
Preserving local gauge invariance with t -channel Regge exchange

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Outline

- Defining the problem
- Regge-trajectory basics
- How *not* to cure the problem
- The origin of the problem
- Gauge invariance basics
- Implementation of **local** gauge invariance
- The cure
- Application: $\gamma + n \rightarrow K^+ + \Sigma^*(1385)^-$
- Summary



Defining the Problem

← time →

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four Feynman-like diagrams for the transition matrix M^μ . Each diagram has an incoming dashed line with momentum q and an outgoing wavy line with momentum k . The other two lines represent nucleons with momenta p' and p .
 - **s-channel**: A solid line connects a vertex labeled 's' to a vertex labeled 'p'.
 - **u-channel**: A solid line connects a vertex labeled 'u' to a vertex labeled 'p'.
 - **t-channel**: A solid line connects a vertex labeled 't' to a vertex labeled 'p'.
 - **interaction current**: A solid line connects a vertex labeled 'p' to a vertex labeled 'p'.



Defining the Problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four terms in the sum:

- s-channel:** A vertex labeled 's' with incoming lines q (dashed) and p' (solid), and outgoing lines k (wavy) and p (solid).
- u-channel:** A vertex labeled 'u' with incoming lines q (dashed) and p (solid), and outgoing lines k (wavy) and p' (solid).
- t-channel:** A vertex labeled 't' with incoming lines q (dashed) and p (solid), and outgoing lines k (wavy) and p' (solid). The vertex is highlighted with a grey box.
- interaction current:** A single vertex with incoming lines q (dashed) and p (solid), and outgoing lines k (wavy) and p' (solid).

Replace t -channel single-meson exchange by **Regge-trajectory exchange**: $\Delta_m F_t \rightarrow \mathcal{P}_m$



Defining the Problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
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 \end{aligned}$$

Diagram 1: s-channel exchange with vertices 's' and a black dot. Incoming lines p' and q, outgoing lines p and k.

 Diagram 2: u-channel exchange with vertices 'u' and a black dot. Incoming lines p' and q, outgoing lines p and k.

 Diagram 3: t-channel exchange with vertex 't' and a black dot. Incoming lines p' and q, outgoing lines p and k.

 Diagram 4: Interaction current with a black dot. Incoming lines p' and q, outgoing lines p and k.

Replace t -channel single-meson exchange by **Regge-trajectory exchange**: $\Delta_m F_t \rightarrow \mathcal{P}_m$

$$\begin{aligned}
 \mathcal{M}^\mu &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \mathcal{P}_m + M_{\text{int}}^\mu
 \end{aligned}$$

Diagram 3: t-channel exchange with vertex 't' and a black dot. Incoming lines p' and q, outgoing lines p and k. A pink arrow points to the t-channel propagator.

\mathcal{P}_m : Regge-trajectory propagator



Defining the Problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{[s-channel diagram]}}_{s\text{-channel}} + \underbrace{\text{[u-channel diagram]}}_{u\text{-channel}} + \underbrace{\text{[t-channel diagram]}}_{t\text{-channel}} + \underbrace{\text{[interaction current diagram]}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four Feynman-like diagrams for the amplitude M^μ . Each diagram has an incoming dashed line with momentum q and an outgoing wavy line with momentum k . The other two external lines are solid lines with momenta p' and p .

- s-channel:** A solid line connects a vertex labeled 'S' (with a circle) to a black vertex. The incoming q and outgoing k lines meet at this black vertex.
- u-channel:** A solid line connects a black vertex to a vertex labeled 'U' (with a circle). The incoming q and outgoing k lines meet at this black vertex.
- t-channel:** A solid line connects a vertex labeled 'T' (with a circle) to a black vertex. The incoming q and outgoing k lines meet at this black vertex.
- interaction current:** A black vertex where the incoming q and outgoing k lines meet, and the solid lines p' and p also meet.

$$k_\mu M^\mu = 0$$

current conserved



Defining the Problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
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Diagram 1: s-channel exchange with vertices S and a black dot. Incoming lines p' (solid), q (dashed); outgoing lines p (solid), k (wavy).
 Diagram 2: u-channel exchange with vertices U and a black dot. Incoming lines p' (solid), q (dashed); outgoing lines p (solid), k (wavy).
 Diagram 3: t-channel exchange with vertex T and a black dot. Incoming lines p' (solid), q (dashed); outgoing lines p (solid), k (wavy).
 Diagram 4: Interaction current with a black dot. Incoming lines p' (solid), q (dashed); outgoing lines p (solid), k (wavy).

$$k_\mu M^\mu = 0$$

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With Reggeized t -channel:

$$\begin{aligned}
 \mathcal{M}^\mu &= \text{Diagram 1} + \text{Diagram 2} + \text{Diagram 3} + \text{Diagram 4} \\
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 Diagram 2: u-channel exchange with vertices U and a black dot. Incoming lines p' (solid), q (dashed); outgoing lines p (solid), k (wavy).
 Diagram 3: Reggeized t-channel exchange with vertex T and a black dot. Incoming lines p' (solid), q (dashed); outgoing lines p (solid), k (wavy). A pink arrow points to the vertex T.
 Diagram 4: Interaction current with a black dot. Incoming lines p' (solid), q (dashed); outgoing lines p (solid), k (wavy).

$$k_\mu \mathcal{M}^\mu \neq 0$$

current not conserved



Defining the Problem

No problem

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram 1}}_{s\text{-channel}} + \underbrace{\text{diagram 2}}_{u\text{-channel}} + \underbrace{\text{diagram 3}}_{t\text{-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
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$$k_\mu M^\mu = 0$$

current conserved

With Reggeized t -channel:

Problem!

$$\begin{aligned}
 \mathcal{M}^\mu &= \text{diagram 1} + \text{diagram 2} + \underbrace{\text{diagram 3}}_{\text{Reggeized } t\text{-channel}} + \text{diagram 4} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \mathcal{P}_m + M_{\text{int}}^\mu
 \end{aligned}$$

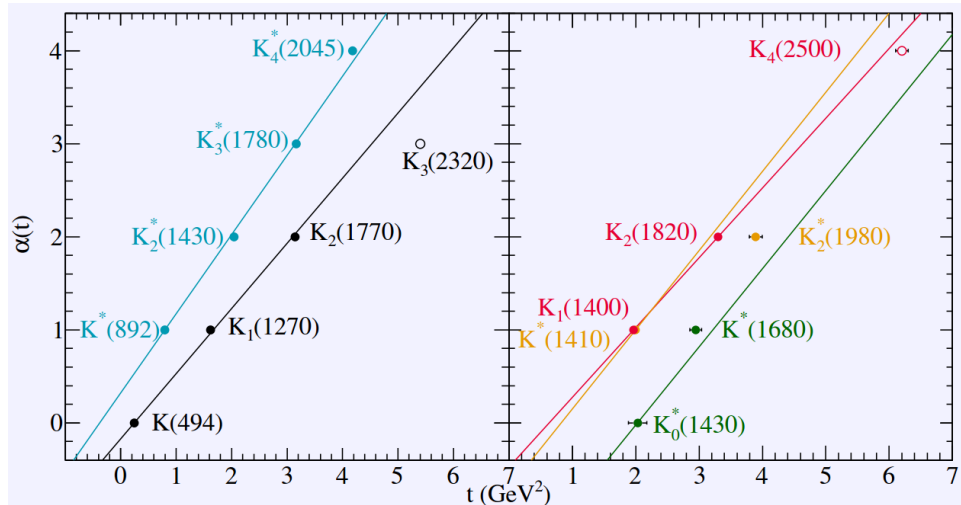
$$k_\mu \mathcal{M}^\mu \neq 0$$

current not conserved



Regge trajectories

Graph stolen from P. Vancraeyveld, PhD Thesis, U. Gent, 2011



Trajectories:

pseudoscalar:

$$\alpha_+(t) = \alpha_0(t),$$

$$\alpha_0(t) = \alpha'_0(t - M_0^2)$$

vector:

