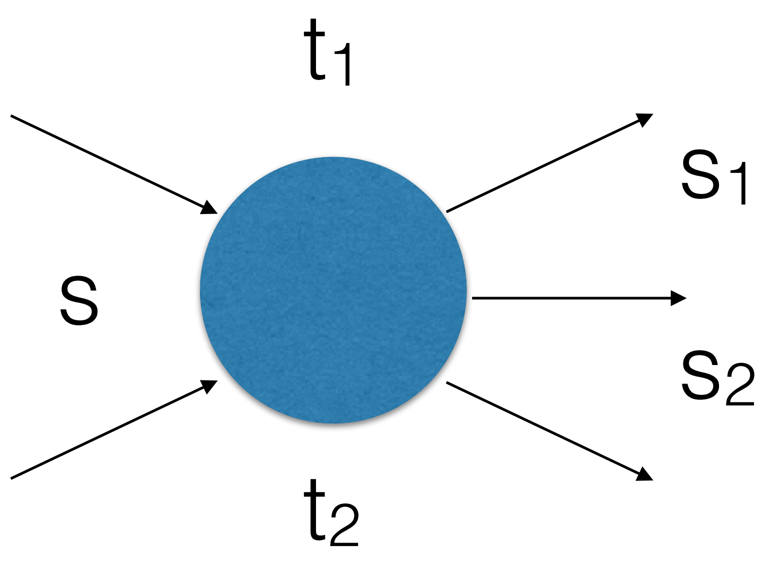


CLAS, GlueX,
MAMI, ELSA,
COMPASS,
BES, LHCb, PANDA,…

Aim to:
Complete understanding of the hadron spectrum and
discover new resonances

JPAC:
Provide theoretical support needed to analyse the data

Amplitude analysis

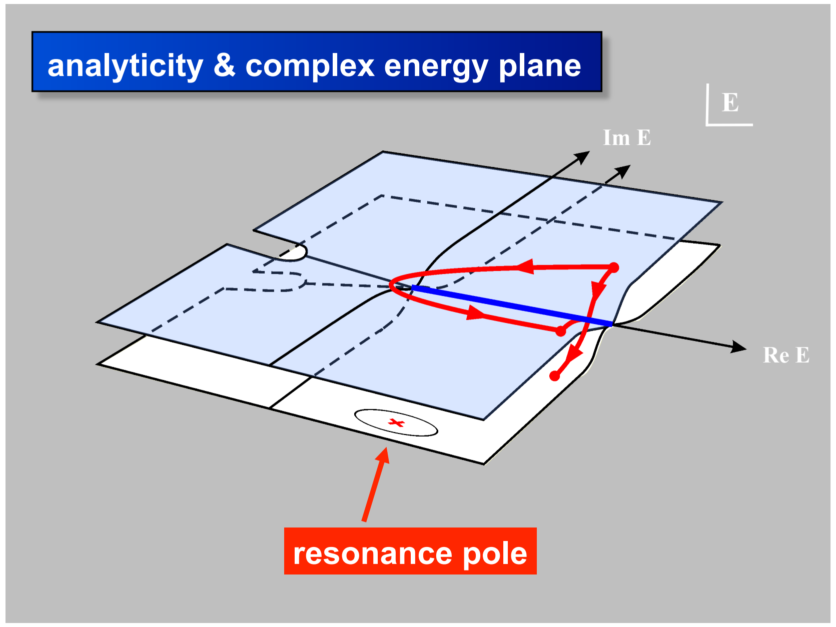


$$d^5\sigma \sim |A(s, s_1, s_2, t_1, t_2, \{\lambda\}|^2$$

$\gamma N \to (MM)N,\ \pi N \to (MM)N,\ etc.$

- Physics of interest (e.g. resonance poles …) resides in *A* evaluated at values of kinematical variables outside the experimentally accessible region

- In **Amplitude analysis** a model of *A* is constructed (based on phys. constraints), fitted to data and continued to regions of interest



Collaborative efforts

Theory

```
double complex function A(gamma,target,recoil,pip,pim,
                          ,lambda_g,lambda_t,lambda_r,
                          params)
implicit double precision (a-h,o-z)
dimension gamma(4)
dimension target(4)
dimension recoil(4)
dimension pip(4),pim(4)
dimension params(100)

 double complex Ampl

  s = (gamma(4)+target(4))**2 - (gamma(1)+target(1))**2
$   - (gamma(2)+target(2))**2 - (gamma(3)+target(3))**2

  s1 = (pip(4)+pim(4))**2 - (pip(1)+pim(1))**2
$    - (pip(2)+pim(2))**2 - (pip(3)+pim(3))**2

  s2 = (pip(4)+recoil(4))**2 - (pip(1)+recoil(1))**2
$    - (pip(2)+recoil(2))**2 - (pip(3)+recoil(3))**2

  t1 = (gamma(4)-pim(4))**2 - (gamma(1)-pim(1))**2
$    - (gamma(2)-pim(2))**2 - (gamma(3)-pim(3))**2

  t2 = (target(4)-recoil(4))**2 - (target(1)-recoil(1))**2
$    - (target(2)-recoil(2))**2 - (target(3)-recoil(3))**2

  call Ath(s,s1,s2,t1,t2,lambda_g,lambda_t,lambda_r,params,Ampl)

  A = Ampl

  return
  end
```

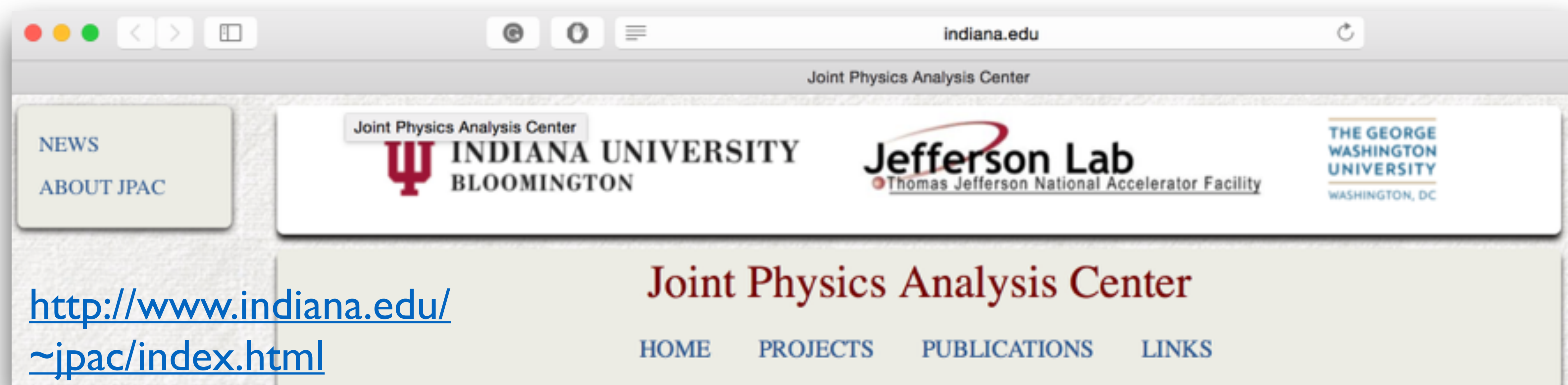
AmpTools
Brought to you by: mashephe

Experiment

Name	Last modified	Size	Description
Parent Directory		-	
g12_data_EBin26_95.txt.gz	05-Aug-2014 08:27	349M	
g12_mc_gen.txt.gz	05-Aug-2014 12:13	385M	
g12_mc_rec.txt.gz	05-Aug-2014 12:13	82M	

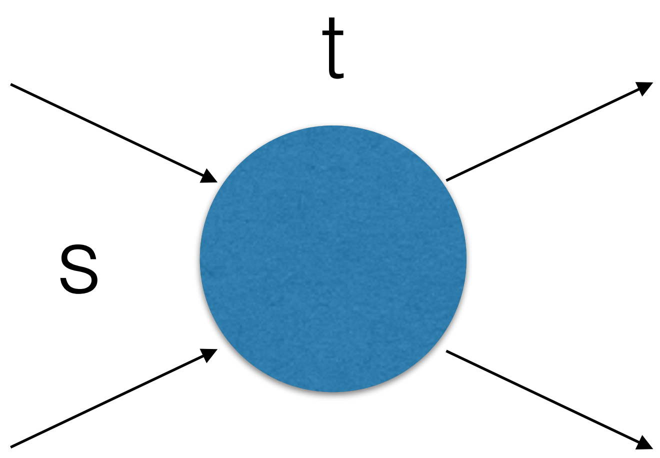
iterative procedure

Amplitudes are
1. fitted on data
2. checked constrains (proba. cons, causality, CPT inv.)
3. continued on sheet II



http://www.indiana.edu/~jpac/index.html

Amplitude analysis vs p.w. Amplitude analysis



$$A(s,t,\{\lambda\}) = \sum_J^{\infty} (2J+1)\, d^J_{\mu,\nu}(\theta_s)\, f^J(s,\{\lambda\})$$

$$\mu = \lambda_1 - \lambda_2, \quad \nu = \bar{\lambda}_1 - \bar{\lambda}_2$$

A(s,t,{λ}): amplitude expressed in terms of kinematical variables

↓

Enter comparison with data

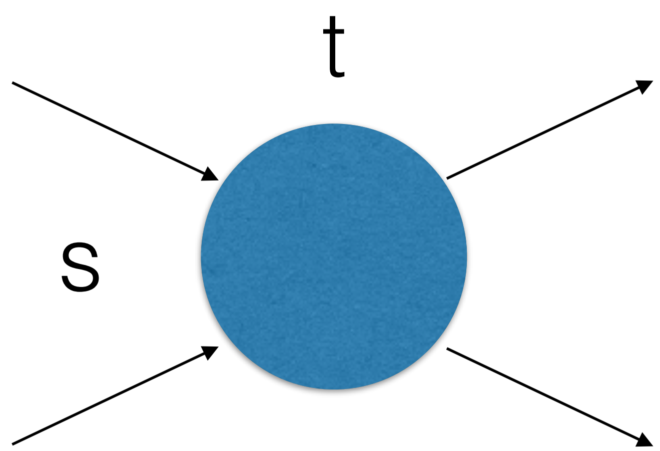
Partial Wave Amplitudes: decomposition in terms of rotational functions

↓

These "diagonalize unitarity" and contain resonance information

Entire dynamical information that does not depend on the underlying theory (e.g. QCD) comes from **unitarity**

