Preserving local gauge invariance with t-channel Regge exchange

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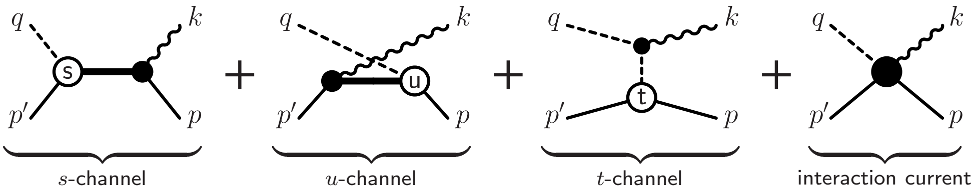
Outline

- Defining the problem
- Regge-trajectory basics
- How *not* to cure the problem
- The origin of the problem
- Gauge invariance basics
- Implementation of local gauge invariance
- The cure
- Application: $\gamma + n \to K^+ + \Sigma^*(1385)^-$
- Summary

Defining the Problem

← time ——

$M^\mu =$



$$= F_s\, S_i\, J_i^\mu + J_f^\mu\, S_f\, F_u + J_m^\mu\, \Delta_m\, F_t + M_{\text{int}}^\mu$$

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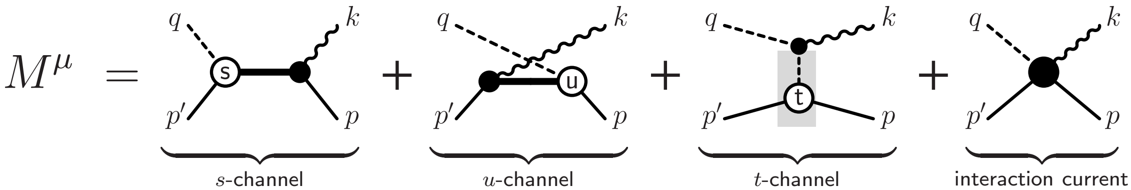

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Replace t-channel single-meson exchange by **Regge-trajectory exchange**: $\Delta_m F_t \to \mathcal{P}_m$

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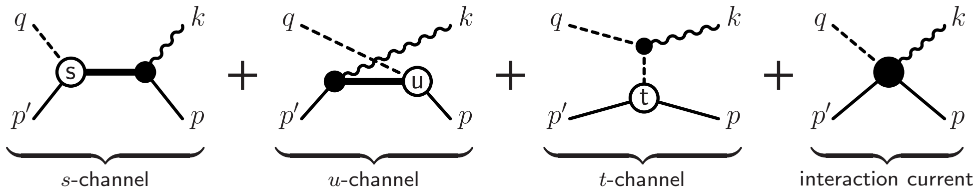

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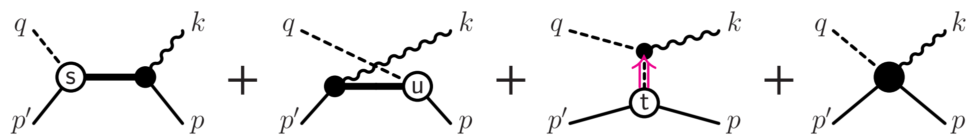

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$\mathcal{P}_m$: Regge-trajectory propagator

Defining the Problem

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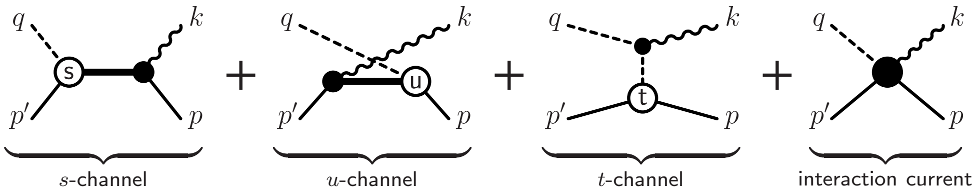

$$k_\mu M^\mu = 0$$

current conserved

$$= F_s S_i J_i^\mu + J_f^\mu S_f F_u + J_m^\mu \Delta_m F_t + M_{\text{int}}^\mu$$

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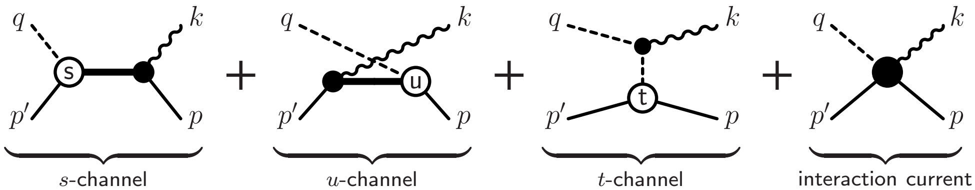

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With Reggeized t-channel:

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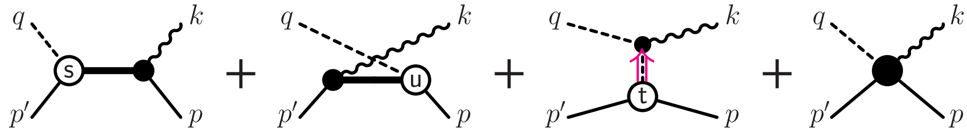

$$k_\mu \mathcal{M}^\mu \neq 0$$

current not conserved

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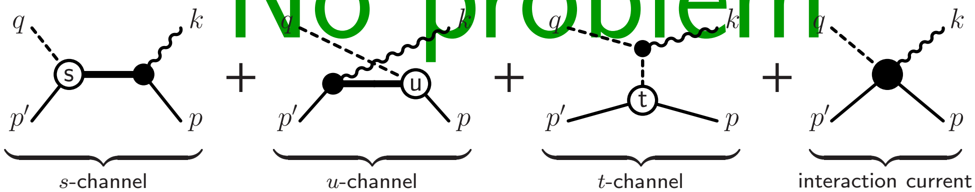
Defining the Problem