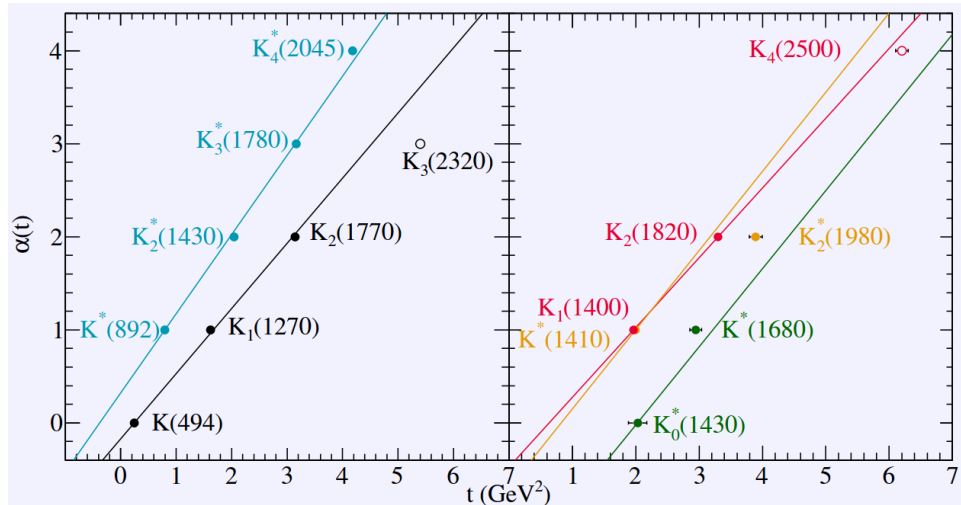
$$\alpha_-(t) = 1 + \alpha_1(t),$$

$$\alpha_1(t) = \alpha'_1(t - M_1^2)$$



Regge trajectories

Graph stolen from P. Vancraeyveld, PhD Thesis, U. Gent, 2011



Trajectories:

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$$\alpha_+(t) = \alpha_0(t),$$

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vector:

$$\alpha_-(t) = 1 + \alpha_1(t),$$

$$\alpha_1(t) = \alpha'_1(t - M_1^2)$$

Regge exchange for t -channel:

$$\Delta_m F_t \rightarrow \mathcal{P}_m(t) = \frac{1}{t - M_m^2} \mathcal{F}_m(t),$$

$$\mathcal{F}_m(t) = \left(\frac{s}{s_{sc}} \right)^{\alpha_m(t)} \frac{N[\alpha_m(t); \eta]}{\Gamma(1 + \alpha_m(t))} \frac{\pi \alpha_m(t)}{\sin(\pi \alpha_m(t))}$$

$m = 0, 1$

$s_{sc} = 1 \text{ GeV}^2$

residual Regge function



Regge trajectories

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residual Regge function

$s_{\text{sc}} = 1 \text{ GeV}$



Regge trajectories

Regge exchange for t -channel:

$$\Delta_m F_t \rightarrow \mathcal{P}_m(t) = \frac{1}{t - M_m^2} \mathcal{F}_m(t),$$

$m = 0, 1$

$$\mathcal{F}_m(t) = \left(\frac{s}{s_{\text{sc}}} \right)^{\alpha_m(t)} \frac{N[\alpha_m(t); \eta]}{\Gamma(1 + \alpha_m(t))} \frac{\pi \alpha_m(t)}{\sin(\pi \alpha_m(t))}$$

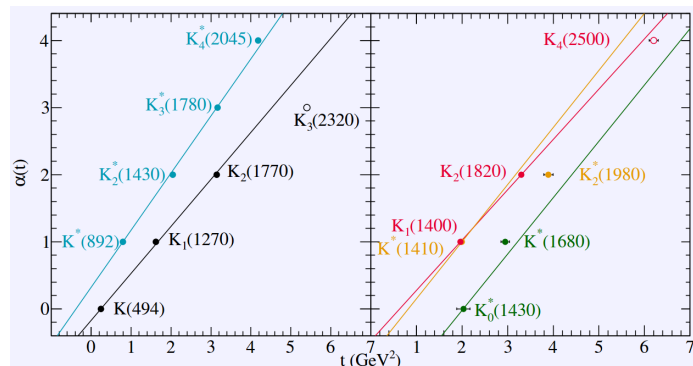
residual Regge function

$s_{\text{sc}} = 1 \text{ GeV}$

Signature function:

$$N[\alpha_m(t); \eta] = \eta + (1 - \eta)e^{-i\pi\alpha_m(t)}$$

$$\eta = \begin{cases} \frac{1}{2}, & \text{pure-signature trajectory} \\ 0, & \text{add trajectories: rotating phase} \\ 1, & \text{subtract trajectories: constant phase} \end{cases}$$



Regge trajectories

Regge exchange for t -channel:

$$\Delta_m F_t \rightarrow \mathcal{P}_m(t) = \frac{1}{t - M_m^2} \mathcal{F}_m(t),$$

$m = 0, 1$

$$\mathcal{F}_m(t) = \left(\frac{s}{s_{\text{sc}}} \right)^{\alpha_m(t)} \frac{N[\alpha_m(t); \eta]}{\Gamma(1 + \alpha_m(t))} \frac{\pi \alpha_m(t)}{\sin(\pi \alpha_m(t))}$$

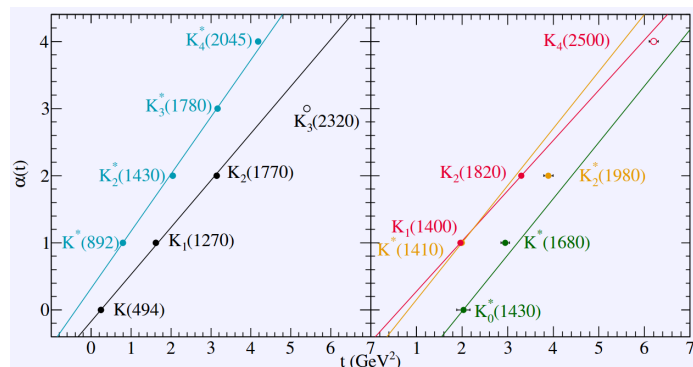
residual Regge function

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$$\eta = \begin{cases} \frac{1}{2}, & \text{pure-signature trajectory} \\ 0, & \text{add trajectories: rotating phase} \\ 1, & \text{subtract trajectories: constant phase} \end{cases}$$