Unitarity defines singularities of partial waves

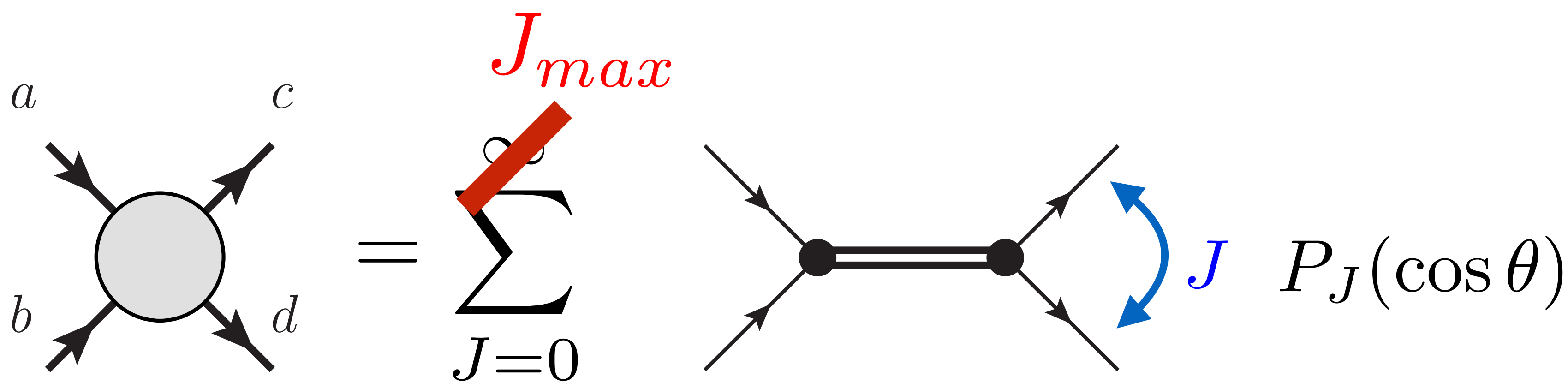


$$A(s,t) = \sum_{J=0}^{\infty} (2J+1)\, P_J(\cos\theta)\, f_J(s)$$

$$\text{Disc}\, f_J(s) = \rho(s)\, f_J(s+)\, f_J(s-)$$

for small s

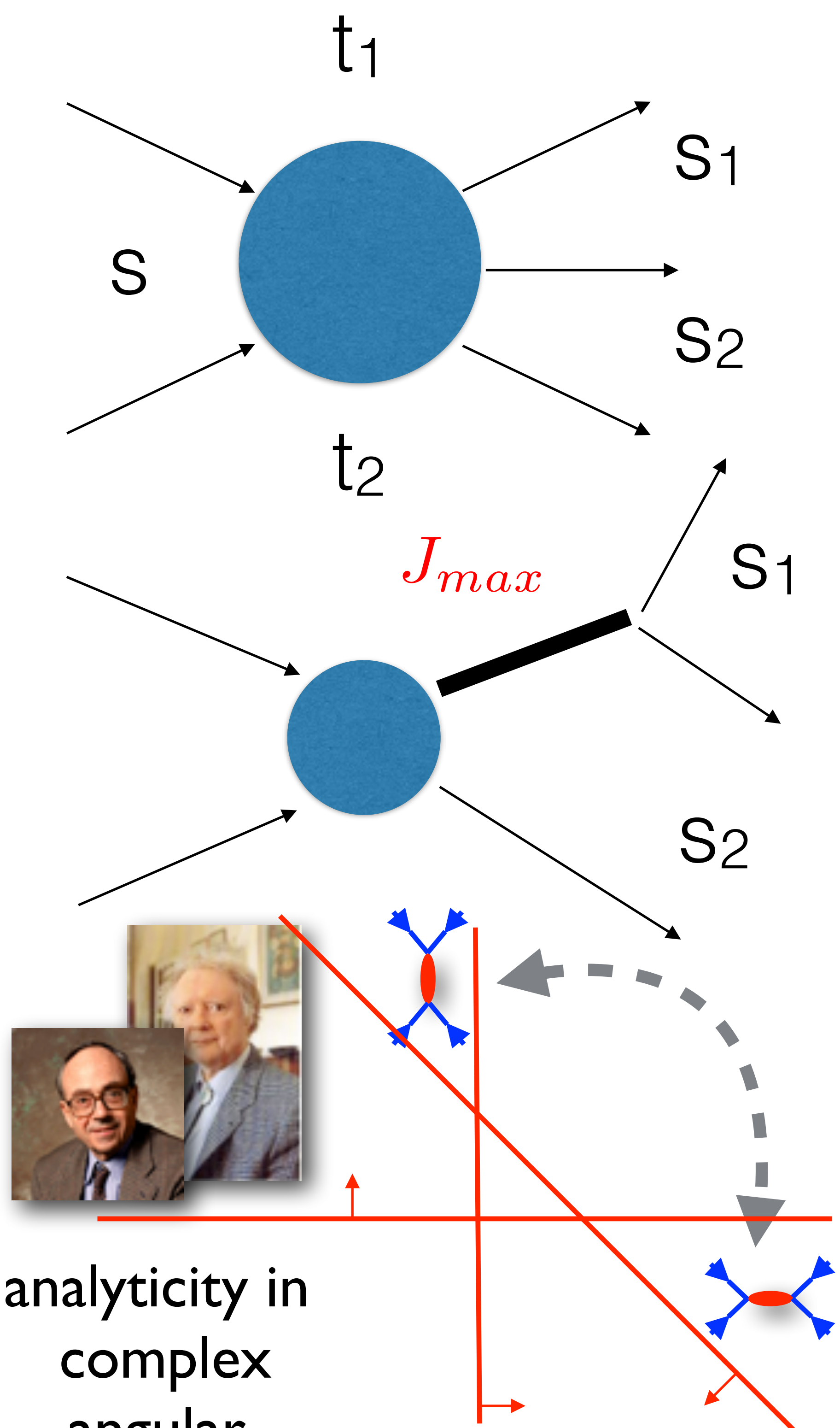
- Isobar model = truncate the partial waves: **Isobars = partial waves**



When is this a bad thing to do?

- For large-s, s-channel unitarity is hopeless. It is the low-*J* t-channel p.w. which become relevant (Regge physics).

Truncated partial wave series

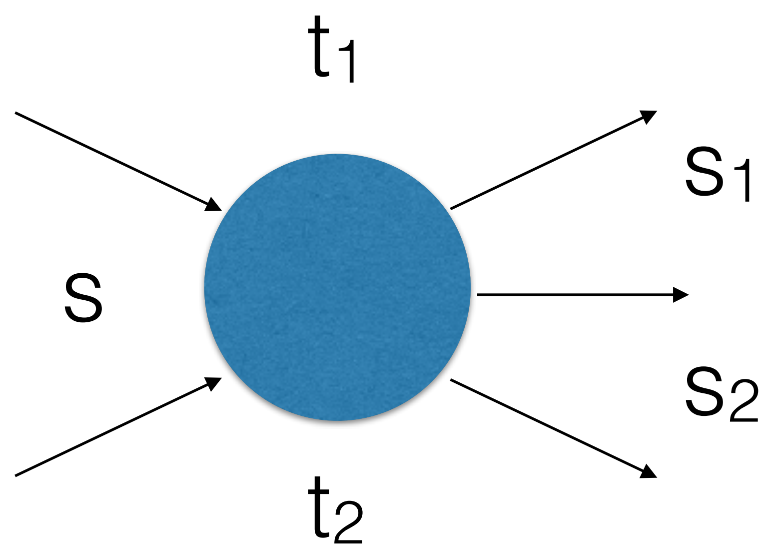


analyticity in complex angular momentum

$$A = \sum_{J_1, J_2, \lambda} d^{J_2}_{\lambda_b - \lambda_t, \lambda - \lambda_r}(\theta_2)\, d^{J_1}_{\lambda 0}(\theta_1)\, e^{i\lambda\phi_1} f_{J_1, J_2, \lambda}(s_1, s)$$

- Suppose the s_1 series is truncated $\sum_{J_1}^{\infty} \to \sum_{J_1}^{J_{max}}$
- Then $A \sim s_2^{J_{max}}$ becomes "wild" for high energies
- The correct behaviour $A \sim s_2^{\alpha} < s_2$ can only emerge if $J_{max} = \infty$
- The "machinery" to account for the contribution to infinite number of terms from cross-channel exchanges is due to Regge and Mandelstam

Isobar model



$$A = \sum_{J_1, J_2, \lambda} d^{J_2}_{\lambda_b - \lambda_t, \lambda - \lambda_r}(\theta_2)\, d^{J_1}_{\lambda 0}(\theta_1)\, e^{i\lambda\phi_1}\, f_{J_1, J_2, \lambda}(s_1, s)$$

If all s_1, s_2, s are small it is OK to truncate

$\eta,\omega,\varphi\to 3\pi$

- Truncate p.w. series

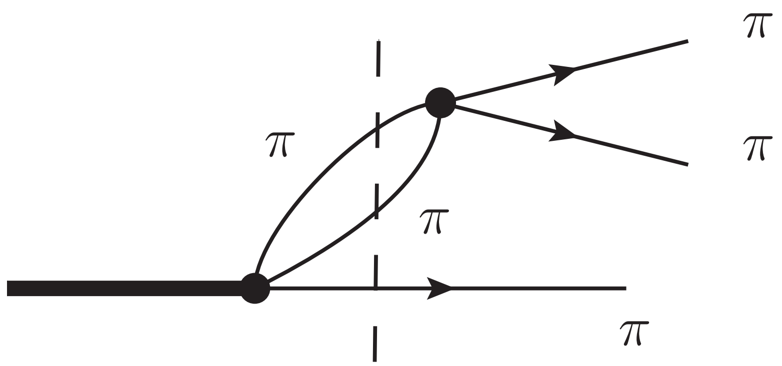
$$A(s,t) = \sum_{J=0}^{\infty \to J_{max}} (2J+1)\, P_J(\cos\theta)\, f_J(s)$$

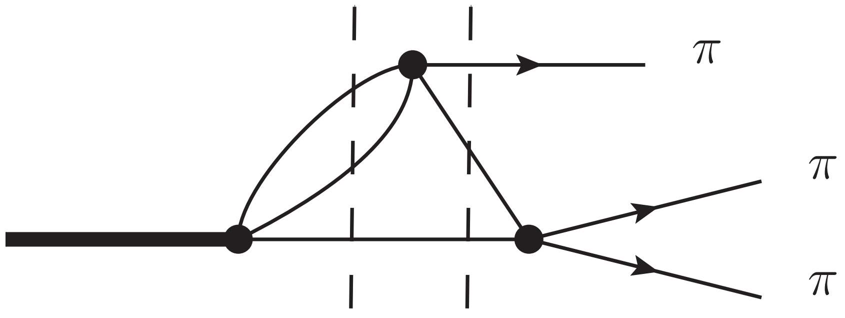
- Reconstruction theorem:
crossing symmetry, analyticity up to NNLO

$$A(s,t,u) = \sum_J^{J_{max}} ...f_J(s) + \sum_J^{J_{max}} ...f_J(t) + \sum_J^{J_{max}} ...f_J(u)$$

ππ scattering
Fuchs, Sazdjian, Stern (1993)

- Unitarity



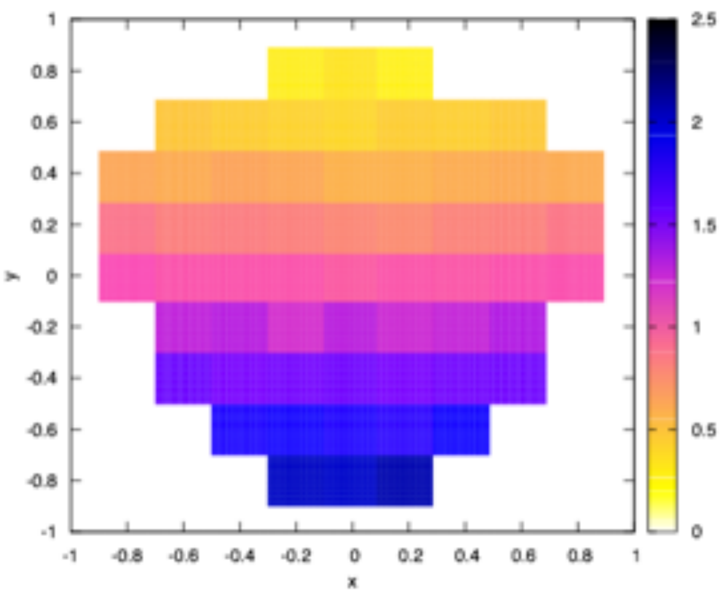


Khuri, Treiman (1960)
Aitchison (1977)

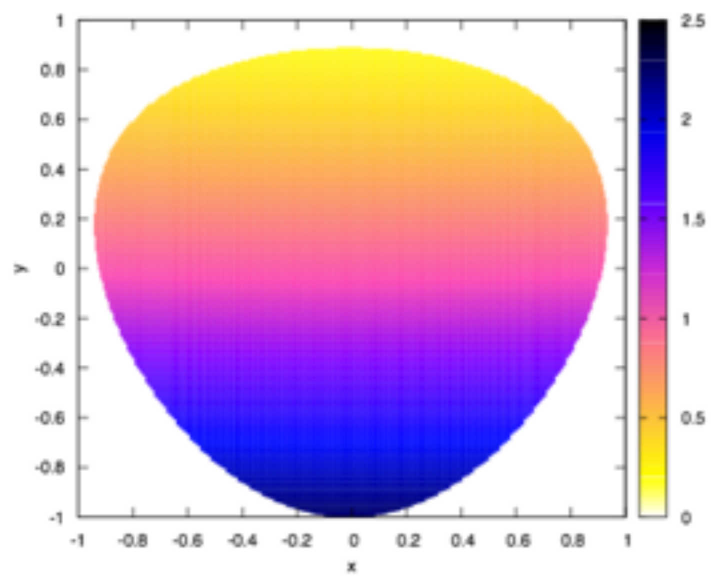
$\eta \to \pi^+ \pi^- \pi^0$

$\eta, \pi \sim$ 

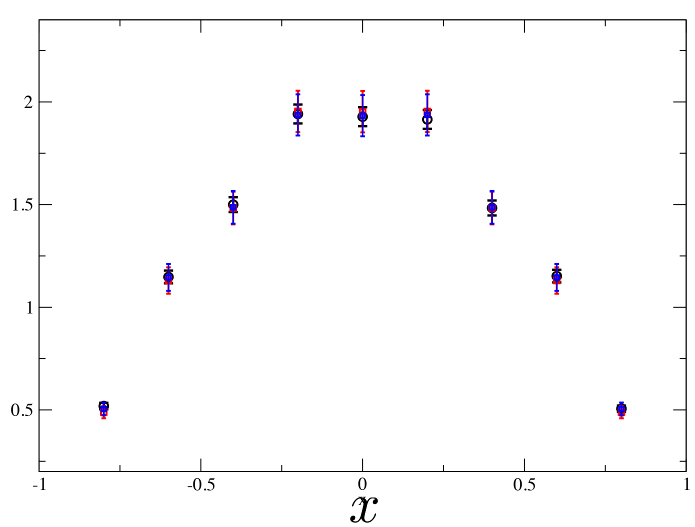
Isospin violating decay: sensitive to quark mass difference

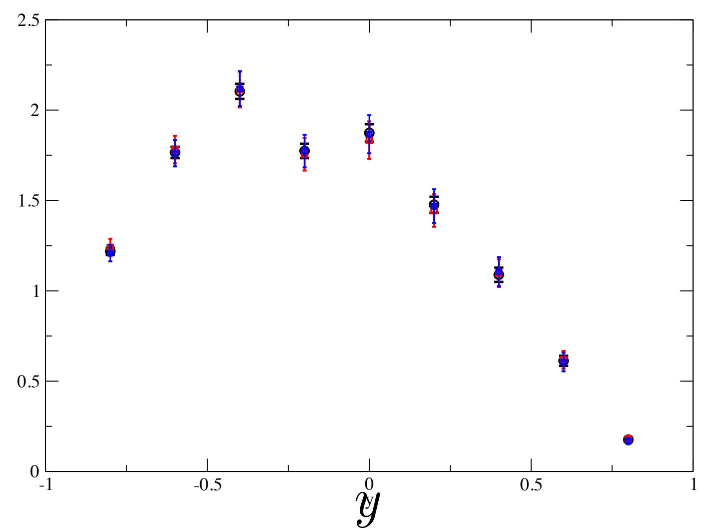


WASA-at-COSY
PRC90 4 045207
1.2×10^7 decays



P. Guo et al (JPAC)
PRD92 5 054016





Case 1:
(L,I)=(0,0), (1,1) - 1 real par.

Case 2:
(L,I)=(0,0), (0,2), (1,1) 2 real par.