No problem

$$M^\mu =$$



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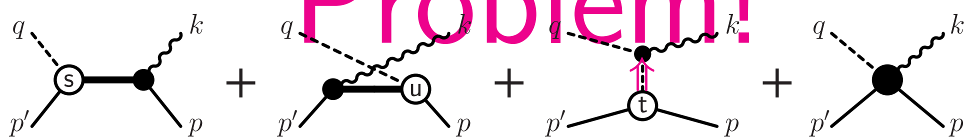
With Reggeized t-channel:

Problem!

$$\mathcal{M}^\mu =$$



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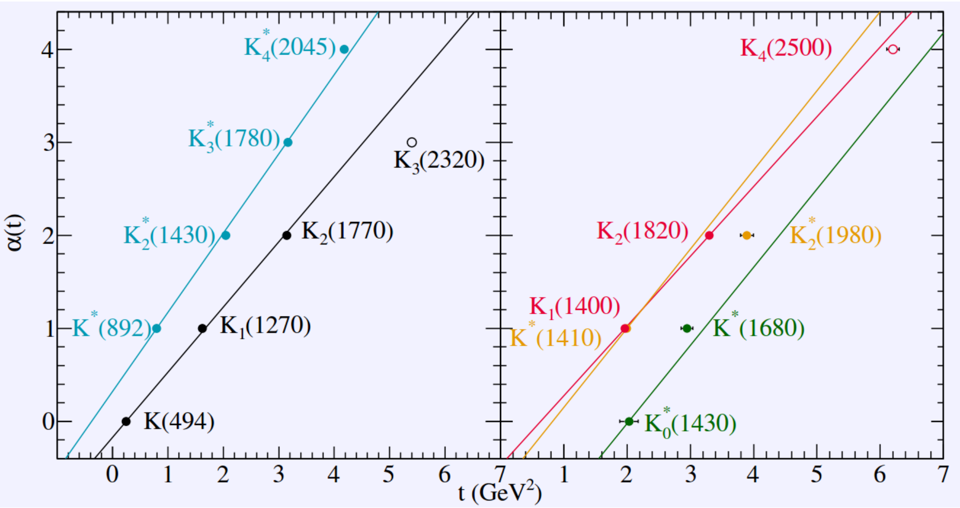
Regge trajectories

Graph stolen from P. Vancraeyveld, PhD Thesis, U. Gent, 2011



Trajectories:

pseudoscalar:

$$\alpha_+(t) = \alpha_0(t) \ ,$$
$$\alpha_0(t) = \alpha'_0\,(t - M_0^2)$$

vector:

$$\alpha_-(t) = 1 + \alpha_1(t) \ ,$$
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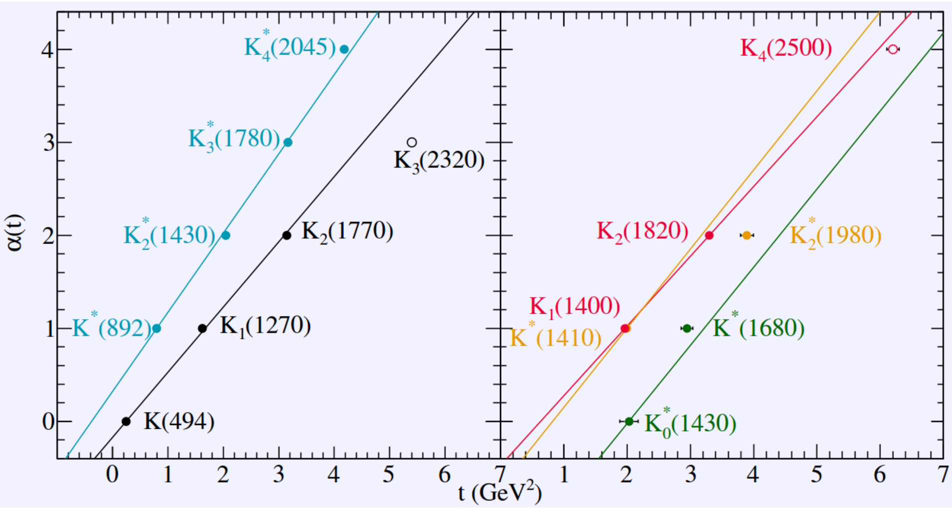

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Regge exchange for t-channel:

$m = 0, 1$

$$\Delta_m F_t \to \mathcal{P}_m(t) = \frac{1}{t - M_m^2} \mathcal{F}_m(t) \ ,$$

$$\mathcal{F}_m(t) = \left(\frac{s}{s_{\rm sc}} \right)^{\alpha_m(t)} \frac{N[\alpha_m(t); \eta]}{\Gamma\big(1 + \alpha_m(t)\big)} \frac{\pi \alpha_m(t)}{\sin\big(\pi \alpha_m(t)\big)}$$

residual Regge function

$s_{\rm sc} = 1$ GeV

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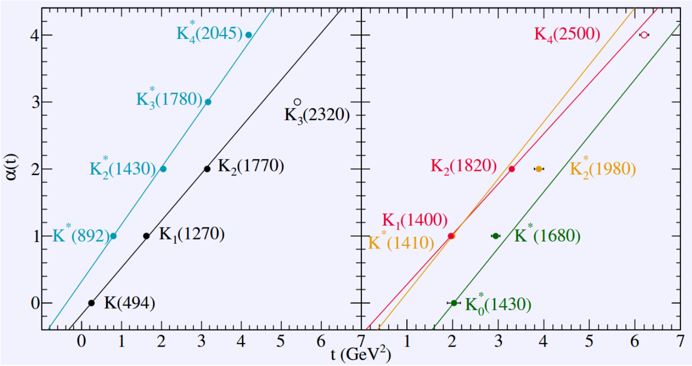
Signature function:

