

# THE ROPER EXCITEMENT

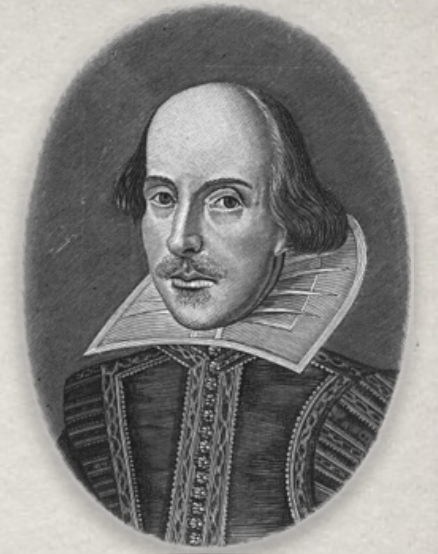
*Nucleon Resonances: From Photoproduction to High Photon Virtualities*  
ECT\* Workshop, October 14, 2015



Bruno El-Bennich

Laboratório de Física Teórica e Computacional  
Universidade Cruzeiro do Sul, São Paulo

Instituto de Física Teórica  
Universidade Estadual Paulista, São Paulo



# Much Excitement About Nothing?

adapted freely from William Shakespeare

- *Observation of the hadron mass spectrum as well as of elastic and transition form factors can be used to study the long-range behavior of QCD's interaction.*
- *Properties of excited hadron states are more sensitive to the long-range behavior of the strong interaction than those of ground states.*

# Quantum Chromodynamics

- QCD is the gauge theory that describes strong interactions.
- Description of interactions between quarks and gluons which form hadrons we observe in Nature.
- The formation of hadronic bound states via constituents is an inherently nonperturbative problem.
- It involves precise knowledge of the infrared (long distance) regime of QCD and the dynamical generation of a constituent quark mass.

## The Lagrangian of QCD

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

# The Lagrangian of QCD

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- The key to complexity in QCD lies the gluon field strength tensor.

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g f^{abc} G_\mu^b G_\nu^c$$

# The Lagrangian of QCD

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i\gamma^\mu \partial_\mu - m) \psi_i - g G_\mu^a \bar{\psi}_i \gamma^\mu T_{ij}^a \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$

- The key to complexity in QCD lies the gluon field strength tensor.

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a + g f^{abc} G_\mu^b G_\nu^c$$

- It generates self-interactions with far-reaching consequences for hadron phenomenology.

This complexity also affects the bare quark-gluon vertex in a nonperturbative manner!