$\chi^2/d.o.f.$	*Case1*	*Case2*
no 3b effects	*1,45*	*0,94*
with 3b effects	*0,96*	*0,9*

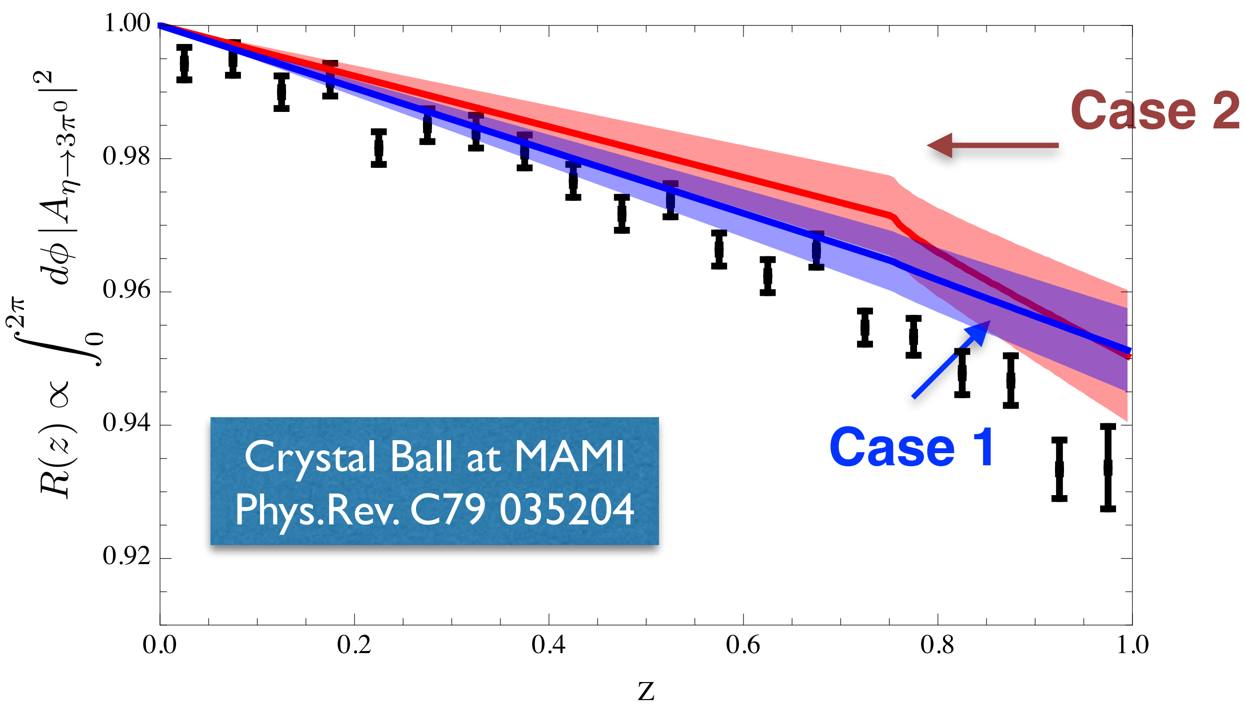
$\eta \to 3\pi^0$

Dalitz plot expansion:

$$|A_{\eta\to 3\pi^0}|^2 \propto 1 + 2\alpha z + 2\beta z^{3/2} \sin\phi$$

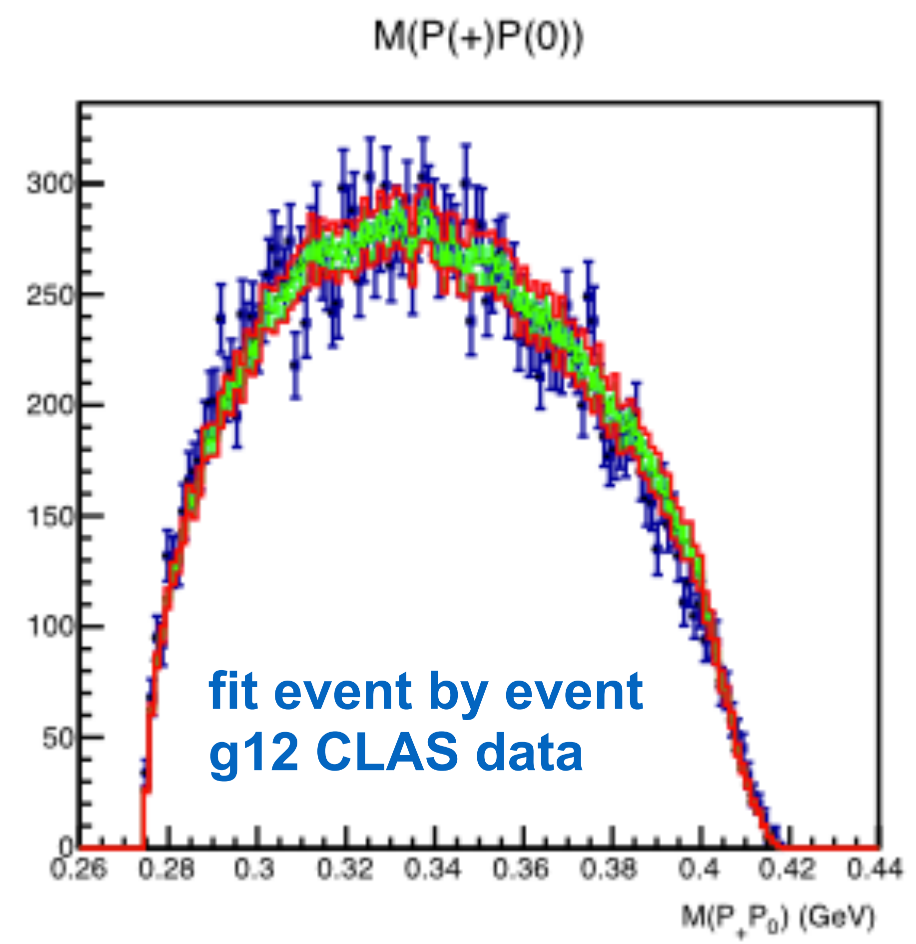
Quark mass double ratio:

$$Q^2 = \frac{m_s^2 - (m_u + m_d)^2/4}{m_d^2 - m_u^2}$$

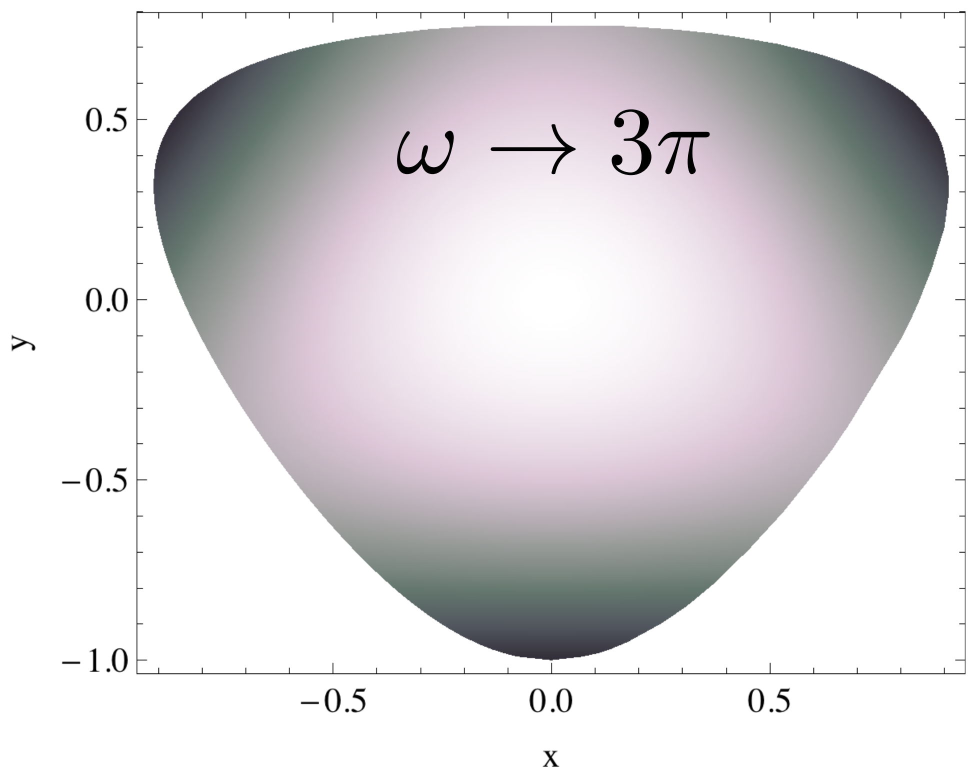


Predictions

WASA@COSY	$\alpha = -0.022 \pm 0.004$ $Q = 21.4 \pm 0.4$
CLAS@CEBAF	**in preparation**
KLOE@DAPHNE	**in preparation**



$\omega,\varphi \to 3\pi$

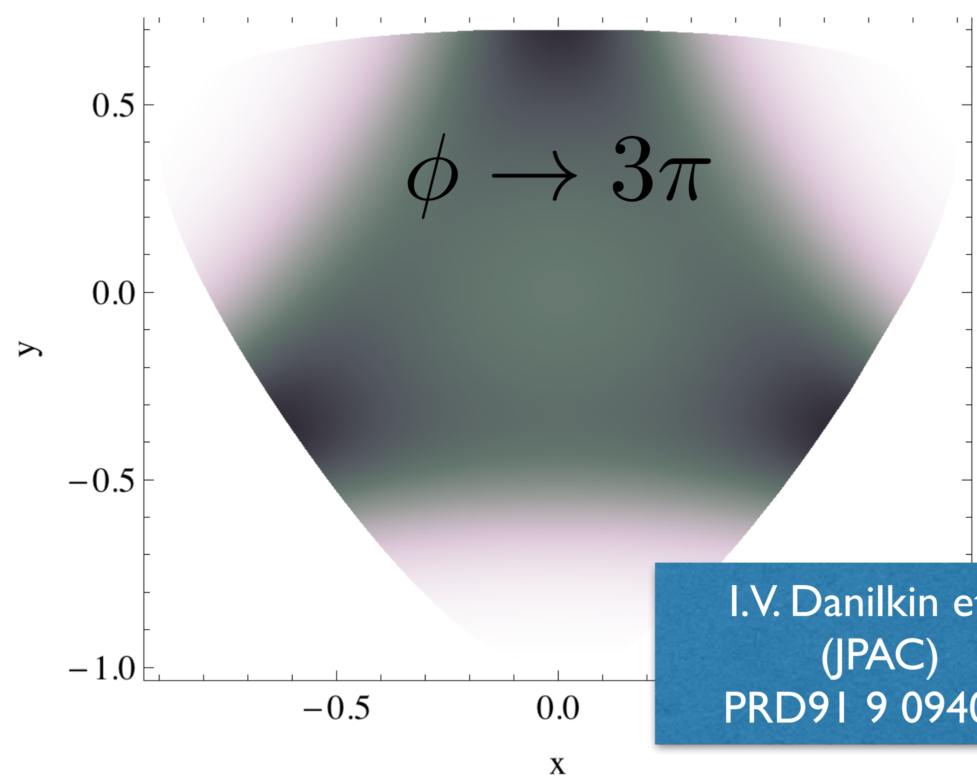


$$\frac{d^2\Gamma}{ds\,dt} \propto |\vec{p}_+ \times \vec{p}_-|^2 |F(s,t)|^2$$

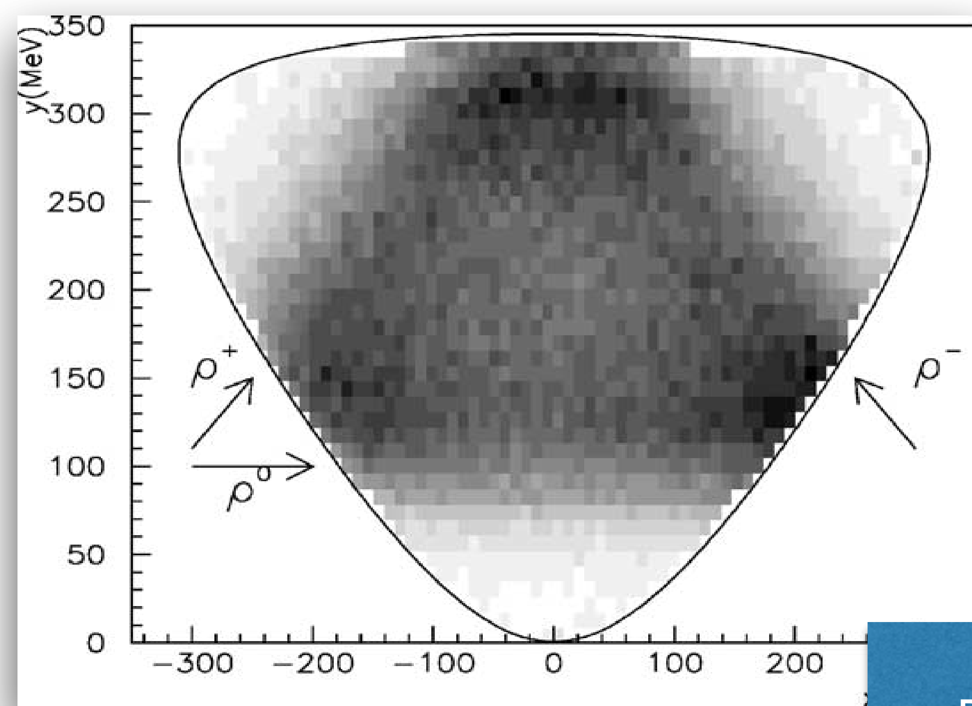
ω→3π: fit event by event g12 CLAS data in progress

Carlos Salgado, Volker Crede, Chris Zeoli, etc.