$$N[\alpha_m(t);\eta] = \eta + (1-\eta)\mathrm{e}^{-i\pi\alpha_m(t)}$$

$$\eta = \begin{cases} \frac{1}{2} \; , & \text{pure-signature trajectory} \\ 0 \; , & \text{add trajectories: rotating phase} \\ 1 \; , & \text{subtract trajectories: constant phase} \end{cases}$$



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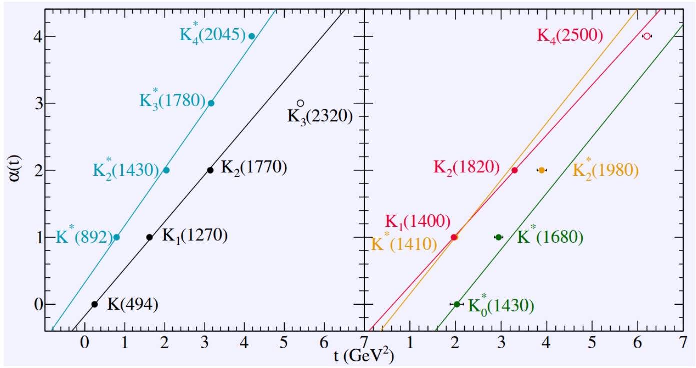


Normalization: $\quad \mathcal{F}_m(M_m^2) = 1$

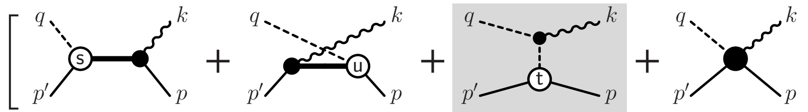
independent of η

Guidal, Laget, and Vanderhaeghen's Recipe

[Nucl. Phys. A 627, 645 (1997)]

Recipe: Take gauge-invariant amplitude M^μ and multiply by *residual function* $\mathcal{F}_m(t)$

$$M^\mu_{\mathrm{GLV}} = M^\mu \times \mathcal{F}_m =$$



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t-channel okay

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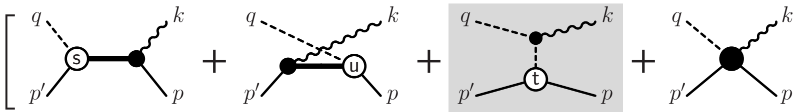

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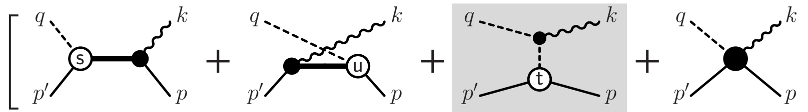

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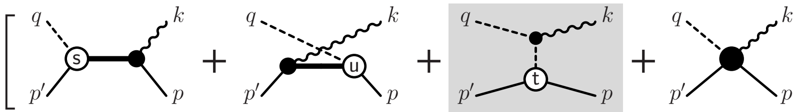

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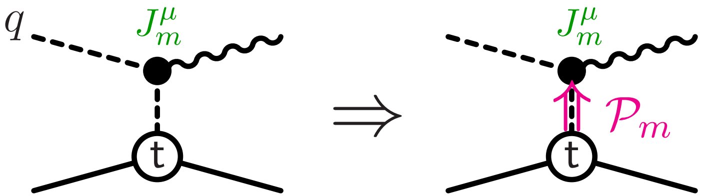
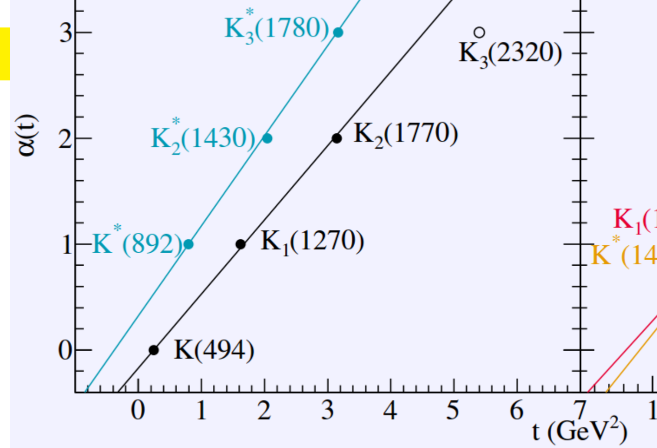
- Very popular
- Quite successful in providing good descriptions of data for many applications
- **Without any dynamical foundation**

Origin of Problem

Reason #1:





Every state in the Regge trajectory appears with same current.