Normalization:

$$\mathcal{F}_m(M_m^2) = 1$$

independent of η



Recipe: Take gauge-invariant amplitude M^μ and multiply by *residual function* $\mathcal{F}_m(t)$

$$M_{\text{GLV}}^\mu = M^\mu \times \mathcal{F}_m = \left[\begin{array}{c} q \\ \text{---} \\ \text{S} \\ \text{---} \\ p' \end{array} \text{---} \text{---} \begin{array}{c} k \\ \text{---} \\ \bullet \\ \text{---} \\ p \end{array} + \begin{array}{c} q \\ \text{---} \\ \bullet \\ \text{---} \\ p' \end{array} \text{---} \text{---} \begin{array}{c} k \\ \text{---} \\ \text{U} \\ \text{---} \\ p \end{array} + \begin{array}{c} q \\ \text{---} \\ \bullet \\ \text{---} \\ p' \end{array} \text{---} \text{---} \begin{array}{c} k \\ \text{---} \\ \bullet \\ \text{---} \\ p \end{array} + \begin{array}{c} q \\ \text{---} \\ \bullet \\ \text{---} \\ p' \end{array} \text{---} \text{---} \begin{array}{c} k \\ \text{---} \\ \bullet \\ \text{---} \\ p \end{array} \right] \times \mathcal{F}_m(t)$$

t-channel okay



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t-channel okay

$$k_\mu M_{\text{GLV}}^\mu = \underbrace{[k_\mu M^\mu]}_{=0} \times \mathcal{F}_m = 0$$



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t-channel okay

$$k_\mu M_{\text{GLV}}^\mu = \underbrace{[k_\mu M^\mu]}_{=0} \times \mathcal{F}_m = 0$$

- Very popular
- Quite successful in providing good descriptions of data for many applications



Recipe: Take gauge-invariant amplitude M^μ and multiply by *residual function* $\mathcal{F}_m(t)$

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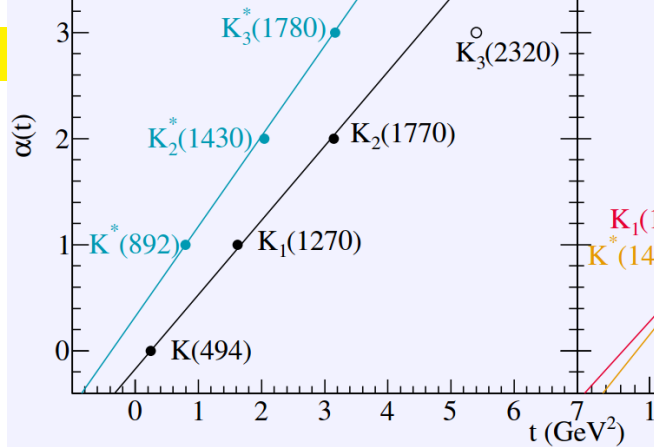
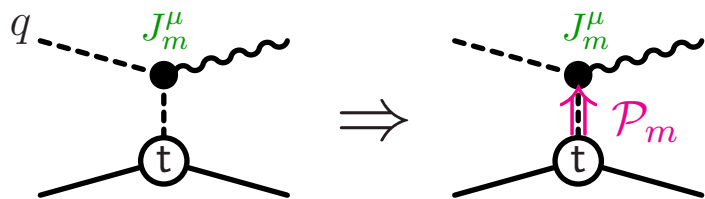
$$k_\mu M_{\text{GLV}}^\mu = \underbrace{[k_\mu M^\mu]}_{=0} \times \mathcal{F}_m = 0$$

- Very popular
- Quite successful in providing good descriptions of data for many applications
- **Without any dynamical foundation**



Origin of Problem

Reason #1:

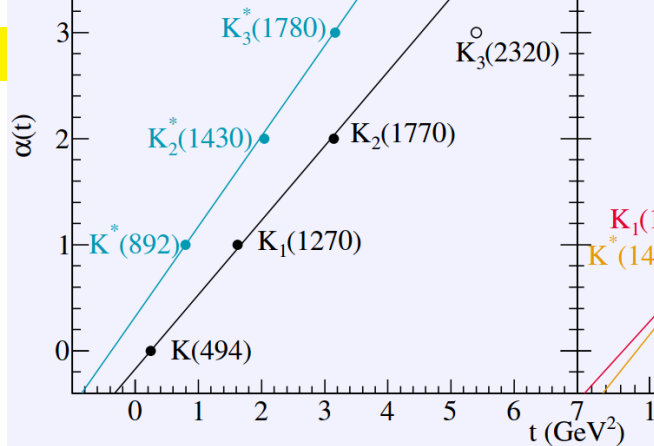
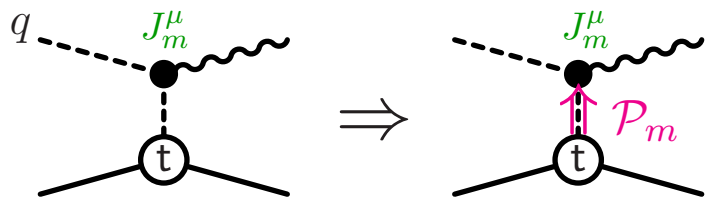


Every state in the Regge trajectory appears with **same** current.



Origin of Problem

Reason #1:



Every state in the Regge trajectory appears with **same** current.

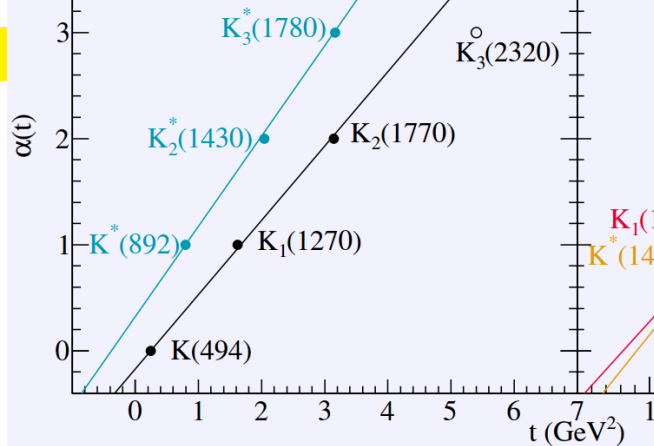
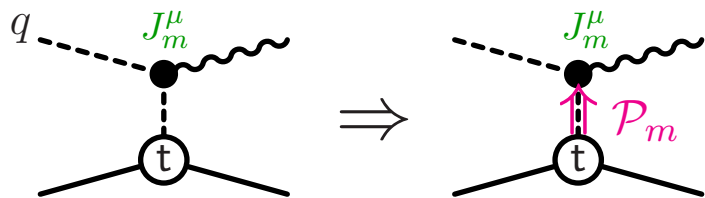
Ward-Takahashi identity:

$$k_\mu J_m^\mu = (q^2 - M_0^2)Q_m - Q_m(t - M_0^2)$$



Origin of Problem

Reason #1:



Every state in the Regge trajectory appears with **same** current.

Ward-Takahashi identity:

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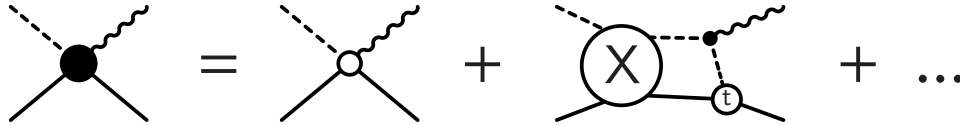


Violates Ward-Takahashi identity for intermediate higher-mass states



Origin of Problem

Reason #2:

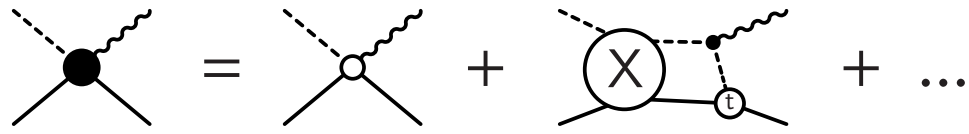


t -Channel exchanges inside interaction current **not Reggeized**.