φ→3π: $\chi^2/d.o.f.$ = 1.11 (no 3b)
= 1.09 (with 3b)

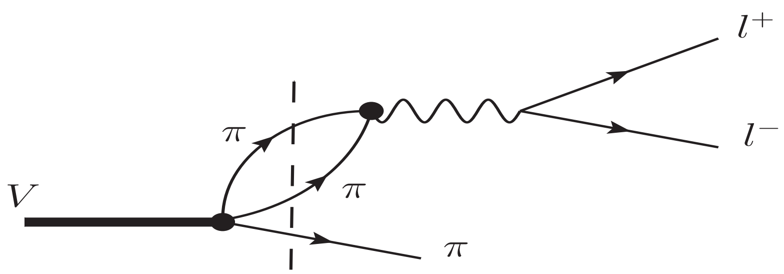


I.V. Danilkin et al (JPAC) PRD91 9 094029

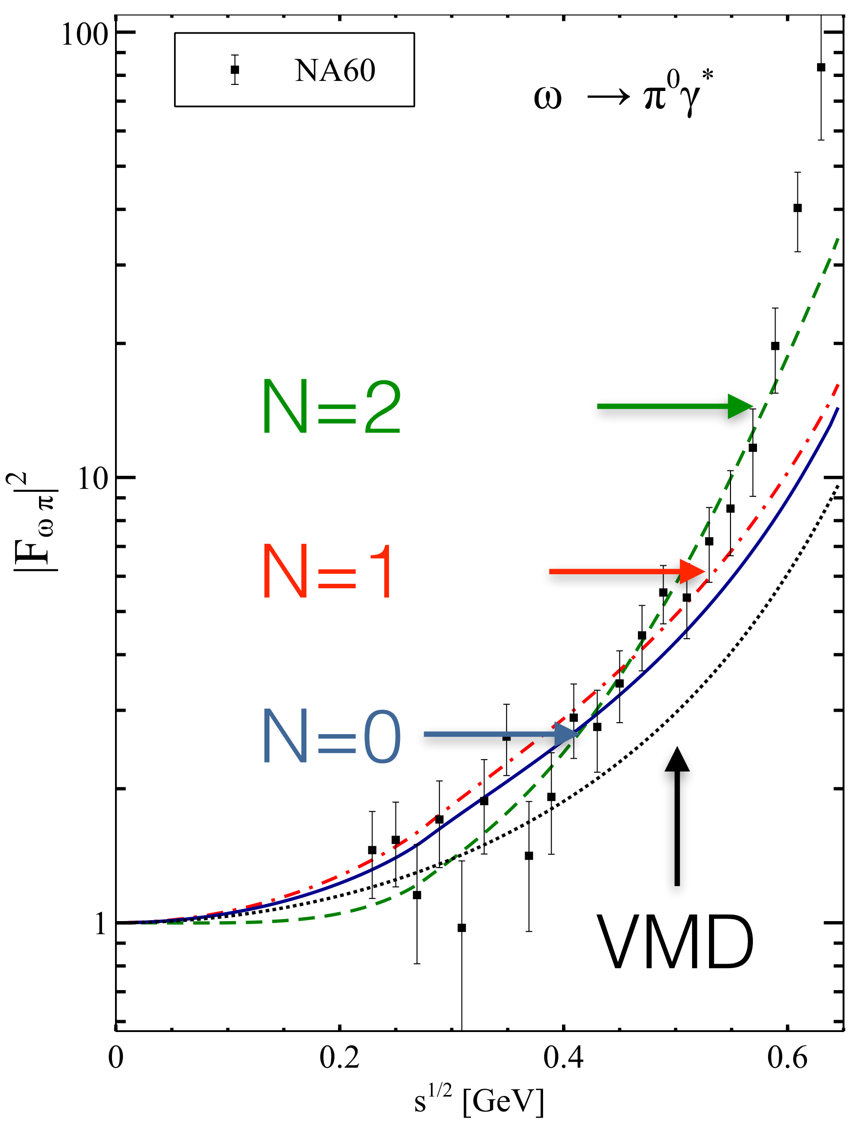


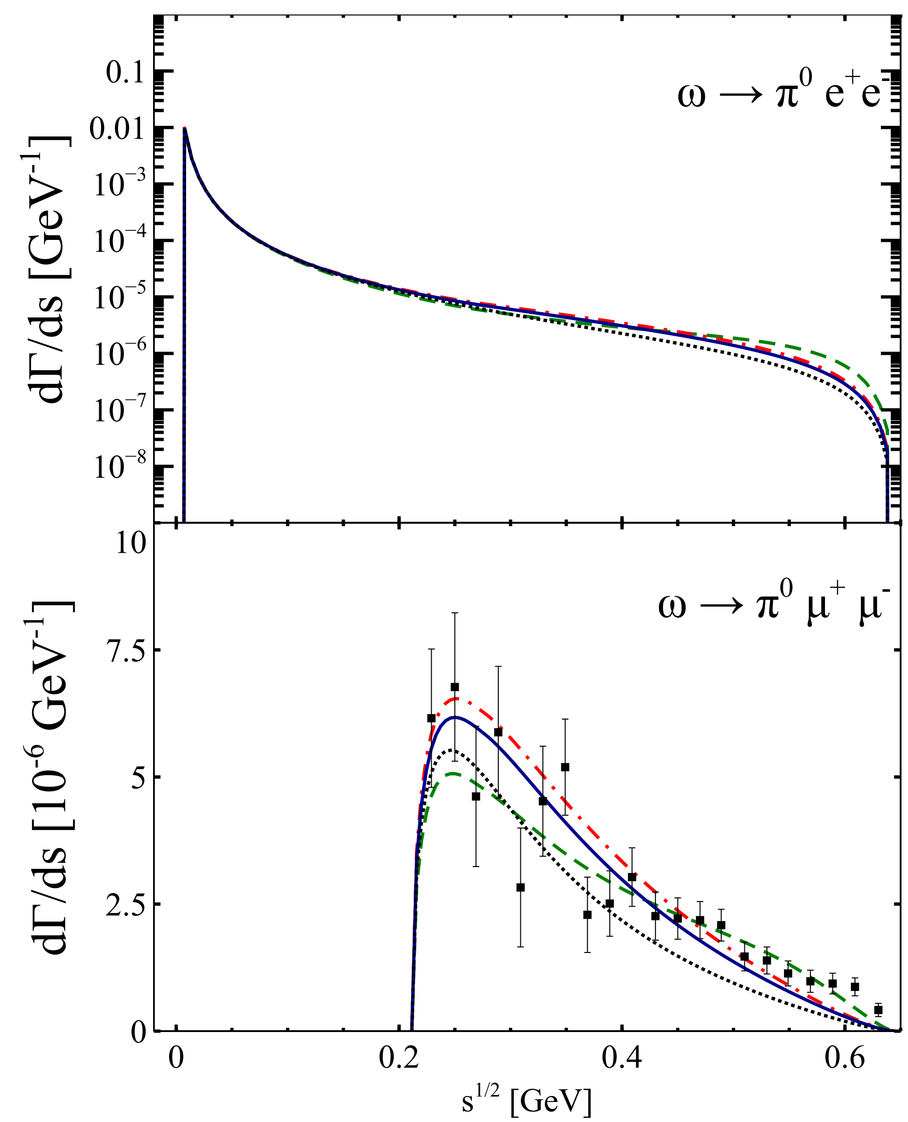
KLOE Coll. PLB 561 55-60

$\omega \to \pi^0 \gamma^*$



$$f_{V\pi}(s) = \int_{s_\pi}^{s_i} \frac{ds'}{\pi} \frac{\text{Disc}\, f_{V\pi}(s')}{s' - s} + \sum_{i=0}^{N} C_i\, \omega(s)^i$$

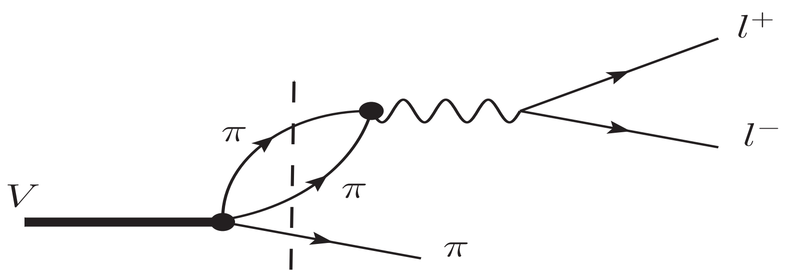




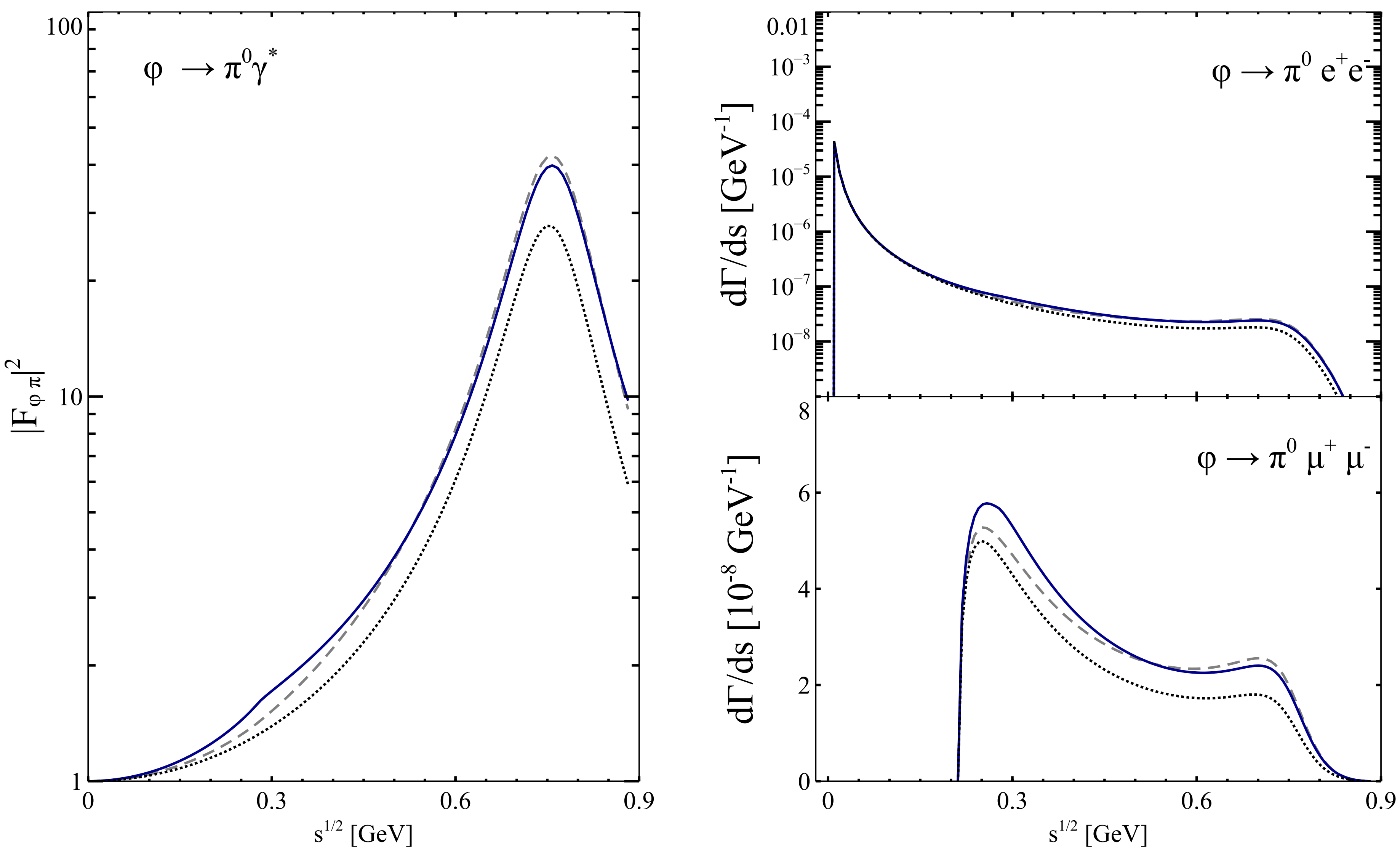
- C_0 fixed from $\Gamma_{exp}(\omega \to \pi\gamma)$
- Nature of the steep rise?

1. Upcoming data from CLAS g12 & MAMI
2. Exp. analysis of $\phi \to \pi\gamma$ is very important

$\varphi \to \pi^0 \gamma^*$



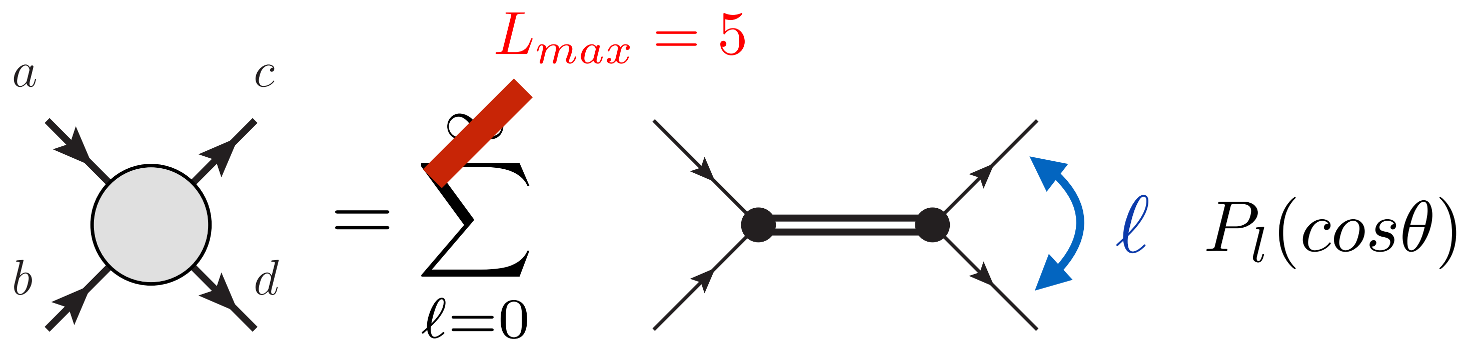
$$f_{V\pi}(s) = \int_{s_\pi}^{s_i} \frac{ds'}{\pi} \frac{\text{Disc}\, f_{V\pi}(s')}{s' - s} + \sum_{i=0}^{N} C_i\, \omega(s)^i$$