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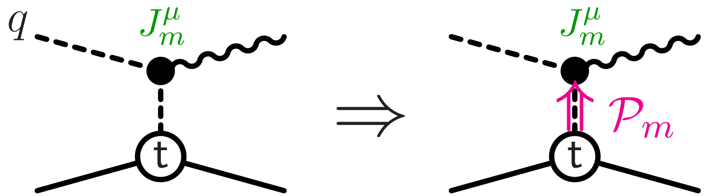

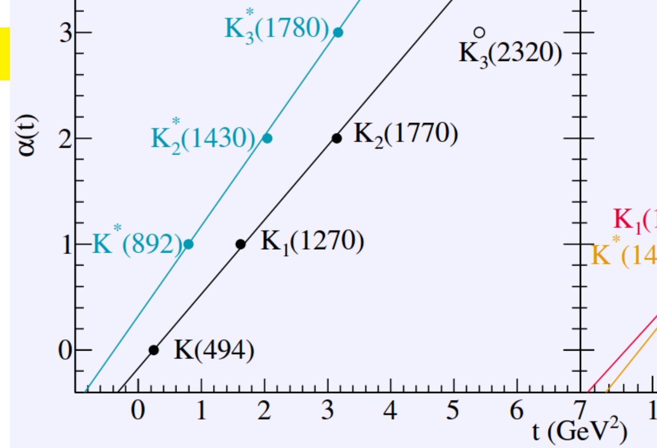

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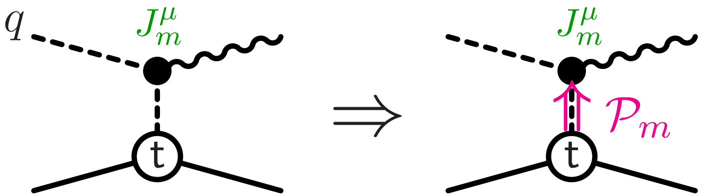

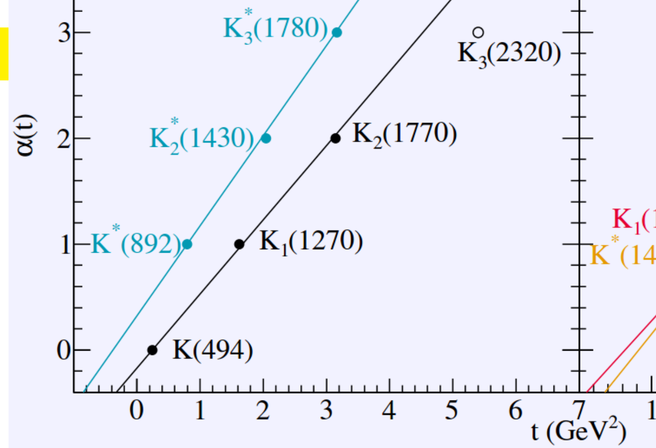

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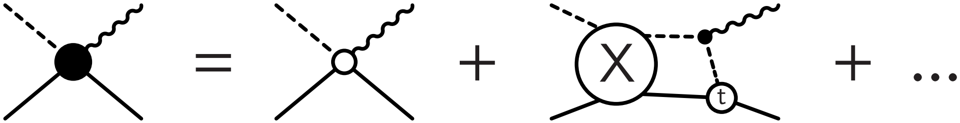
⇒ Violates Ward-Takahashi identity for intermediate higher-mass states

Origin of Problem

Reason #2:



t-Channel exchanges inside interaction current not Reggeized.

Origin of Problem

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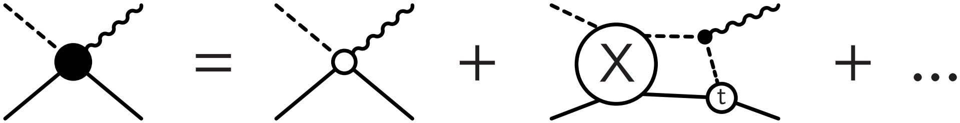

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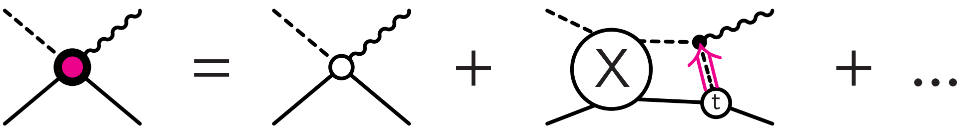
Needed: Consistent treatment



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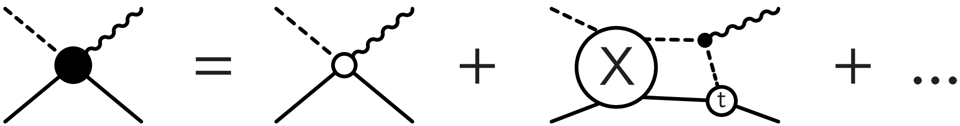


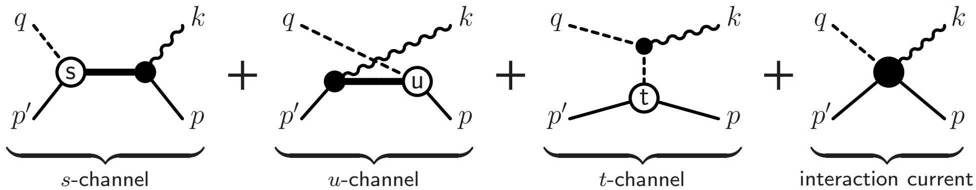
t-Channel exchanges inside interaction current not Reggeized.