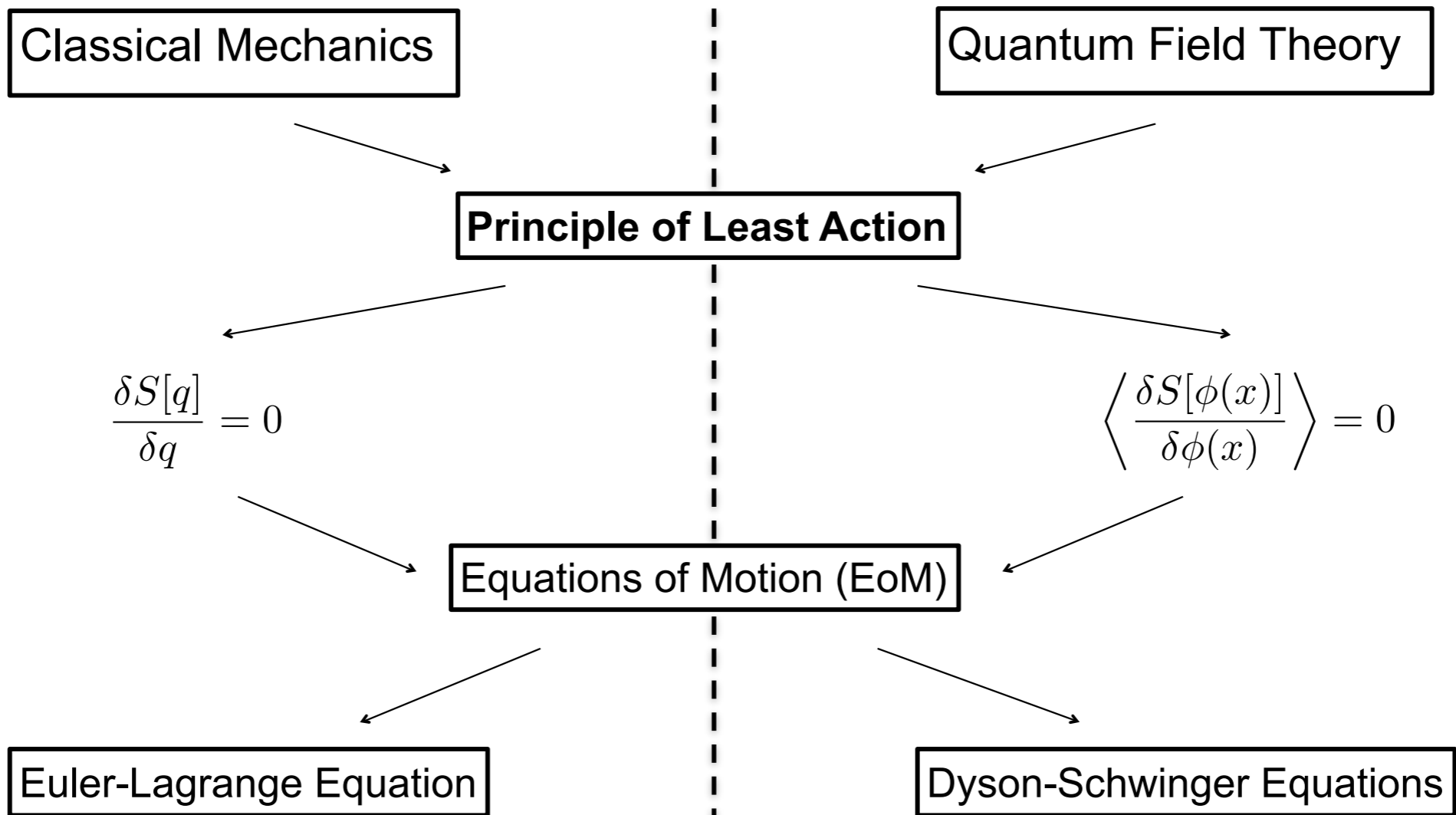
Por favor  
não perturbe

Please, do not disturb

Por favor, no molestar

Ποι παρακαλώ, μη ενοχώ

# Nonperturbative Continuum Tools for QCD



courtesy of Sishue Qin



# QCD's Dyson-Schwinger Equations

The propagator can be obtained from QCD's **gap equation**: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of **infinitely many** coupled equations:

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) := i\gamma \cdot p A(p^2) + B(p^2)$$

$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a(q,p)$$

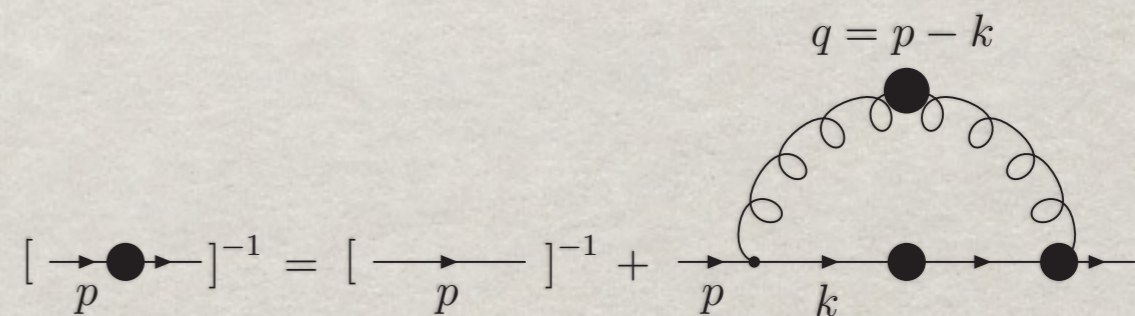
with the *running* mass function  $M(p^2) = B(p^2)/A(p^2)$ .

- $D_{\mu\nu}$  : dressed-gluon propagator
- $\Gamma_\nu^a(q,p)$  : dressed quark-gluon vertex
- $Z_2$  : quark wave function renormalization constant
- $Z_1$  : quark-gluon vertex renormalization constant

each satisfies  
it's own DSE

$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma \cdot p + m(\zeta)$$

where  $\zeta$  is the renormalization point.



# QCD's Dyson-Schwinger Equations

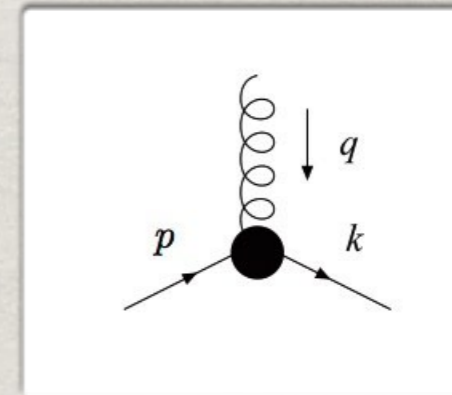
The propagator can be obtained from QCD's **gap equation**: the Dyson-Schwinger equation (DSE) for the dressed-fermion self-energy, which involves the set of **infinitely many** coupled equations:

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m^{\text{bm}}) + \Sigma(p) := i\gamma \cdot$$

$$\Sigma(p) = Z_1 \int^{\Lambda} \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_{\mu}$$

$$\Gamma_{\nu}^a(q, p)$$

with the *running* mass function  $M(p^2) =$



each satisfies its own DSE

- $D_{\mu\nu}$  : dressed-gluon propagator
- $\Gamma_{\nu}^a(q, p)$  : dressed quark-gluon vertex
- $Z_2$  : quark wave function renormalization constant
- $Z_1$  : quark-gluon vertex renormalization constant

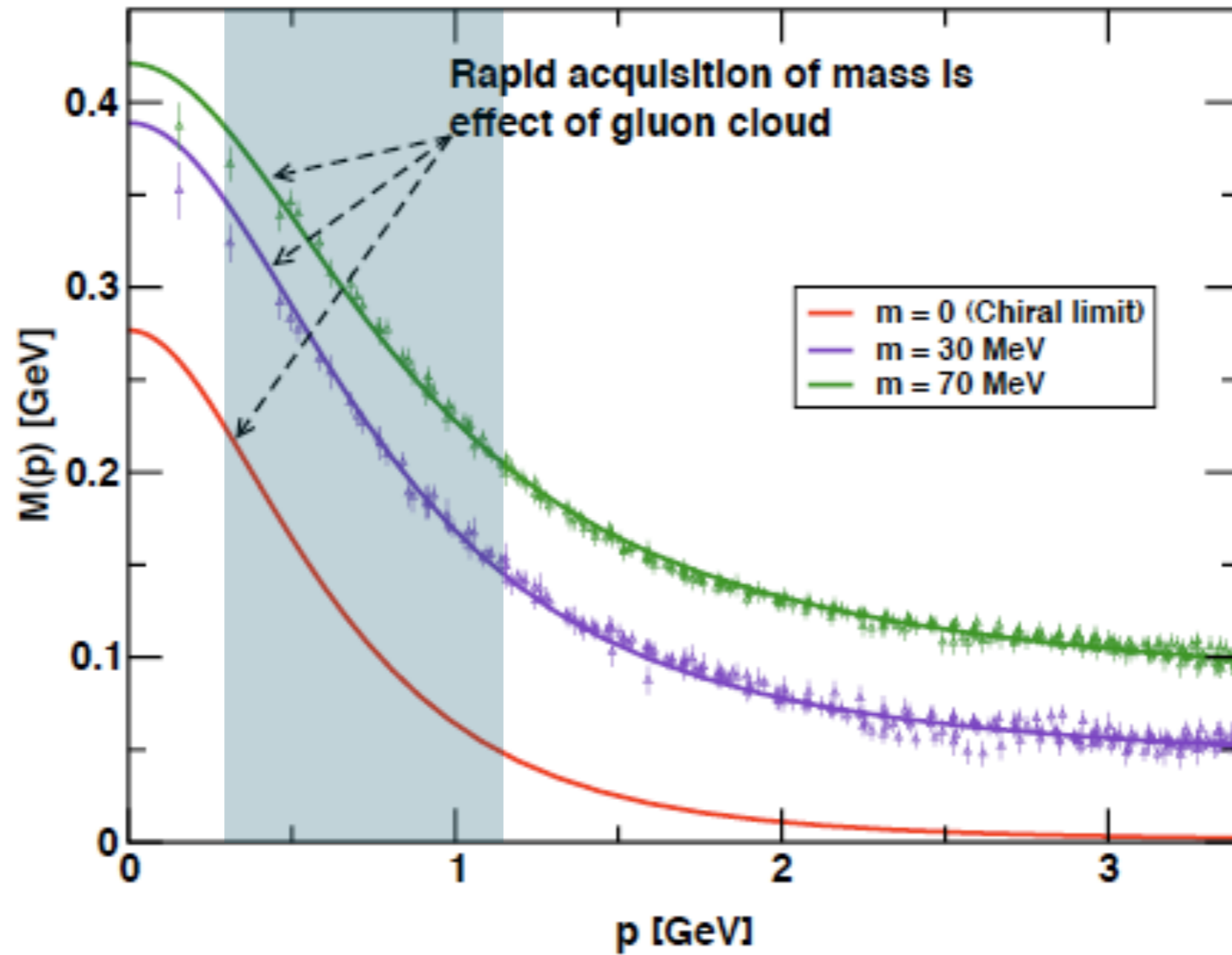
$$S^{-1}(p)|_{p^2=\zeta^2} = i\gamma \cdot p + m(\zeta)$$

where  $\zeta$  is the renormalization point.