- C_0 fixed from $\Gamma_{exp}(\phi \to \pi\gamma)$
- **Grey:** no 3b effects

$\bar{K}N$ scattering (resonance region)

- P.w. analysis



- Coupled channel unitarity:

$$\bar{K}N, \pi\Sigma, \pi\Lambda, \eta\Lambda, \eta\Sigma, \pi\Sigma(1385), \pi\Lambda(1520), \bar{K}\Delta(1232), \bar{K}^*N, \sigma\Lambda, \sigma\Sigma$$

- Resonances and backgrounds are incorporated through analytic K-matrices → search for poles in the complex s-plane

- Right threshold behaviour (angular momentum barrier)

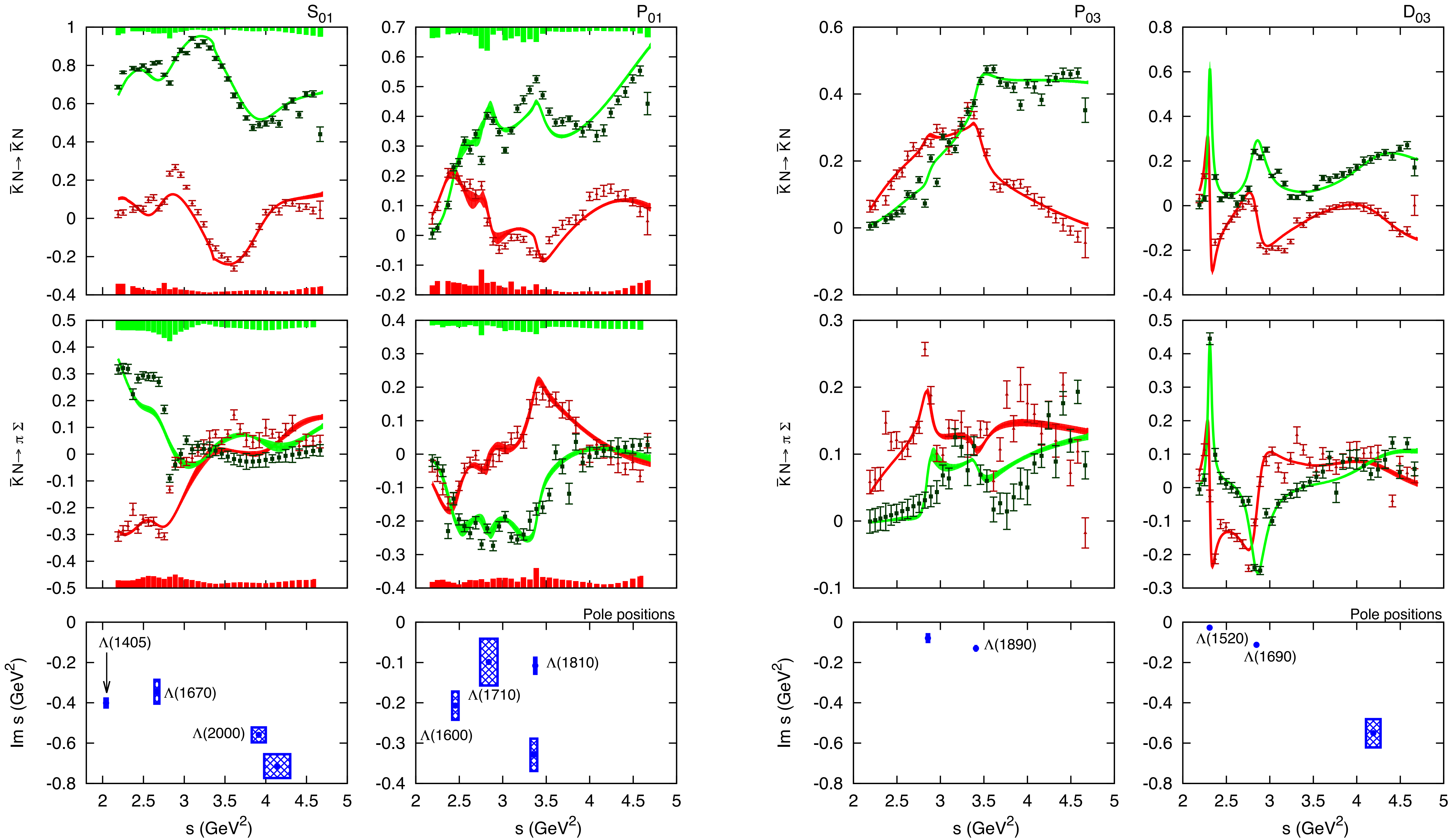
C. Fernandez-Ramirez et.al. (JPAC) (in preparation)

- Fit single-energy p.w. up to *J*=7/2 and 2.15 GeV

C. Zhang et al., PRC 88 035205

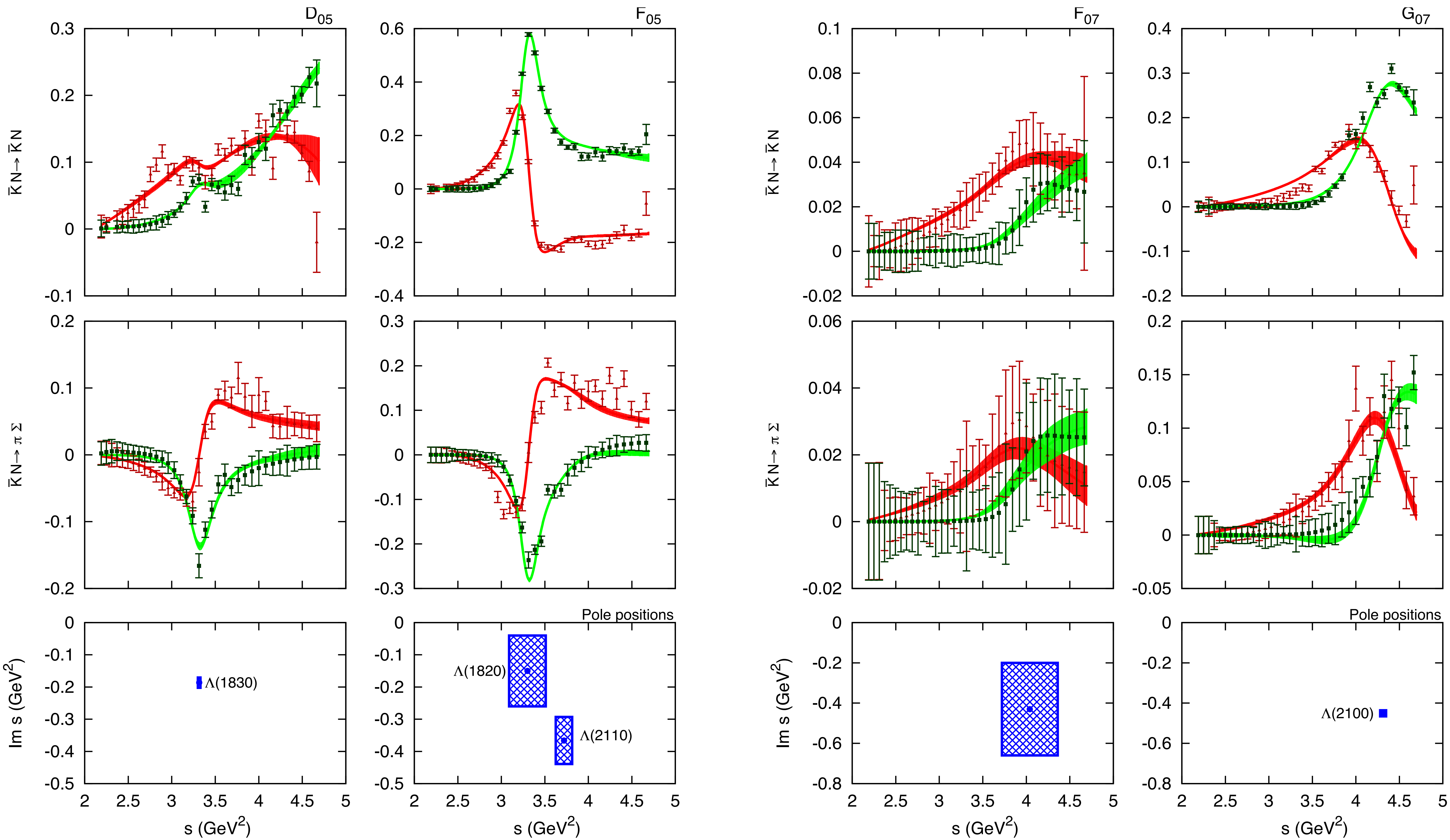
KN scattering (resonance region)

$[l_{I\,2J}]$



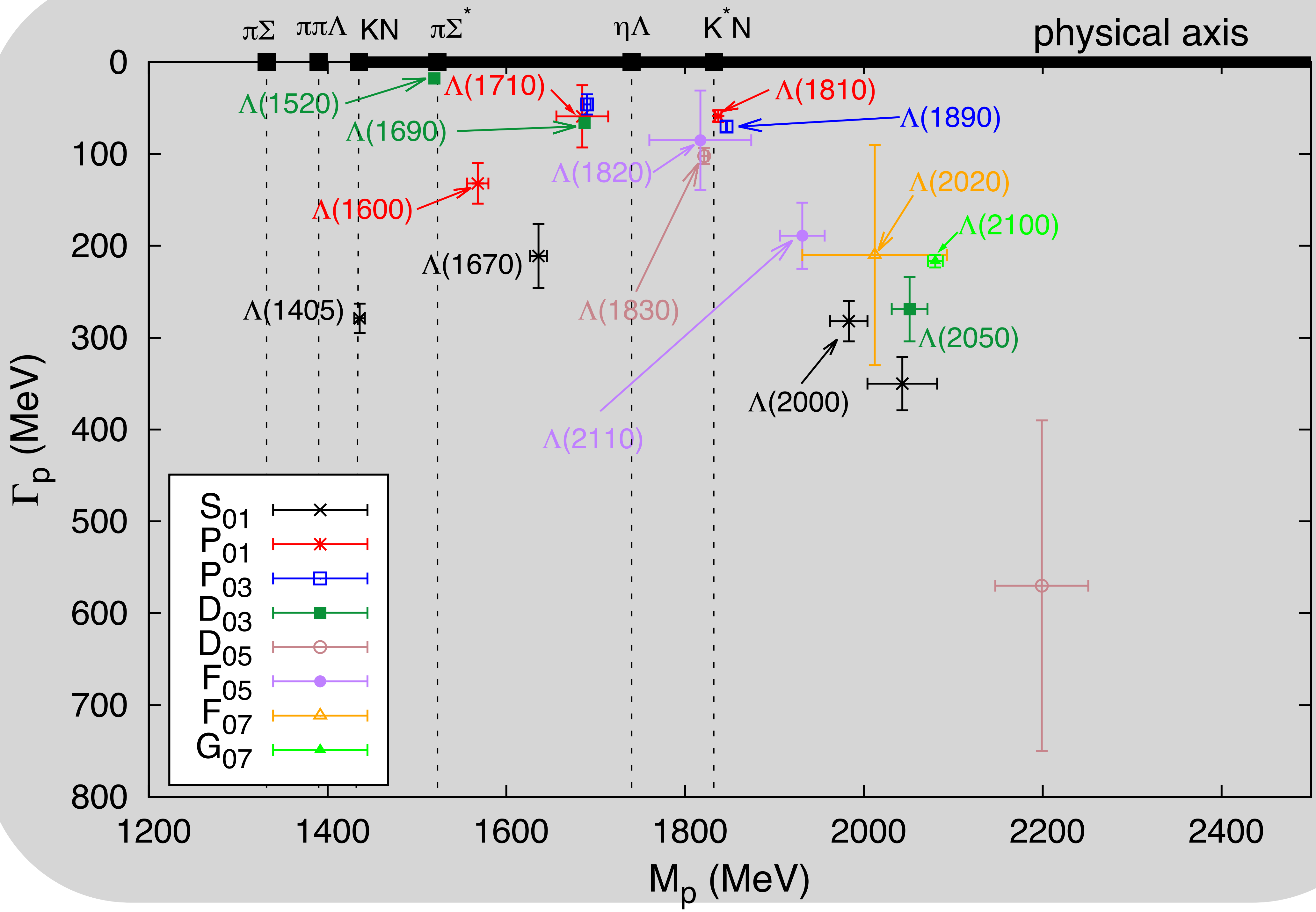
KN scattering (resonance region)

$[l_{I\,2J}]$

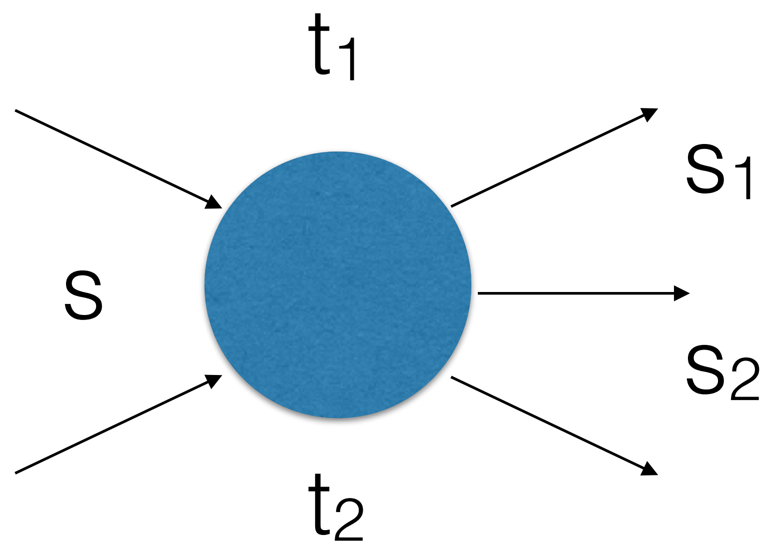


KN scattering (resonance region)