Needed: Consistent treatment

$\Longrightarrow$ Interaction-current contribution must be Reggeized as well

Gauge Invariance

$$M^\mu =$$



$$= F_s\, S_i\, J_i^\mu \;+\; J_f^\mu\, S_f\, F_u \;+\; J_m^\mu\, \Delta_m\, F_t \;+\; M_{\text{int}}^\mu$$

Global gauge invariance

$$\boxed{k_\mu M^\mu = 0}$$

all external hadrons on-shell

$$\Phi \to \Phi\, \mathrm{e}^{-i\Lambda}$$

Gauge Invariance

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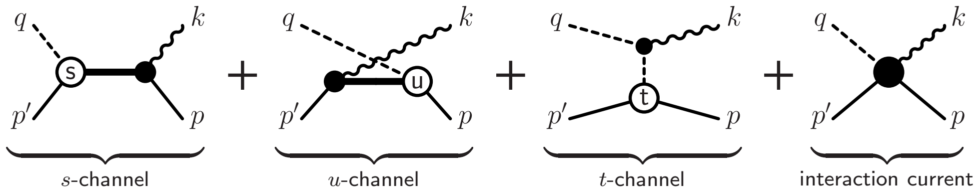

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conserved current $\quad \Rightarrow \quad$ implies charge conservation

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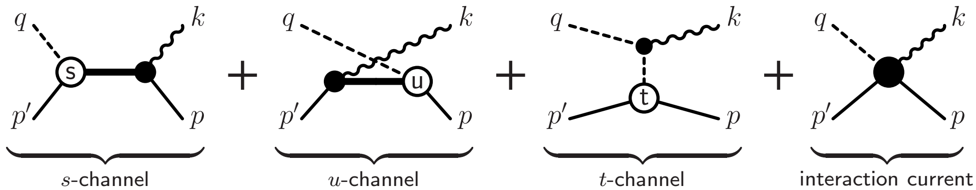

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Global gauge invariance $\boxed{k_\mu M^\mu = 0}$ all external hadrons on-shell

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Fixing global gauge invariance does not mean internal damage is fixed as well

Gauge Invariance

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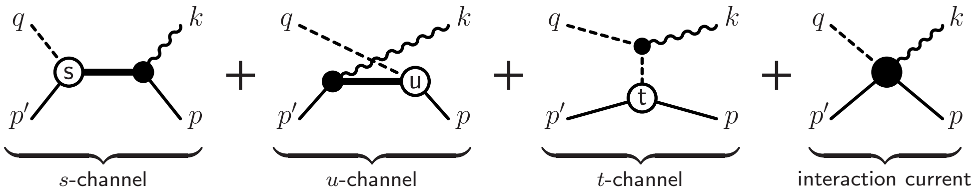

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Local gauge invariance

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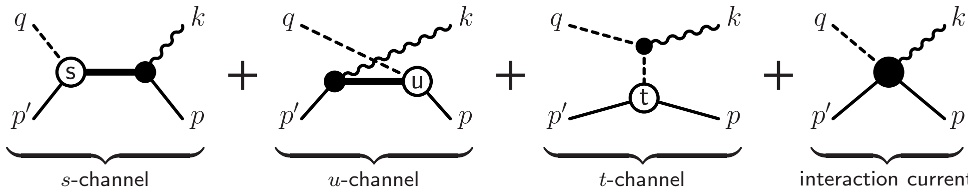

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Generalized Ward-Takahashi identities (gWTI)

$$k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

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off-shell relations

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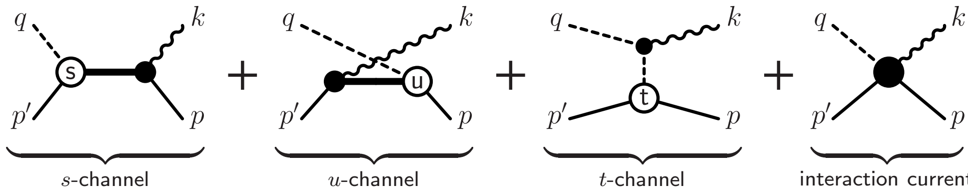
$M^\mu =$ 

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off-shell relations

local gauge invariance $\Rightarrow$ **implies existence of e.m. field**

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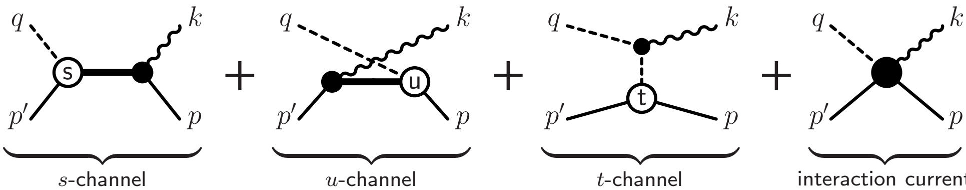

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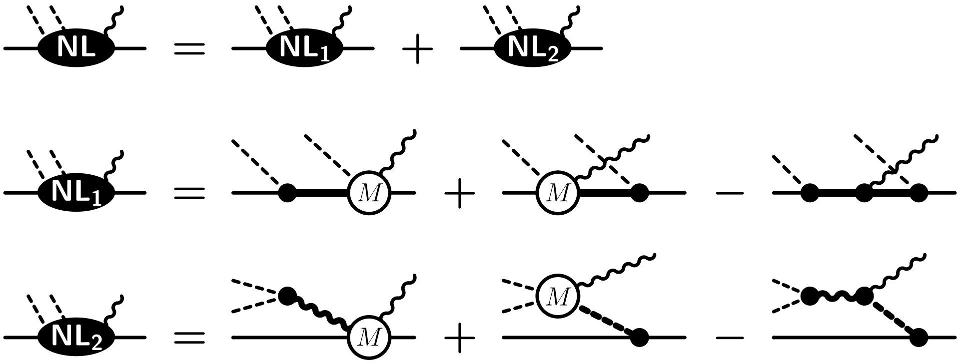
off-shell relations

local gauge invariance $\Rightarrow$ implies existence of e.m. field

Without gWTI underlying e.m. field is damaged

Practical Relevance of Local Gauge Invariance

Example: Two-pion production at the no-loop level



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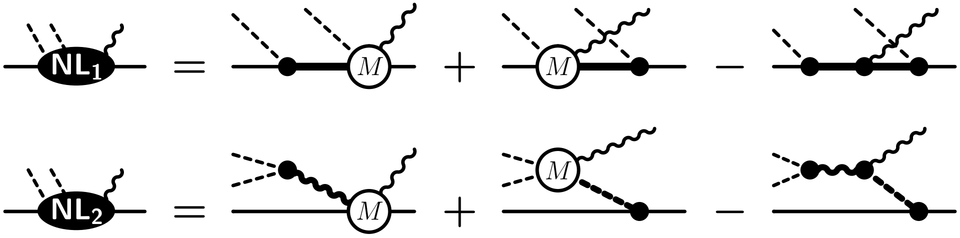