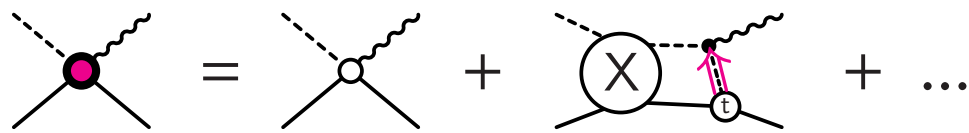
Origin of Problem

Reason #2:



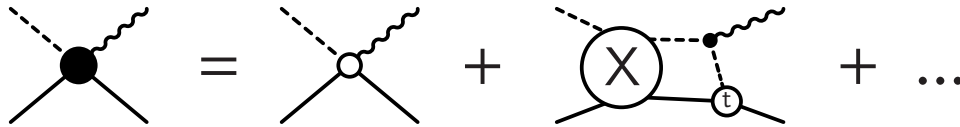
t -Channel exchanges inside interaction current **not Reggeized**.

Needed: Consistent treatment



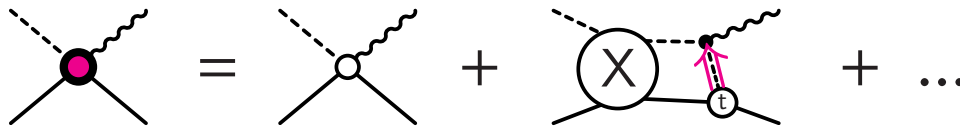
Origin of Problem

Reason #2:



t -Channel exchanges inside interaction current **not Reggeized**.

Needed: Consistent treatment



Interaction-current contribution must be Reggeized as well



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram 1}}_{s\text{-channel}} + \underbrace{\text{diagram 2}}_{u\text{-channel}} + \underbrace{\text{diagram 3}}_{t\text{-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four terms in the sum:

- s-channel:** A solid line with a white circle 's' on the left and a black circle on the right. A dashed line 'q' enters from the top left, and a wavy line 'k' exits from the top right. Two solid lines 'p'' and 'p' enter from the bottom left and bottom right respectively.
- u-channel:** A solid line with a black circle on the left and a white circle 'u' on the right. A dashed line 'q' enters from the top left, and a wavy line 'k' exits from the top right. Two solid lines 'p'' and 'p' enter from the bottom left and bottom right respectively.
- t-channel:** A solid line with a white circle 't' on the left and a black circle on the right. A dashed line 'q' enters from the top left, and a wavy line 'k' exits from the top right. Two solid lines 'p'' and 'p' enter from the bottom left and bottom right respectively.
- interaction current:** A single black circle. A dashed line 'q' enters from the top left, and a wavy line 'k' exits from the top right. Two solid lines 'p'' and 'p' enter from the bottom left and bottom right respectively.

Global gauge invariance

$$k_\mu M^\mu = 0$$

all external hadrons on-shell

$$\Phi \rightarrow \Phi e^{-i\Lambda}$$



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram 1}}_{s\text{-channel}} + \underbrace{\text{diagram 2}}_{u\text{-channel}} + \underbrace{\text{diagram 3}}_{t\text{-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
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The diagrams show four terms in the sum:

- s-channel:** A dashed line with momentum q and a solid line with momentum p' meet at a white circle labeled 's'. A solid line with momentum p and a wavy line with momentum k meet at a black circle.
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- t-channel:** A dashed line with momentum q and a solid line with momentum p meet at a white circle labeled 't'. A solid line with momentum p' and a wavy line with momentum k meet at a black circle.
- interaction current:** A dashed line with momentum q and a solid line with momentum p meet at a black circle. A solid line with momentum p' and a wavy line with momentum k also meet at this black circle.

Global gauge invariance

$$k_\mu M^\mu = 0$$

all external hadrons on-shell

$$\Phi \rightarrow \Phi e^{-i\Lambda}$$

conserved current \Rightarrow implies charge conservation



Gauge Invariance

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 M^\mu &= \underbrace{\text{diagram 1}}_{s\text{-channel}} + \underbrace{\text{diagram 2}}_{u\text{-channel}} + \underbrace{\text{diagram 3}}_{t\text{-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four terms in the sum:

- s-channel:** A solid line with a white circle 's' on the left and a black circle on the right. Incoming lines are dashed (q) and solid (p'). Outgoing lines are solid (k) and solid (p).
- u-channel:** A solid line with a black circle on the left and a white circle 'u' on the right. Incoming lines are dashed (q) and solid (p'). Outgoing lines are solid (k) and solid (p).
- t-channel:** A solid line with a black circle on the left and a white circle 't' on the right. Incoming lines are dashed (q) and solid (p'). Outgoing lines are solid (k) and solid (p).
- interaction current:** A single black circle. Incoming lines are dashed (q) and solid (p'). Outgoing lines are solid (k) and solid (p).

Global gauge invariance

$$k_\mu M^\mu = 0$$

all external hadrons on-shell

$$\Phi \rightarrow \Phi e^{-i\Lambda}$$

conserved current \Rightarrow implies charge conservation

Fixing global gauge invariance does **not** mean internal damage is fixed as well



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four Feynman diagrams for the amplitude M^μ . Each diagram has an incoming dashed line with momentum q and an outgoing wavy line with momentum k . The other two external lines have momenta p' and p .

- s-channel:** A solid line connects a vertex labeled 's' to a vertex labeled '●'. The wavy line is attached to the '●' vertex.
- u-channel:** A solid line connects a vertex labeled '●' to a vertex labeled 'u'. The wavy line is attached to the '●' vertex.
- t-channel:** A solid line connects a vertex labeled 't' to a vertex labeled '●'. The wavy line is attached to the '●' vertex.
- interaction current:** A single vertex labeled '●' where all four lines meet.

Local gauge invariance

$$\Phi \rightarrow \Phi e^{-i\lambda(x)}$$



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

Diagram 1 (s-channel): A dashed line with momentum q and a solid line with momentum p' meet at a vertex labeled 's'. A solid line with momentum p and a wavy line with momentum k meet at another vertex.

Diagram 2 (u-channel): A dashed line with momentum q and a solid line with momentum p meet at a vertex labeled 'u'. A solid line with momentum p' and a wavy line with momentum k meet at another vertex.

Diagram 3 (t-channel): A dashed line with momentum q and a solid line with momentum p meet at a vertex labeled 't'. A solid line with momentum p' and a wavy line with momentum k meet at another vertex.

Diagram 4 (interaction current): A dashed line with momentum q and a solid line with momentum p meet at a vertex. A solid line with momentum p' and a wavy line with momentum k meet at another vertex.