$[l_{I\,2J}]$



Regge physics

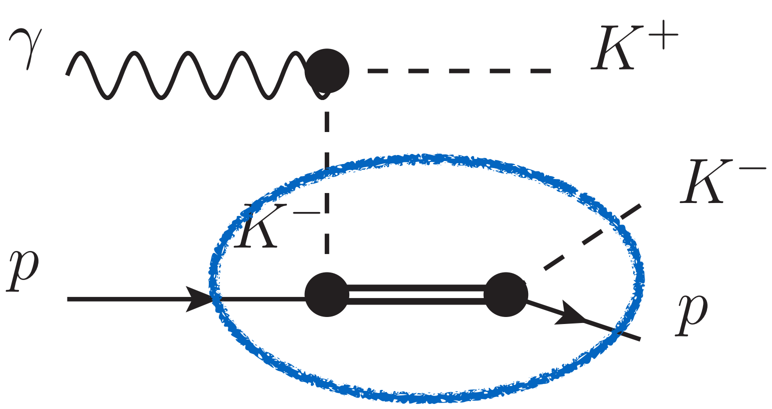


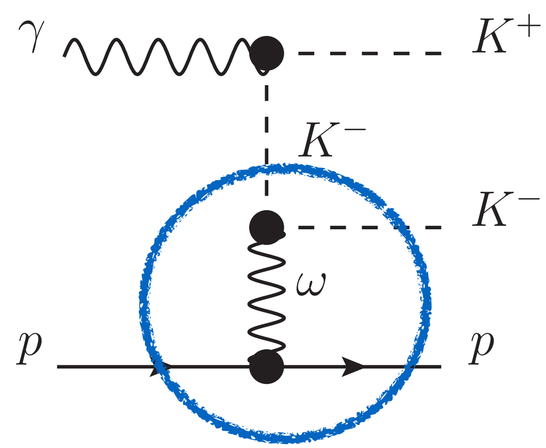
$$A = \sum_{J_1, J_2, \lambda} d^{J_2}_{\lambda_b - \lambda_t, \lambda - \lambda_r}(\theta_2)\, d^{J_1}_{\lambda 0}(\theta_1)\, e^{i\lambda\phi_1} f_{J_1, J_2, \lambda}(s_1, s)$$

If all s_1, s_2, s are large it is NOT OK to truncate

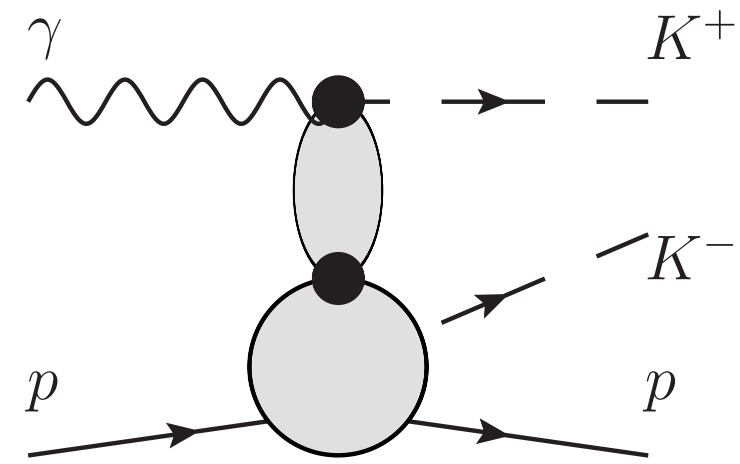
$\gamma p \to K^+ K^- p$

Deck model





B5 model



$$K^- p \to K^- p$$

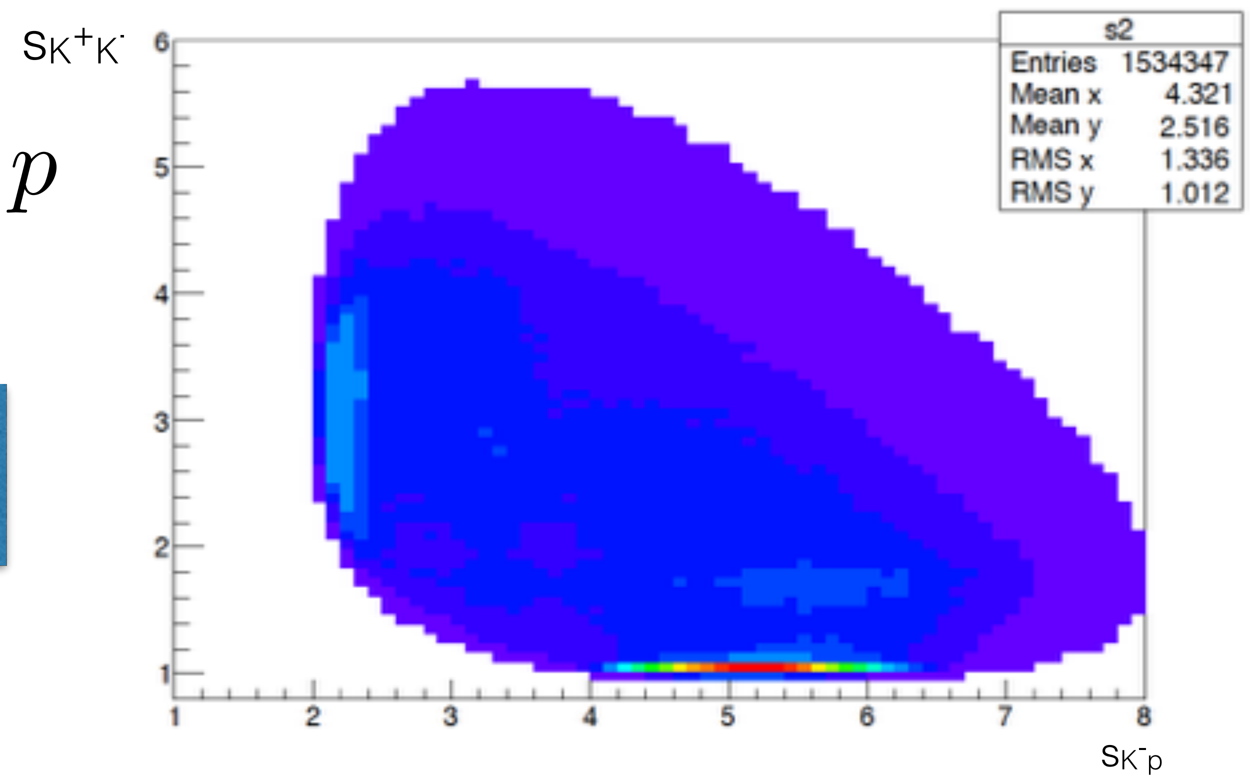
low energy fit

C. Fernandez-Ramirez et.al. (JPAC) (in preparation)

$$K^- p \to K^- p$$

high energy fit

V. Mathieu et.al. (JPAC) (in preparation)

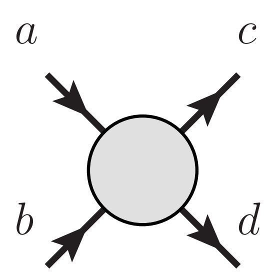


Preliminary CLAS g12

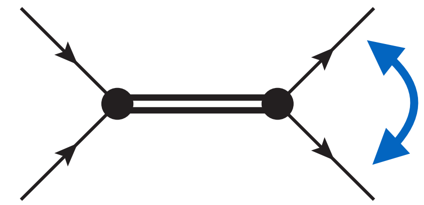
Analytical continuation between the two regions via dispersion relations (FESR)

FESR: $\pi N \to \pi N$

P.w. analysis



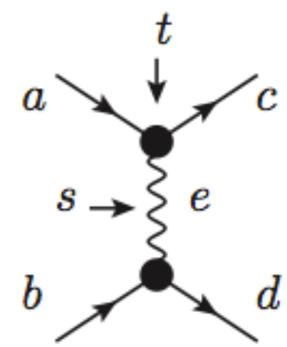
$$= \sum_{\ell=0}^{\infty \to L_{max}}$$



$\ell \quad P_\ell(\cos\theta)$

Sum over Regge poles + background integral

$=$



$\beta(t) s^{\alpha(t)}$

$$+ \mathcal{O}\left(\frac{1}{\sqrt{s}}\right) \; ?$$

One can take advantages of both: analyticity implies FESR

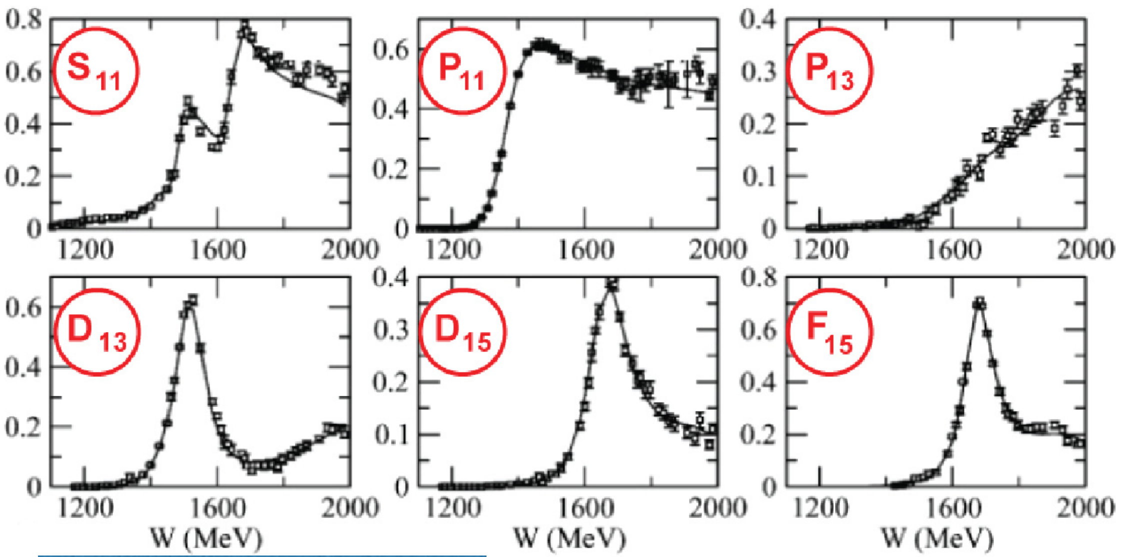
$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\; A^{(-)}(\nu', t) \nu'^{2k} d\nu' = \beta(t) \frac{\Lambda^{\alpha_\rho(t)+2k+1}}{\alpha_\rho(t) + 2k + 1}$$

$$\mathrm{Im}\; A^{(-)}(\nu, t) \longrightarrow \beta(t) \nu^{\alpha_\rho(t)}, \quad \nu = \frac{s-u}{4m} > \Lambda$$

FESR: πN→πN

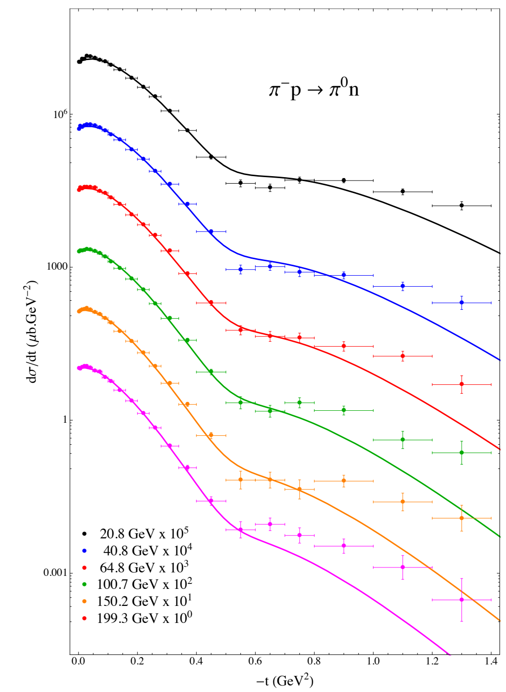
$$\int_{\nu_0}^{\Lambda} \mathrm{Im}\ A^{(-)}(\nu',t)\nu'^{2k}d\nu' = \beta(t)\frac{\Lambda^{\alpha_\rho(t)+2k+1}}{\alpha_\rho(t)+2k+1}$$

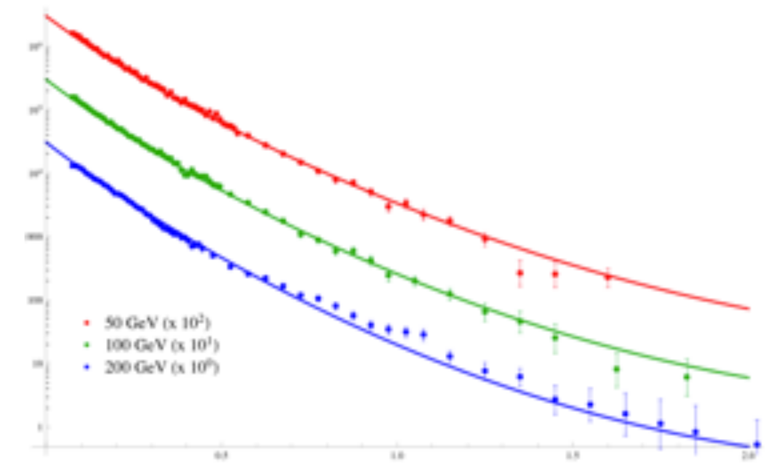
p.w. analysis



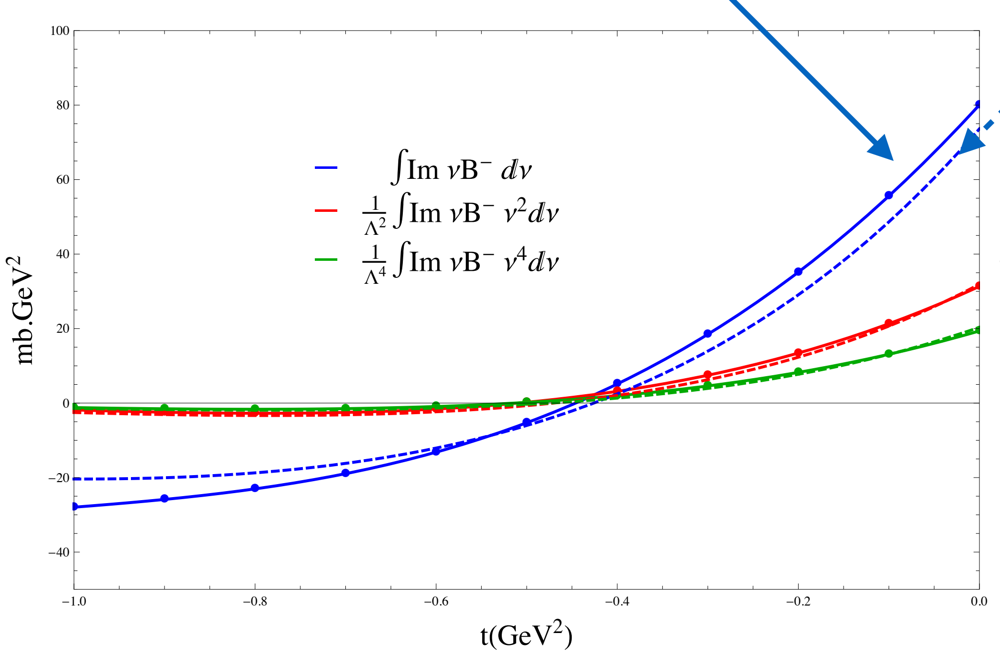
SAID, Workman et.al.