Without gWTI, this amplitude will not be gauge invariant

Generalized Ward-Takahashi Identities

$$(1) \qquad k_\mu M^\mu = (q^2 - M_m^2) Q_m F_t + S_f^{-1}(p') Q_f F_u - F_s Q_i S_i^{-1}(p)$$

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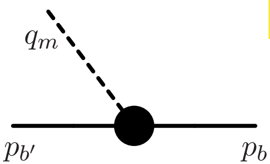

Hadronic vertex

$$F(p_{b'}, p_b) = \boldsymbol{G}(q_m)\, \boldsymbol{\tau}\, f(q_m^2, p_{b'}^2, p_b^2)$$

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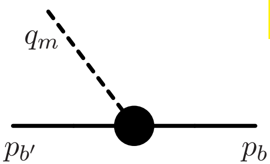

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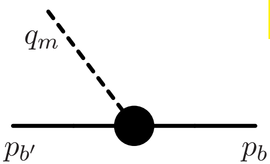

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$\Rightarrow$ Determine C^μ such that (3) is true

Auxiliary Contact Current C^μ

HH, Nakayama, Krewald, PRC74, 045202 (2006)

Non-singular

$$C^\mu = -e_m(2q-k)^\mu \frac{f_t - 1}{t - M_m^2}\left(\delta_s f_s + \delta_u f_u - \delta_s\delta_u f_s f_u\right)$$

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where

$$\delta_x = \begin{cases} 1 & \text{channel contributes} \\ 0 & \text{channel does not contribute} \end{cases} \qquad x = s, u, t$$

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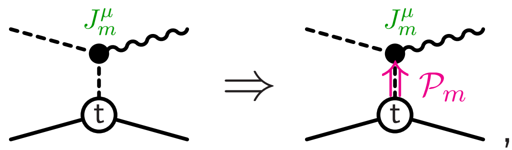
Four-divergence: $k_\mu C^\mu = e_m f_t + e_f f_u - e_i f_s$

ensures correct four-divergence for M^μ_{int}

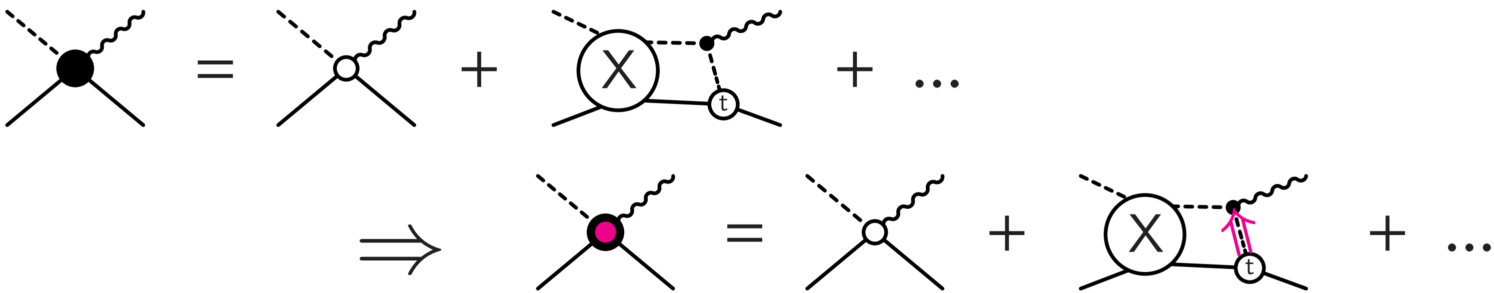
Back to Regge . . .

Reggeizing Final-state Interaction

Reggeize both t-channnel,



and FSI contribution,



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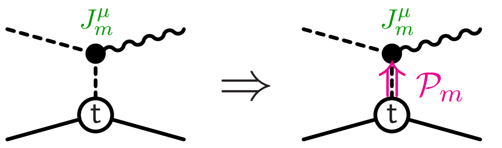

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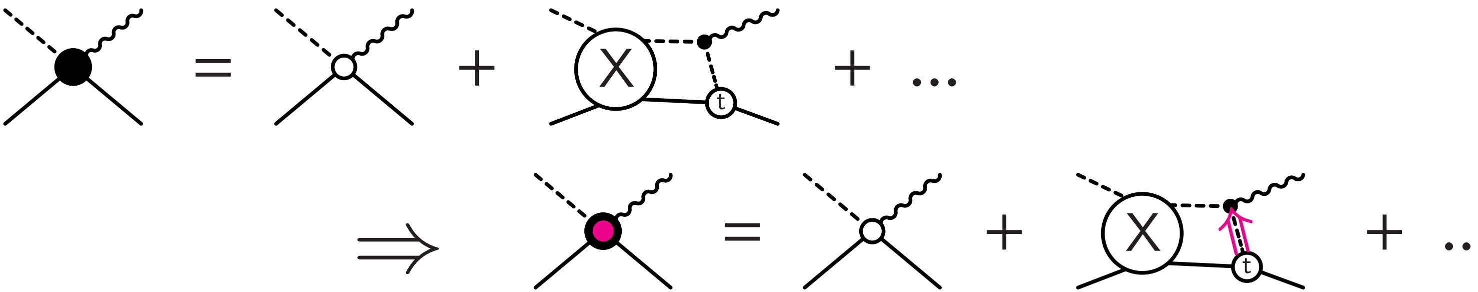
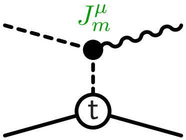