Local gauge invariance

$$\Phi \rightarrow \Phi e^{-i\lambda(x)}$$

Generalized Ward-Takahashi identities (gWTI)

$$k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

$$k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m (t - M_m^2)$$

$$k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$$

off-shell relations



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{Diagram 1}}_{s\text{-channel}} + \underbrace{\text{Diagram 2}}_{u\text{-channel}} + \underbrace{\text{Diagram 3}}_{t\text{-channel}} + \underbrace{\text{Diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four Feynman-like diagrams for the transition amplitude M^μ . Each diagram has an incoming fermion line with momentum p' and an outgoing fermion line with momentum p . A wavy boson line with momentum q and index μ is attached to the fermion lines. Diagram 1 (s-channel) has a vertex S on the incoming line and a vertex i on the outgoing line. Diagram 2 (u-channel) has a vertex f on the incoming line and a vertex u on the outgoing line. Diagram 3 (t-channel) has a vertex m on the incoming line and a vertex t on the outgoing line. Diagram 4 (interaction current) has a vertex i on the incoming line and a vertex i on the outgoing line.

Local gauge invariance

$$\Phi \rightarrow \Phi e^{-i\lambda(x)}$$

Generalized Ward-Takahashi identities (gWTI)

$$k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

$$k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m (t - M_m^2)$$

$$k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$$

off-shell relations

local gauge invariance \Rightarrow implies existence of e.m. field



Gauge Invariance

$$\begin{aligned}
 M^\mu &= \underbrace{\text{diagram 1}}_{\text{s-channel}} + \underbrace{\text{diagram 2}}_{\text{u-channel}} + \underbrace{\text{diagram 3}}_{\text{t-channel}} + \underbrace{\text{diagram 4}}_{\text{interaction current}} \\
 &= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu
 \end{aligned}$$

The diagrams show four Feynman-like diagrams for the transition amplitude M^μ . Each diagram has an incoming dashed line with momentum q and an outgoing wavy line with momentum k . The other two external lines have momenta p' and p . The diagrams are: 1) s-channel with a vertex labeled 's', 2) u-channel with a vertex labeled 'u', 3) t-channel with a vertex labeled 't', and 4) a direct interaction current vertex.

Local gauge invariance

$$\Phi \rightarrow \Phi e^{-i\lambda(x)}$$

Generalized Ward-Takahashi identities (gWTI)

$$\begin{aligned}
 k_\mu M^\mu &= (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p) \\
 k_\mu J_m^\mu &= (q^2 - M_m^2) Q_m - Q_m (t - M_m^2) \\
 k_\mu M_{\text{int}}^\mu &= Q_m F_t + Q_f F_u - F_s Q_i
 \end{aligned}$$

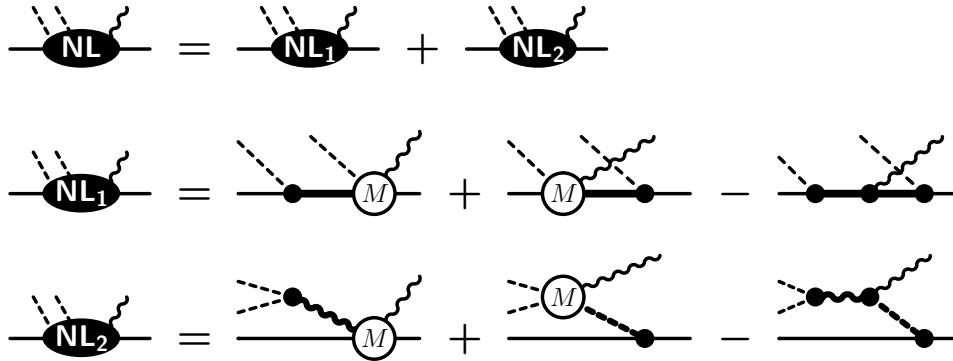
off-shell relations

local gauge invariance \Rightarrow implies existence of e.m. field

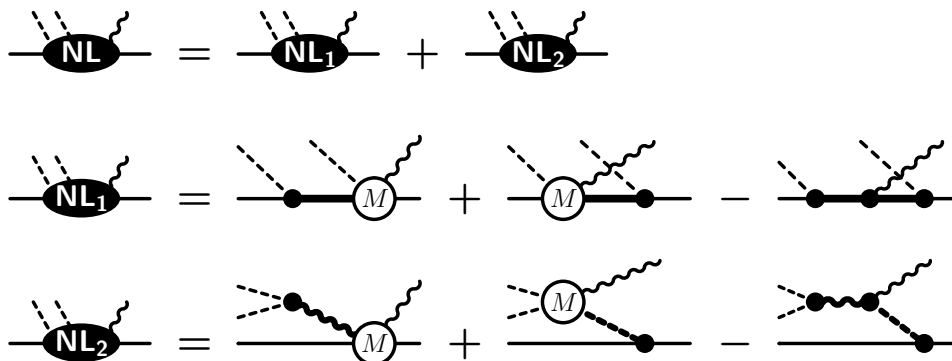
Without gWTI underlying e.m. field is damaged



Example: Two-pion production at the no-loop level



Example: Two-pion production at the no-loop level:



Without gWTI, this amplitude will not be gauge invariant



Generalized Ward-Takahashi Identities

$$(1) \quad k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

$$(2) \quad k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m(t - M_m^2)$$

$$(3) \quad k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$$



Generalized Ward-Takahashi Identities

$$(1) \quad k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

$$(2) \quad k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m(t - M_m^2) \quad \text{trivial}$$

$$(3) \quad k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$$

Only two relations are independent \Rightarrow Use (2) & (3)

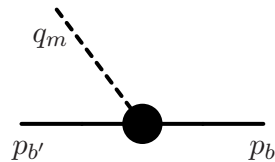


Generalized Ward-Takahashi Identities

- (1) $k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$
- (2) $k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m(t - M_m^2)$ trivial
- (3) $k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$

Only two relations are independent \Rightarrow Use (2) & (3)

Hadronic vertex



$$F(p_{b'}, p_b) = \mathbf{G}(q_m) \boldsymbol{\tau} f(q_m^2, p_{b'}^2, p_b^2)$$

$$\begin{cases} f_s(s) = f(M_m^2, M_{b'}^2, s) \\ f_u(u) = f(M_m^2, u, M_b^2) \\ f_t(t) = f(t, M_{b'}^2, M_b^2) \end{cases}$$

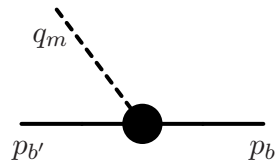


Generalized Ward-Takahashi Identities

- (1) $k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$
- (2) $k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m(t - M_m^2)$ trivial
- (3) $k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$

Only two relations are independent \Rightarrow Use (2) & (3)

Hadronic vertex



$$F(p_{b'}, p_b) = \mathbf{G}(q_m) \boldsymbol{\tau} f(q_m^2, p_{b'}^2, p_b^2)$$

$$\begin{cases} f_s(s) = f(M_m^2, M_{b'}^2, s) \\ f_u(u) = f(M_m^2, u, M_b^2) \\ f_t(t) = f(t, M_{b'}^2, M_b^2) \end{cases}$$

Interaction-current Ansatz:

$$M_{\text{int}}^\mu = m_c^\mu f_t(t) + \mathbf{G}(q) \mathbf{C}^\mu + T_{\text{int}}^\mu$$

$$k_\mu T_{\text{int}}^\mu \equiv 0$$

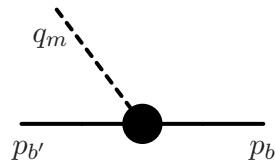


Generalized Ward-Takahashi Identities

- (1) $k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$
- (2) $k_\mu J_m^\mu = (q^2 - M_m^2) Q_m - Q_m(t - M_m^2)$ trivial
- (3) $k_\mu M_{\text{int}}^\mu = Q_m F_t + Q_f F_u - F_s Q_i$