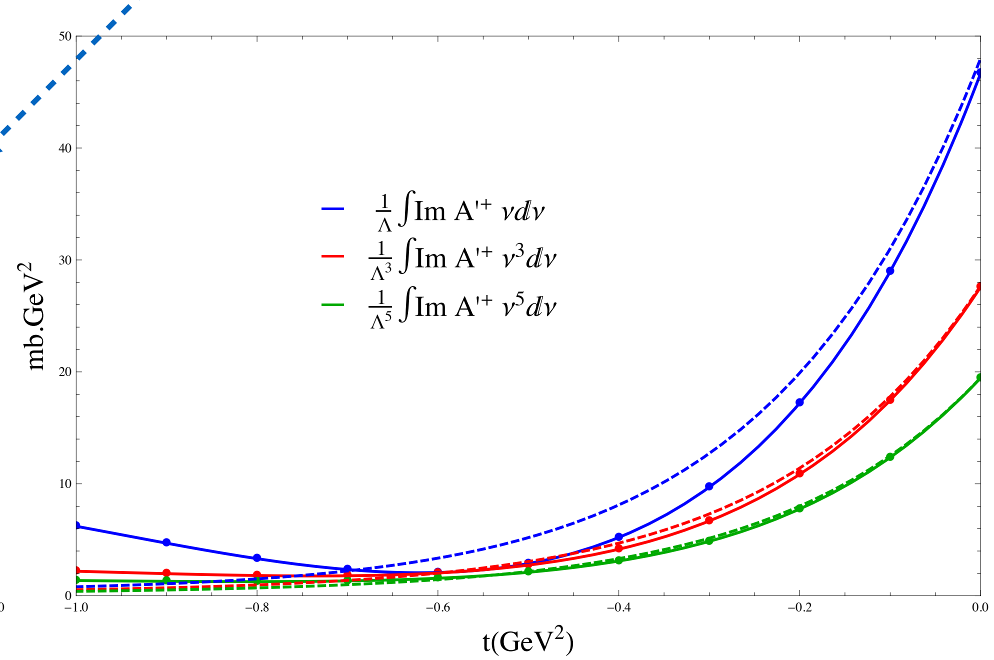
sum over Regge poles





V. Mathieu et.al.
(JPAC)
arXiv:1506.01764



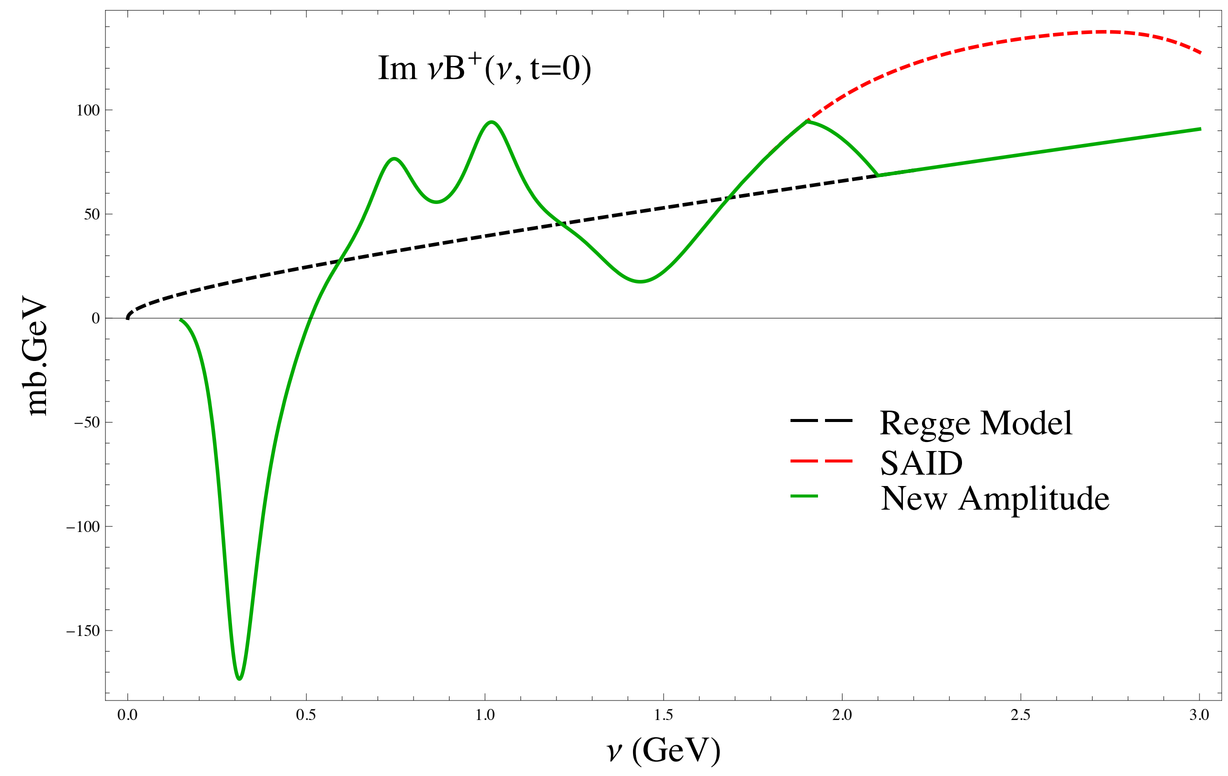


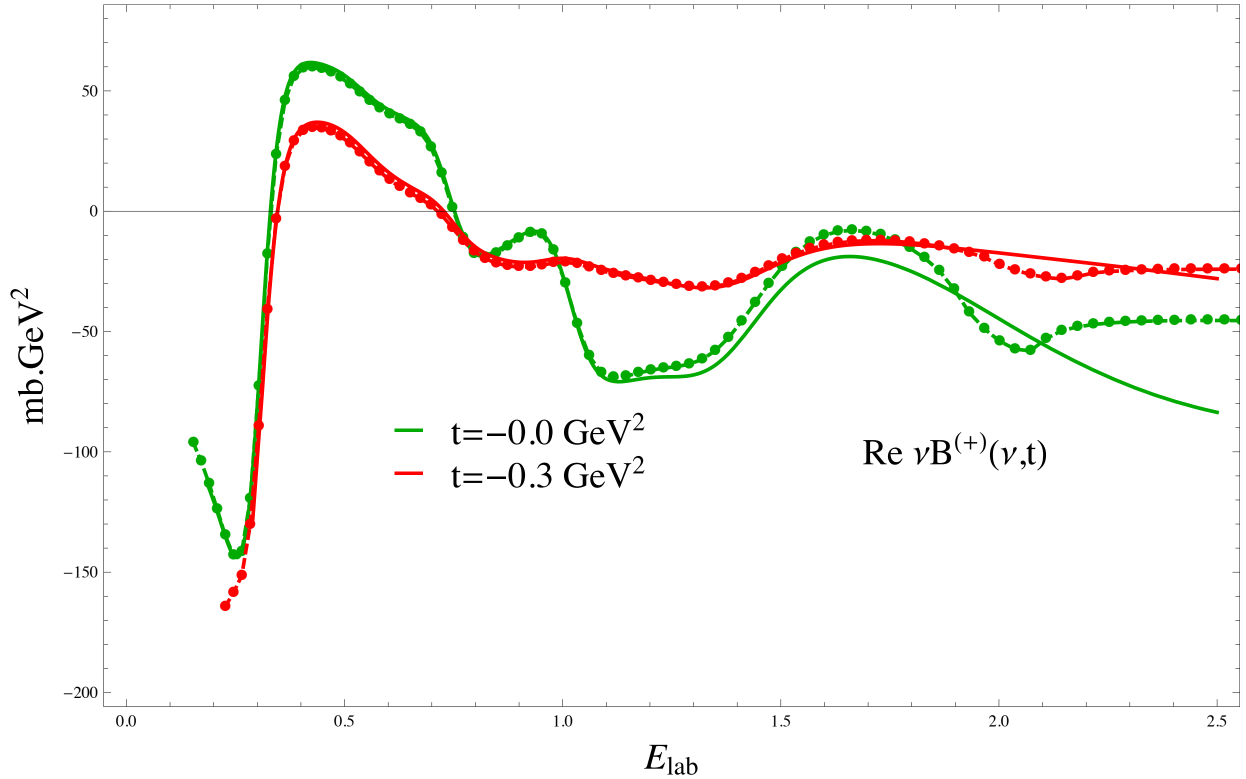
FESR: $\pi N \to \pi N$

- Construct Im(A(s,t)) from [s_0, ∞] via FESR
Reconstruct Re(A(s,t)) from dispersion relation

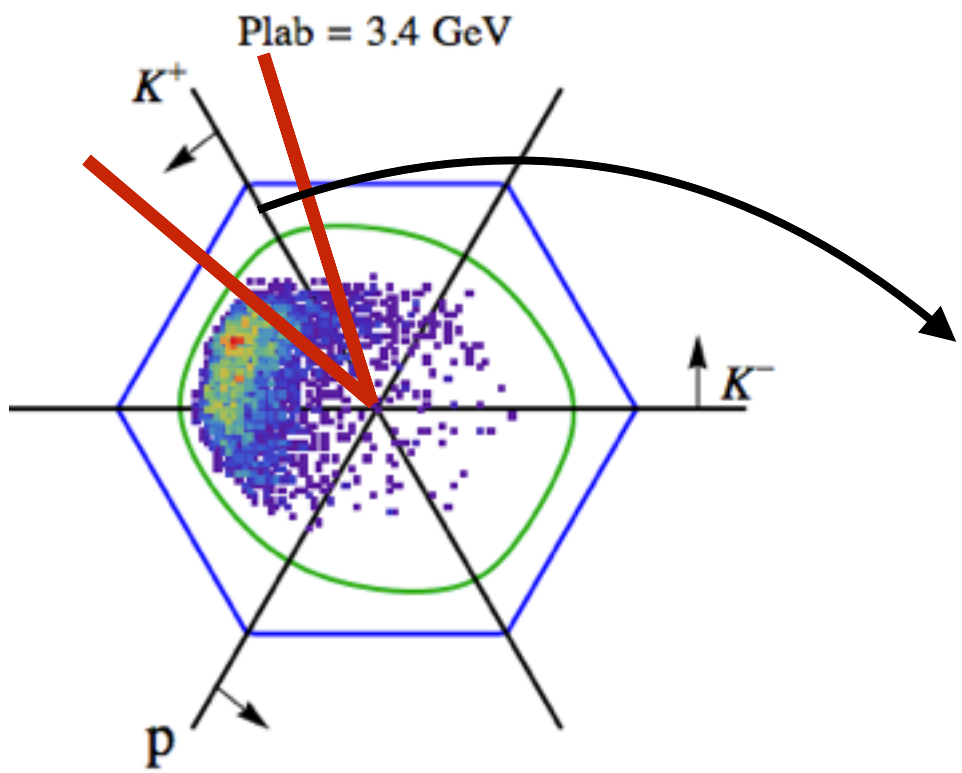
$$A^{(-)}(\nu,t) = \frac{2\nu}{\pi}\int_{\nu_0}^{\infty}\frac{\text{Im } A^{(-)}(\nu',t)}{\nu'^2-\nu^2}d\nu'$$

- Excellent Match between
Re(SAID) Solid lines and Re(Reconstructed) Dashed-Dotted line