Not necessary to calculate FSI loops $\Rightarrow$ modify C^μ instead

Reggeizing t-Channel

Before Reggeization

$$J^\mu_m \frac{G\tau}{t - M_m^2} f_t =$$



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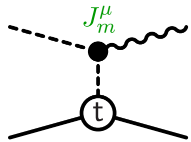
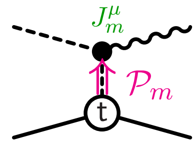


$$\Rightarrow$$



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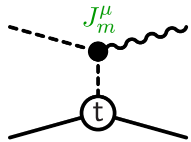

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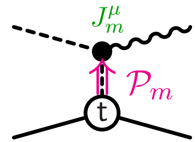

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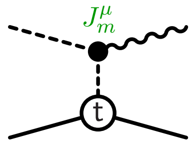

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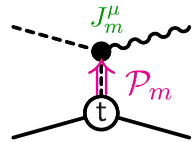

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Reggeization corresponds to an effective prescription for hadronic form factor

To preserve local gauge invariance,
replace f_t by Regge residual function $\mathcal{F}_t$ everywhere

The Cure: Modified Auxiliary Contact Current $\mathcal{C}^\mu$

$$\boxed{\mathcal{M}^\mu_{\text{int}} = m^\mu_c\,\mathcal{F}_t + \boldsymbol{G}(q)\mathcal{C}^\mu + T^\mu_{\text{int}}}$$

$$k_\mu T^\mu_{\text{int}} \equiv 0$$

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Provides correct generalized Ward-Takahashi identity:

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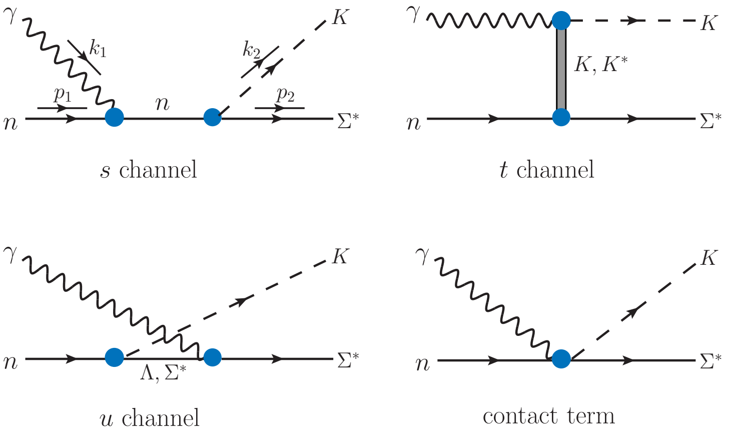
$\Rightarrow$ **Production current locally gauge invariant**

Application: $\gamma + n \to K^{+} + \Sigma^{*}(1385)^{-}$



Compared with data from
CLAS: P. Mattione (CLAS Collaboration),
Int. J. Phys. Conf. Series **26**, 1460101, (2014);
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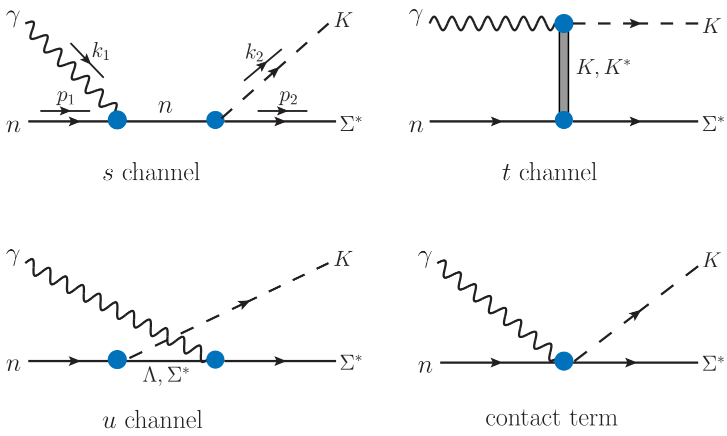

For gauge-invariance considerations, the s-channel is irrelevant.

Interpolation between f_t and $\mathcal{F}_t$:

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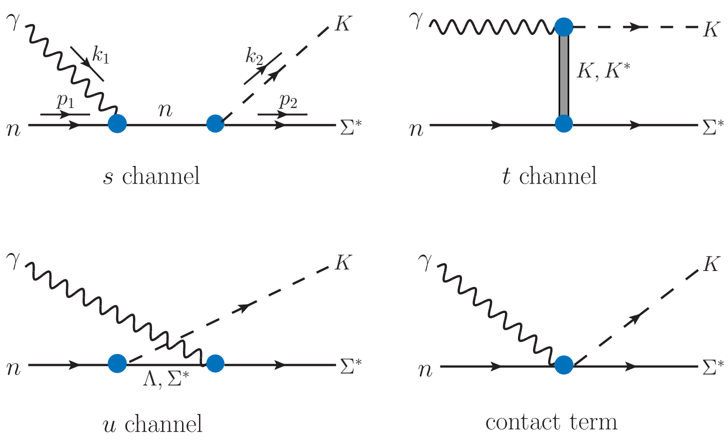