Only two relations are independent \Rightarrow Use (2) & (3)

Hadronic vertex



$$F(p_{b'}, p_b) = \mathbf{G}(q_m) \boldsymbol{\tau} f(q_m^2, p_{b'}^2, p_b^2)$$

$$\begin{cases} f_s(s) = f(M_m^2, M_{b'}^2, s) \\ f_u(u) = f(M_m^2, u, M_b^2) \\ f_t(t) = f(t, M_{b'}^2, M_b^2) \end{cases}$$

Interaction-current Ansatz:

$$M_{\text{int}}^\mu = m_c^\mu f_t(t) + \mathbf{G}(q) C^\mu + T_{\text{int}}^\mu$$

$$k_\mu T_{\text{int}}^\mu \equiv 0$$

\Rightarrow Determine C^μ such that (3) is true



Non-singular

$$\begin{aligned}
 C^\mu = & -e_m(2q - k)^\mu \frac{f_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\
 & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t f_t + \delta_s f_s - \delta_t \delta_s f_t f_s) \\
 & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t f_t - \delta_u \delta_t f_u f_t)
 \end{aligned}$$



Non-singular

$$\begin{aligned}
 C^\mu = & -e_m(2q - k)^\mu \frac{f_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\
 & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t f_t + \delta_s f_s - \delta_t \delta_s f_t f_s) \\
 & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t f_t - \delta_u \delta_t f_u f_t)
 \end{aligned}$$

where

$$\delta_x = \begin{cases} 1 & \text{channel contributes} \\ 0 & \text{channel does not contribute} \end{cases} \quad x = s, u, t$$



Non-singular

$$\begin{aligned}
 C^\mu = & -e_m(2q - k)^\mu \frac{f_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\
 & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t f_t + \delta_s f_s - \delta_t \delta_s f_t f_s) \\
 & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t f_t - \delta_u \delta_t f_u f_t)
 \end{aligned}$$

where

$$\delta_x = \begin{cases} 1 & \text{channel contributes} \\ 0 & \text{channel does not contribute} \end{cases} \quad x = s, u, t$$

Charge conservation: $Q_m \tau + Q_f \tau - \tau Q_i = e_m + e_f - e_i = 0$



Non-singular

$$\begin{aligned}
 C^\mu = & -e_m(2q - k)^\mu \frac{f_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\
 & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t f_t + \delta_s f_s - \delta_t \delta_s f_t f_s) \\
 & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t f_t - \delta_u \delta_t f_u f_t)
 \end{aligned}$$

where

$$\delta_x = \begin{cases} 1 & \text{channel contributes} \\ 0 & \text{channel does not contribute} \end{cases} \quad x = s, u, t$$

Charge conservation: $Q_m \tau + Q_f \tau - \tau Q_i = e_m + e_f - e_i = 0$

Four-divergence: $k_\mu C^\mu = e_m f_t + e_f f_u - e_i f_s$

ensures correct four-divergence for M_{int}^μ

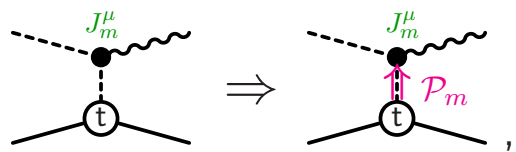


Back to Regge . . .

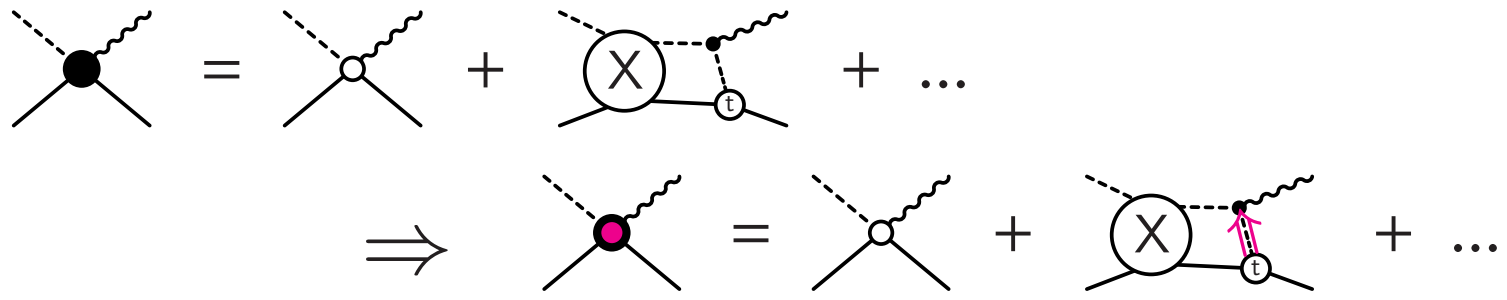


Reggeizing Final-state Interaction

Reggeize both t -channel,

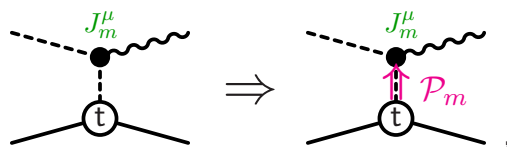


and FSI contribution,

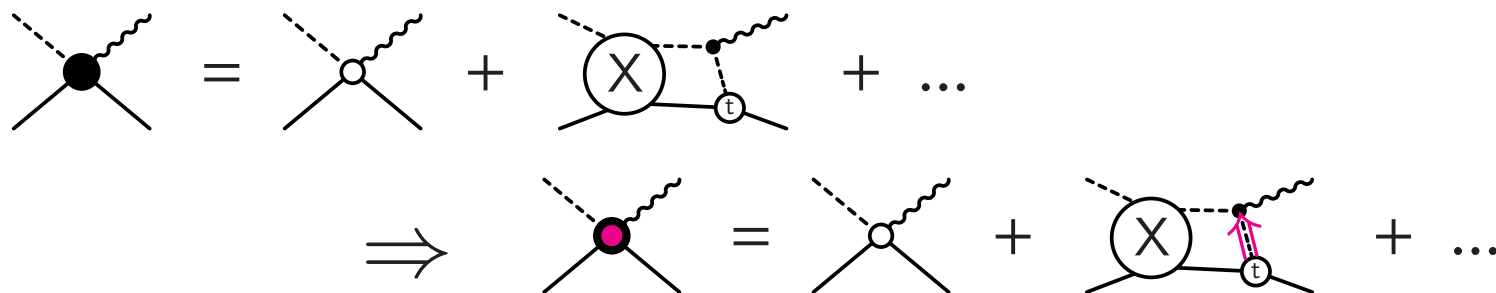


Reggeizing Final-state Interaction

Reggeize both t -channel,



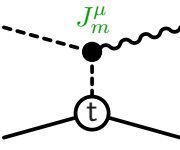
and FSI contribution,



Not necessary to calculate FSI loops \Rightarrow modify C^μ instead



Before Reggeization

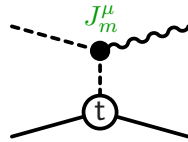
$$J_m^\mu \frac{G\tau}{t - M_m^2} f_t =$$