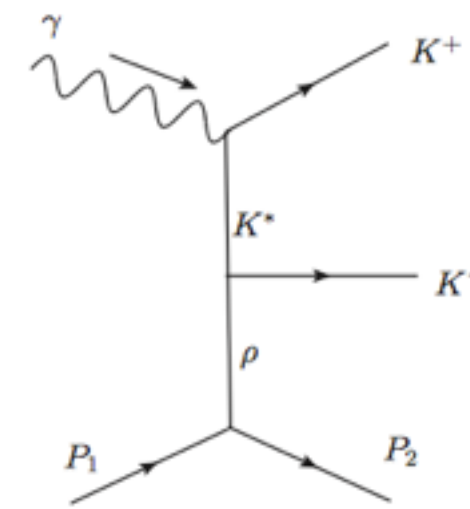


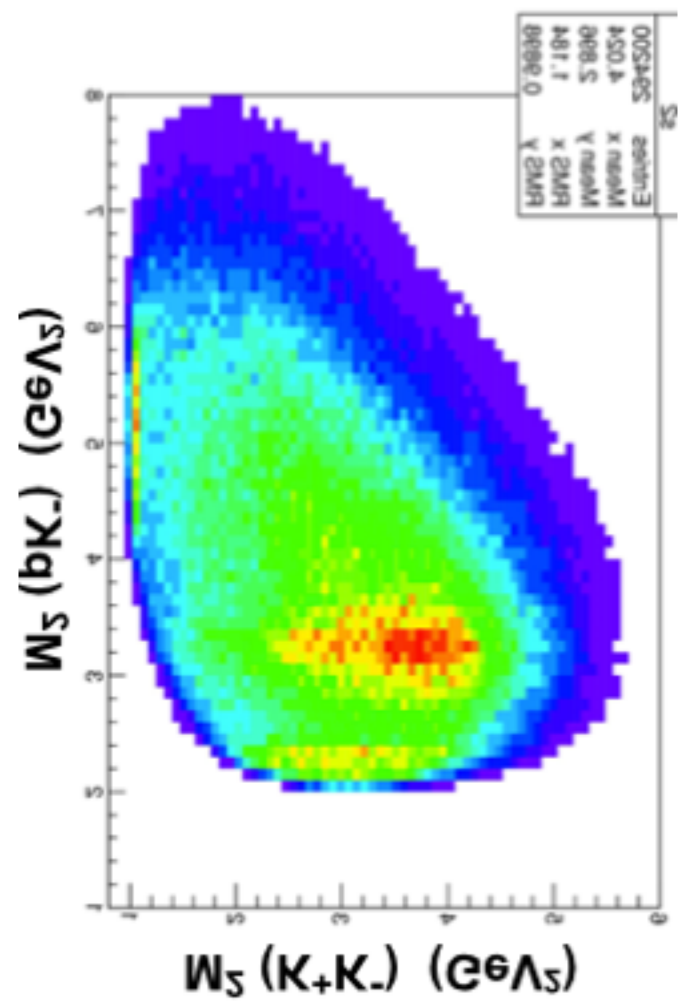


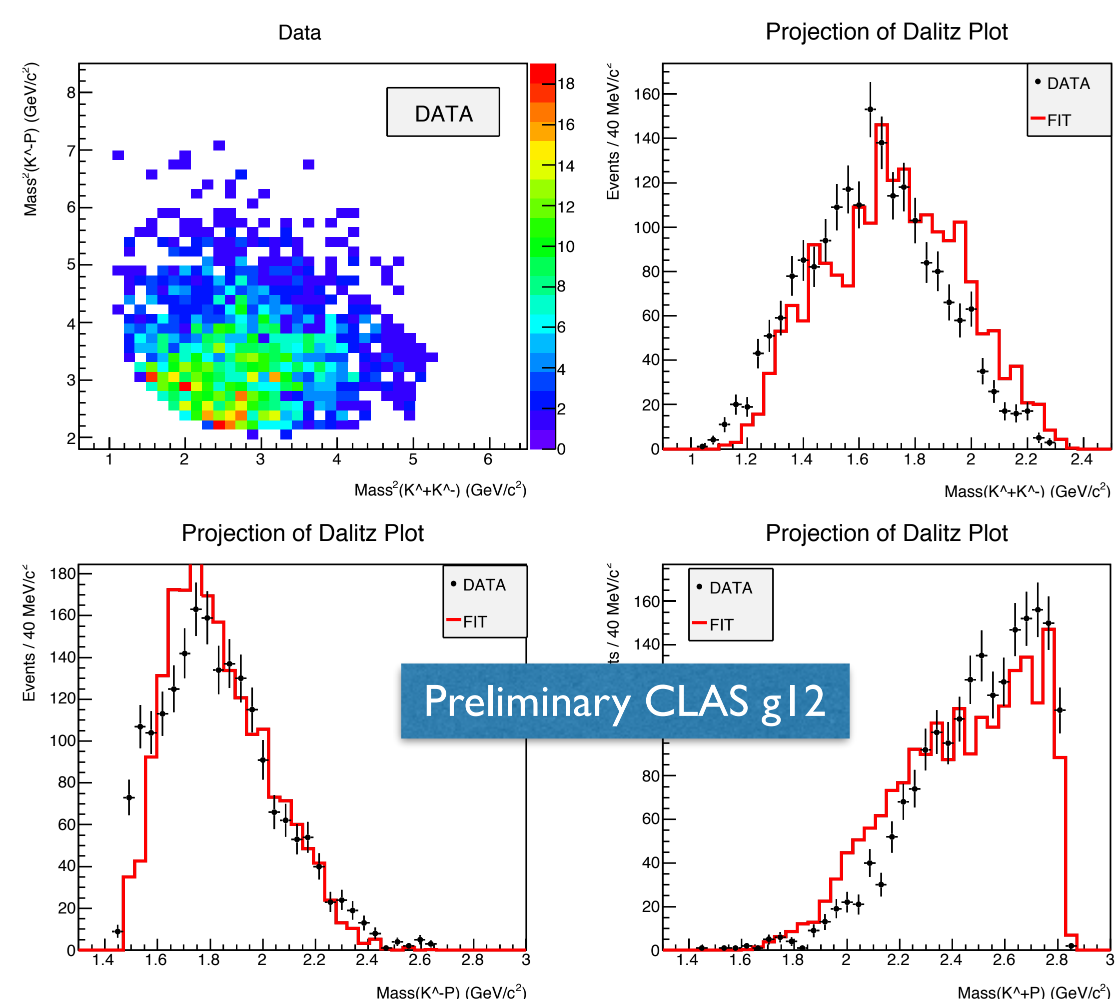
$\gamma p \to K^+K^-p$: double Regge limit



Double Regge limit







Preliminary CLAS g12

M. Shi et.al.
(JPAC)
PRD91 3 034007

*Prospects for γNN**

$$\gamma^{(*)} N \to \pi N,\ \gamma^{(*)} N \to (\pi\pi)\, N$$

- FESR calculation for $\gamma N \to \pi N$ and extension to $\gamma^* N \to \pi N$:
  Use it as a constraint from high energy
- Final state interactions between $\pi\pi N$ (Khuri Treiman dispersive calculation)
- Extension of Dual models to $\pi\pi N$

Joint Physics Analysis Center (JPAC)

Low energy, unitarity etc.

$\eta \to \pi^+\pi^-\pi^0$	PRD92 5 054016
$\omega, \phi \to \pi^+\pi^-\pi^0$	PRD91 9 094029
$\to \gamma^*\pi^0$	
$KN \to KN$	in preparation

"Technology"

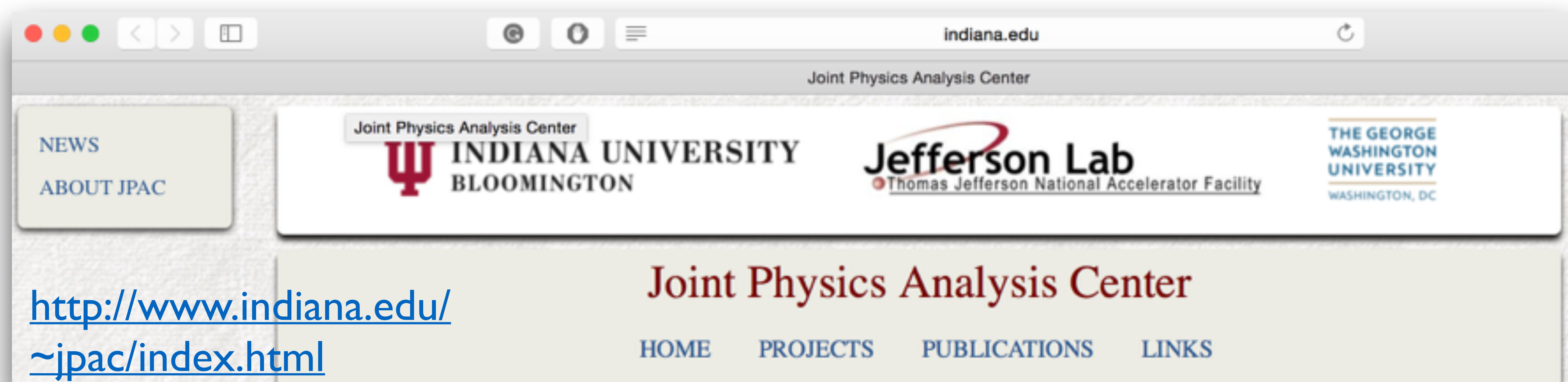
Khuri-Treiman eq.	arXiv:1409.8652
Traingle singl. & XYZ	PLB747 410-416
	arXiv:1510.01789
	arXiv:1510.00695

Regge & FESR

$\gamma p \to \pi^0 p$	arXiv:1505.02321
$\pi N \to \pi N$	arXiv:1506.01764
$KN \to KN$	in preparation

Dual models

$\gamma p \to K^+K^-p$	PRD91 3 034007
$J/\psi\,(\psi') \to 3\pi$	PLB 737 283-288

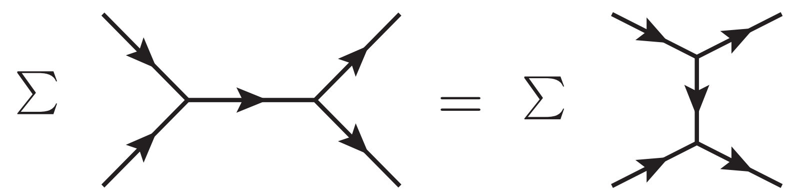


http://www.indiana.edu/~jpac/index.html

Spares

$J/\Psi \to 3\pi$

Dual model



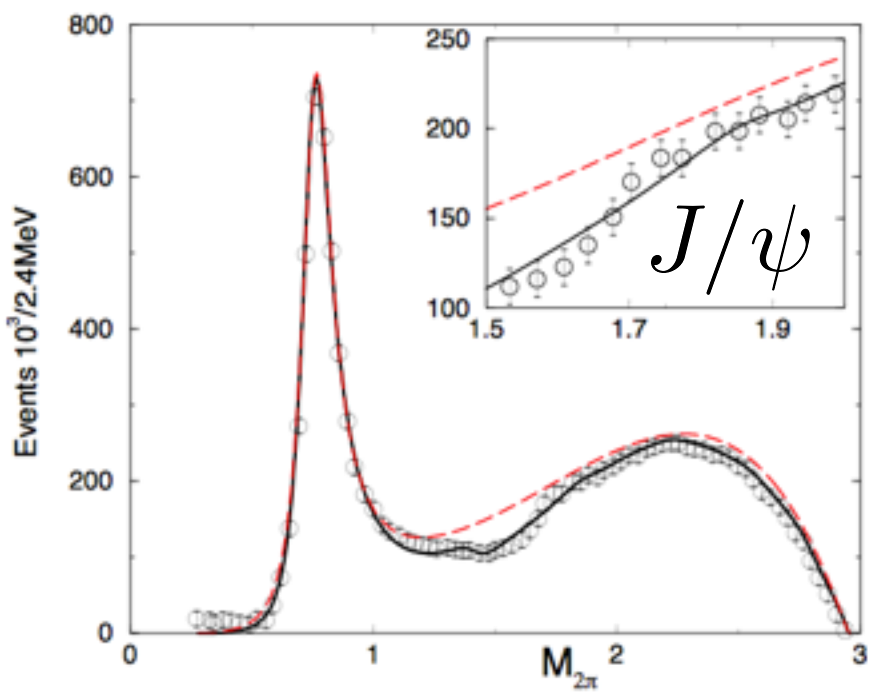
$$A(s,t) = \sum_{n,m} c_{n,m} \frac{\Gamma(n-\alpha_s)\Gamma(n-\alpha_t)}{\Gamma(n+m-\alpha_s-\alpha_t)}$$

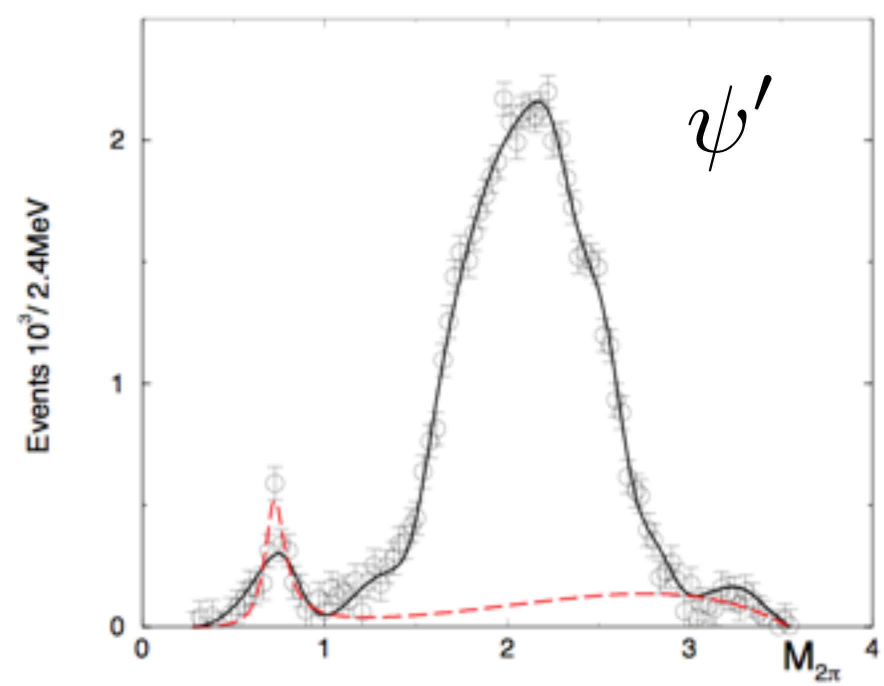
Parameters:

trajectory $\alpha(s)$

couplings $c_{n,m}$

$J/\psi, \psi' \to 3\pi$





A. Szczepaniak
M. Pennington
arXiv:1403.5782