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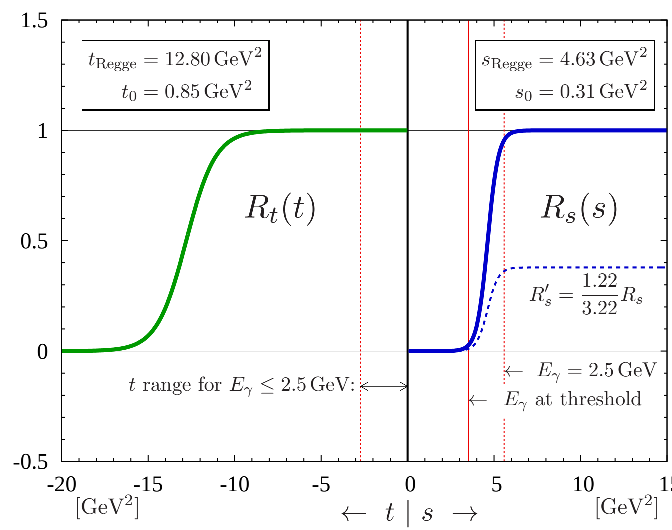

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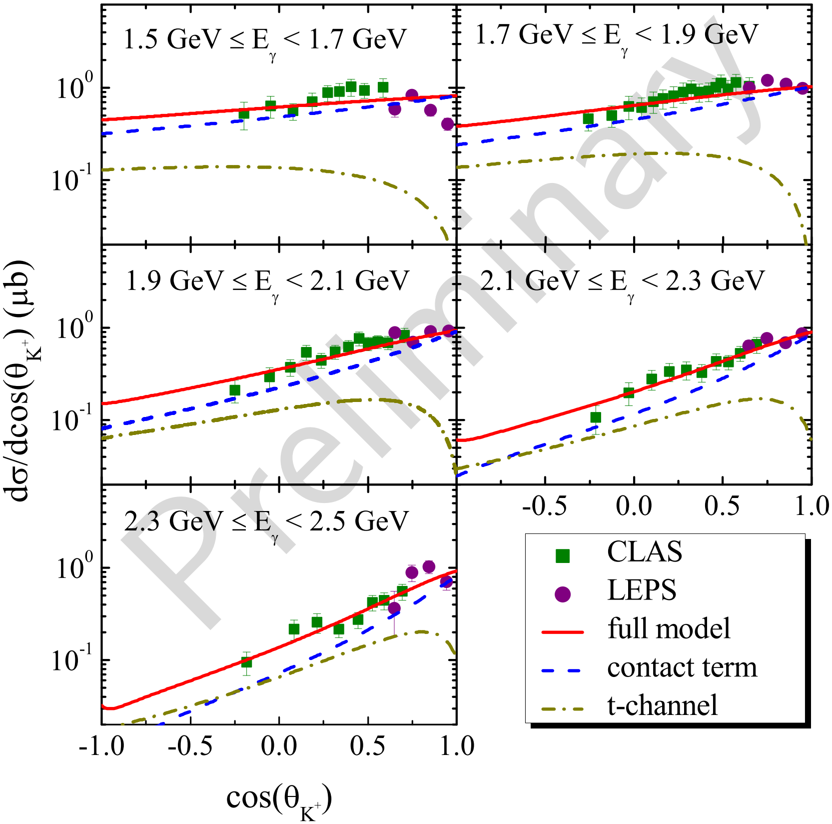

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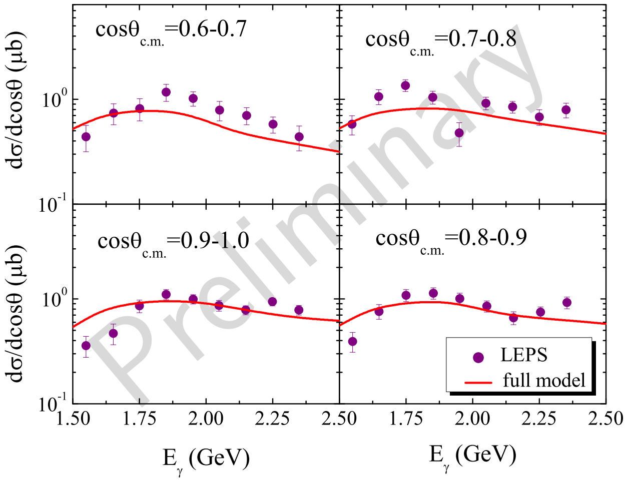

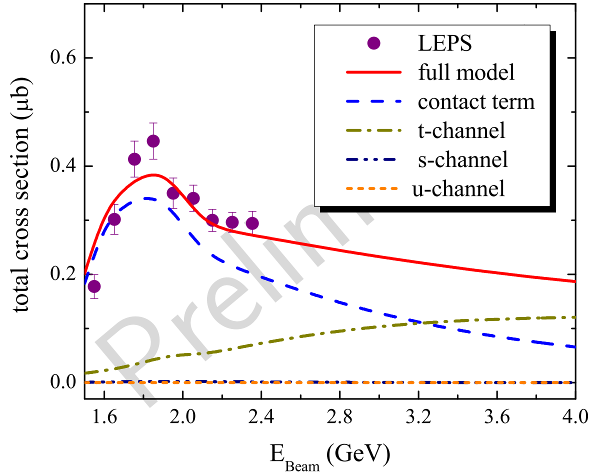

Summary

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- Reggeization of t-channel leads to violation of gauge invariance due to *inconsistent* implementation of Reggeization
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