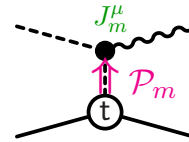
Reggeizing t -Channel

Before Reggeization

$$J_m^\mu \frac{G\tau}{t - M_m^2} f_t =$$



\Rightarrow



$=$

$$J_m^\mu \frac{G\tau}{t - M_m^2} \mathcal{F}_t$$

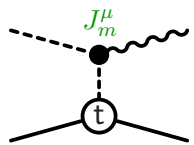
After Reggeization



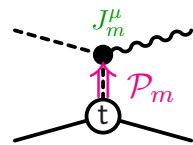
Reggeizing t -Channel

Before Reggeization

$$J_m^\mu \frac{G\tau}{t - M_m^2} f_t \quad \uparrow$$



\Rightarrow



$=$

After Reggeization

$$J_m^\mu \frac{G\tau}{t - M_m^2} \mathcal{F}_t \quad \uparrow$$

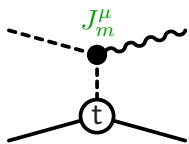
Reggeization corresponds to an effective prescription for hadronic form factor



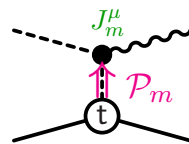
Reggeizing t -Channel

Before Reggeization

$$J_m^\mu \frac{G\tau}{t - M_m^2} f_t =$$



\Rightarrow



$=$

After Reggeization

$$J_m^\mu \frac{G\tau}{t - M_m^2} \mathcal{F}_t$$

Reggeization corresponds to an effective prescription for hadronic form factor

To preserve local gauge invariance,
replace f_t by Regge residual function \mathcal{F}_t everywhere



The Cure: Modified Auxiliary Contact Current \mathcal{C}^μ

$$\mathcal{M}_{\text{int}}^\mu = m_c^\mu \mathcal{F}_t + \mathbf{G}(q) \mathcal{C}^\mu + T_{\text{int}}^\mu$$

$$k_\mu T_{\text{int}}^\mu \equiv 0$$

Non-singular

$$\begin{aligned} \mathcal{C}^\mu = & -e_m(2q - k)^\mu \frac{\mathcal{F}_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\ & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t \mathcal{F}_t + \delta_s f_s - \delta_t \delta_s \mathcal{F}_t f_s) \\ & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t \mathcal{F}_t - \delta_u \delta_t f_u \mathcal{F}_t) \end{aligned}$$



The Cure: Modified Auxiliary Contact Current \mathcal{C}^μ

$$\mathcal{M}_{\text{int}}^\mu = m_c^\mu \mathcal{F}_t + \mathbf{G}(q) \mathcal{C}^\mu + T_{\text{int}}^\mu$$

$$k_\mu T_{\text{int}}^\mu \equiv 0$$

$$\begin{aligned} \mathcal{C}^\mu = & -e_m(2q - k)^\mu \frac{\mathcal{F}_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\ & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t \mathcal{F}_t + \delta_s f_s - \delta_t \delta_s \mathcal{F}_t f_s) \\ & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t \mathcal{F}_t - \delta_u \delta_t f_u \mathcal{F}_t) \end{aligned}$$

Provides correct generalized Ward-Takahashi identity:

$$k_\mu \mathcal{M}^\mu = (q^2 - M_m^2) Q_m \hat{\mathcal{F}}_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

Non-singular



The Cure: Modified Auxiliary Contact Current \mathcal{C}^μ

$$\mathcal{M}_{\text{int}}^\mu = m_c^\mu \mathcal{F}_t + \mathbf{G}(q) \mathcal{C}^\mu + T_{\text{int}}^\mu$$

$$k_\mu T_{\text{int}}^\mu \equiv 0$$

Non-singular

$$\begin{aligned} \mathcal{C}^\mu = & -e_m(2q - k)^\mu \frac{\mathcal{F}_t - 1}{t - M_m^2} (\delta_s f_s + \delta_u f_u - \delta_s \delta_u f_s f_u) \\ & - e_f(2p' - k)^\mu \frac{f_u - 1}{u - M_f^2} (\delta_t \mathcal{F}_t + \delta_s f_s - \delta_t \delta_s \mathcal{F}_t f_s) \\ & - e_i(2p + k)^\mu \frac{f_s - 1}{s - M_i^2} (\delta_u f_u + \delta_t \mathcal{F}_t - \delta_u \delta_t f_u \mathcal{F}_t) \end{aligned}$$

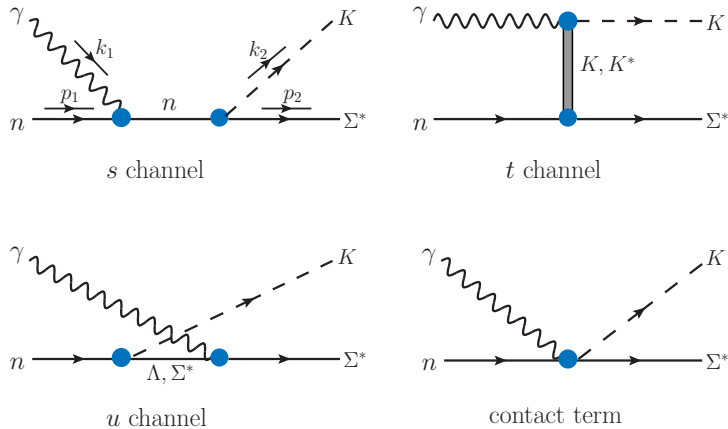
Provides correct generalized Ward-Takahashi identity:

$$k_\mu \mathcal{M}^\mu = (q^2 - M_m^2) Q_m \hat{\mathcal{F}}_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

⇒ **Production current locally gauge invariant**



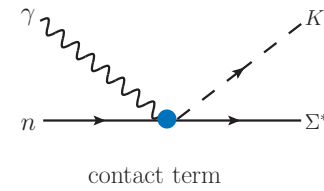
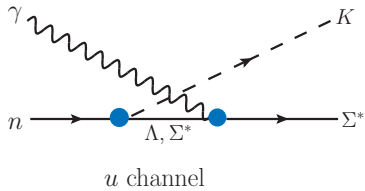
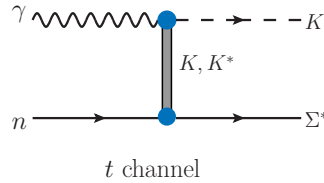
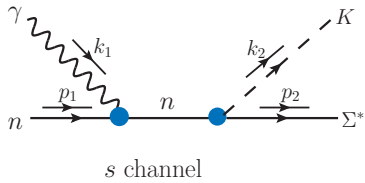
Application: $\gamma + n \rightarrow K^+ + \Sigma^*(1385)^-$



Compared with data from
 CLAS: P. Mattione (CLAS Collaboration),
 Int. J. Phys. Conf. Series **26**, 1460101, (2014);
 LEPS: K. Hicks et al. (LEPS Collaboration),
 Phys. Rev. Lett. **102**, 012501 (2009).



Application: $\gamma + n \rightarrow K^+ + \Sigma^*(1385)^-$



For gauge-invariance considerations, the s -channel is irrelevant.

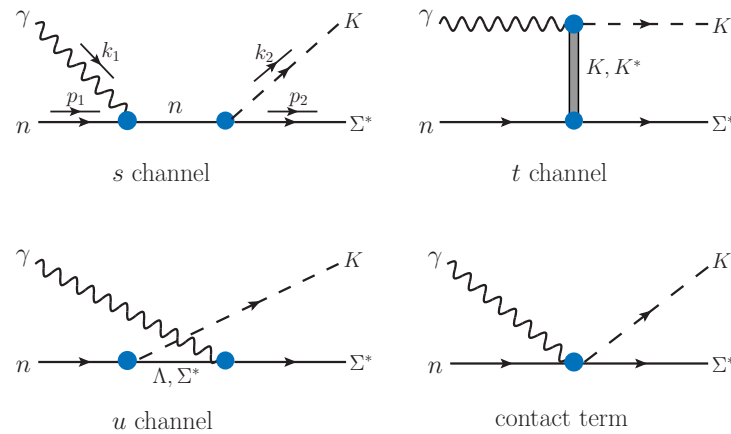
Interpolation between f_t and \mathcal{F}_t :

$$\tilde{f}_t(t) = \mathcal{F}_t(t) R_s + f_t(t) (1 - R_s)$$

Compared with data from
 CLAS: P. Mattione (CLAS Collaboration),
 Int. J. Phys. Conf. Series **26**, 1460101, (2014);
 LEPS: K. Hicks et al. (LEPS Collaboration),
 Phys. Rev. Lett. **102**, 012501 (2009).



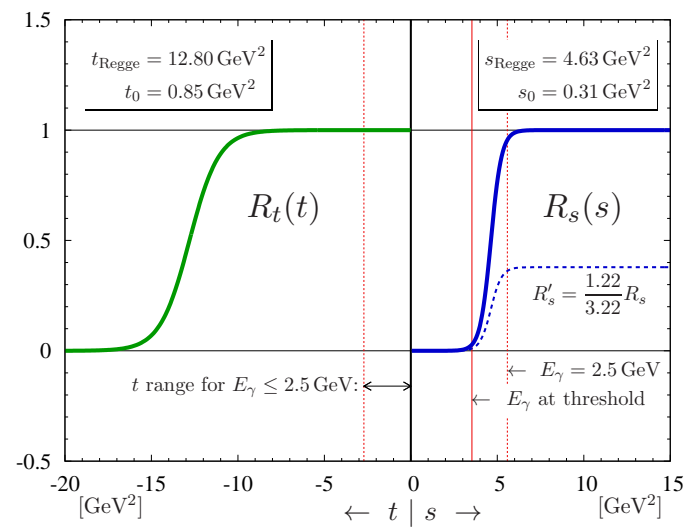
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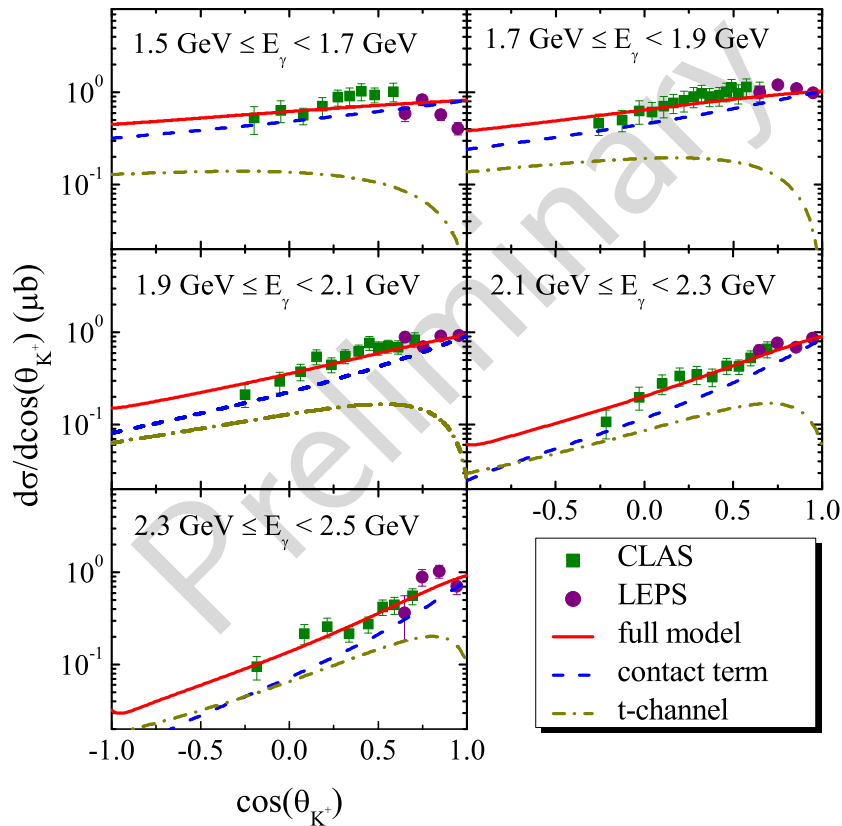
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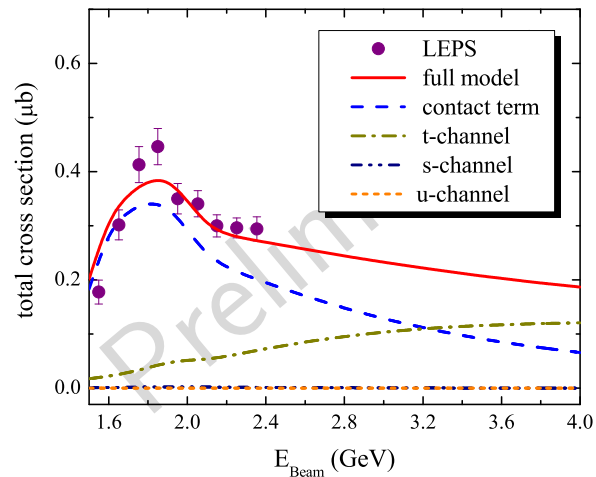
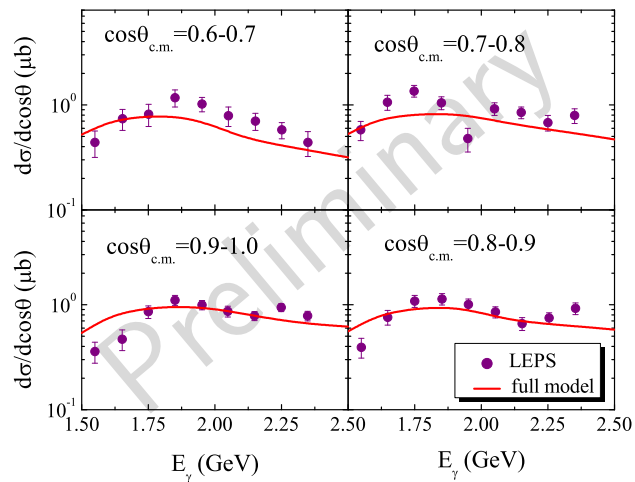
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Summary

- Implementation of Regge phenomenology for t -channel exchange corresponds to replacing usual phenomenological form factor f_t by Regge residual function \mathcal{F}_t
- Reggeization of t -channel leads to violation of gauge invariance due to *inconsistent* implementation of Reggeization
- Interaction-current M_{int}^μ needs to be Reggeized as well
- Global gauge invariance not a good starting point ~~[GLV]~~
- Correct dynamical basis provided by **generalized Ward-Takahashi identities** as they follow from **local** gauge invariance
- The cure: Modify auxiliary contact current C^μ
- Application to $\gamma + n \rightarrow K^+ + \Sigma^*(1385)^-$ at energies up to 2.5 GeV requires mixing of conventional and Reggeized t -channel to provide acceptable χ^2
- Method can easily be applied to Reggeizing u -channel as well



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Thank you!