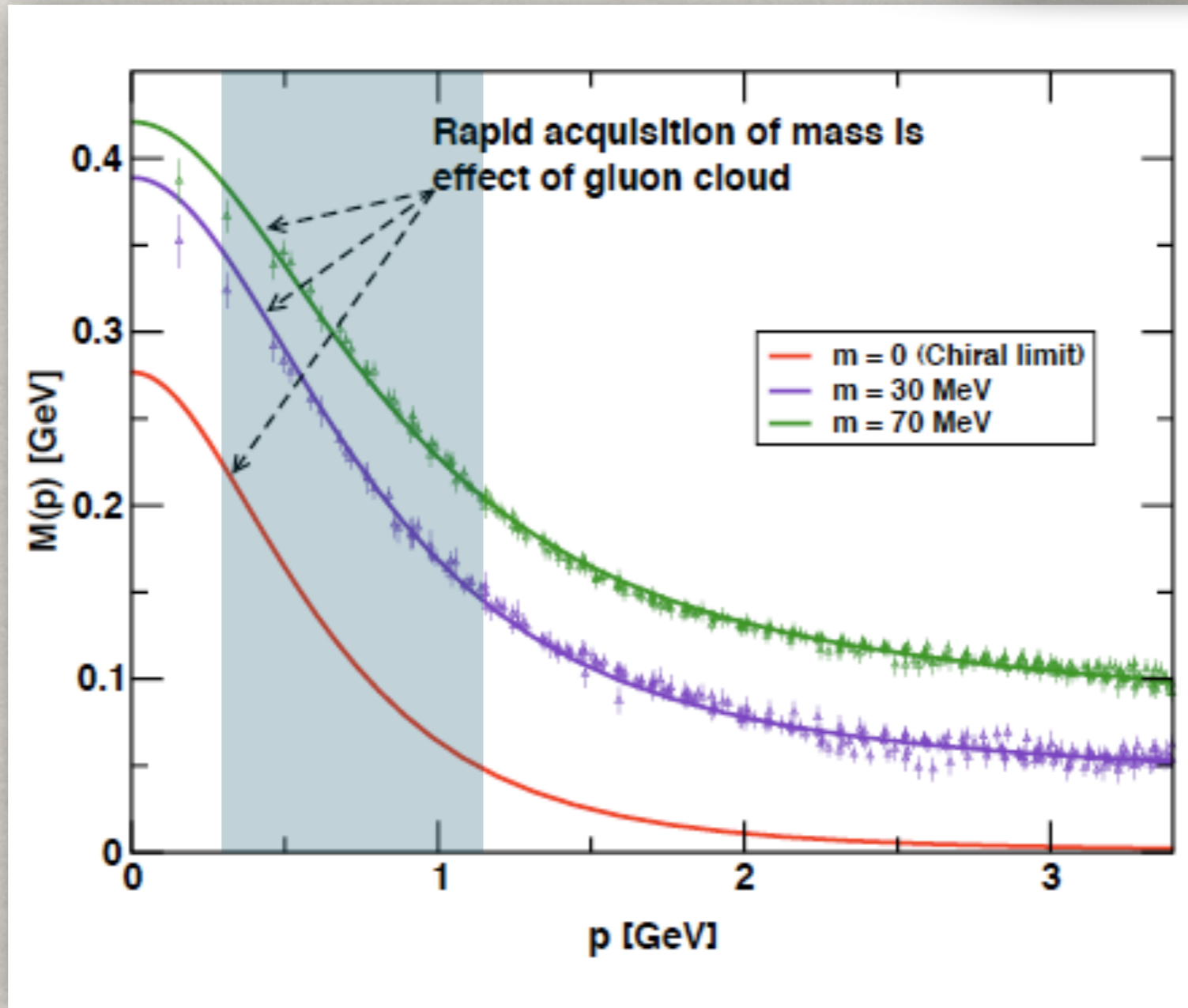
$$[\text{quark line}]^{-1} = [\text{bare quark line}]^{-1} + [\text{quark line with gluon loop}]^{-1}$$

$q = p - k$

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



⇒ For light quarks the Higgs mechanism is almost irrelevant!

## Motivation: Connection with Real World



## Motivation: Connection with Real World

- How does one incorporate the dressed-quark mass function  $M(p^2)$  in study of mesons and baryons? Behavior of  $M(p^2)$  is essentially a quantum field theoretical effect.
- In quantum field theory a meson(nucleon) appears as a pole in the four(six)-point quark Green functions amplitude.
- Residue is proportional to meson's Bethe-Salpeter or nucleon's Faddeev amplitude.
- Poincaré covariant Bethe-Salpeter/Faddeev equation sum all possible exchanges and interactions that can take place between dressed-quarks ( $Q^2 \gg M^2$ ).

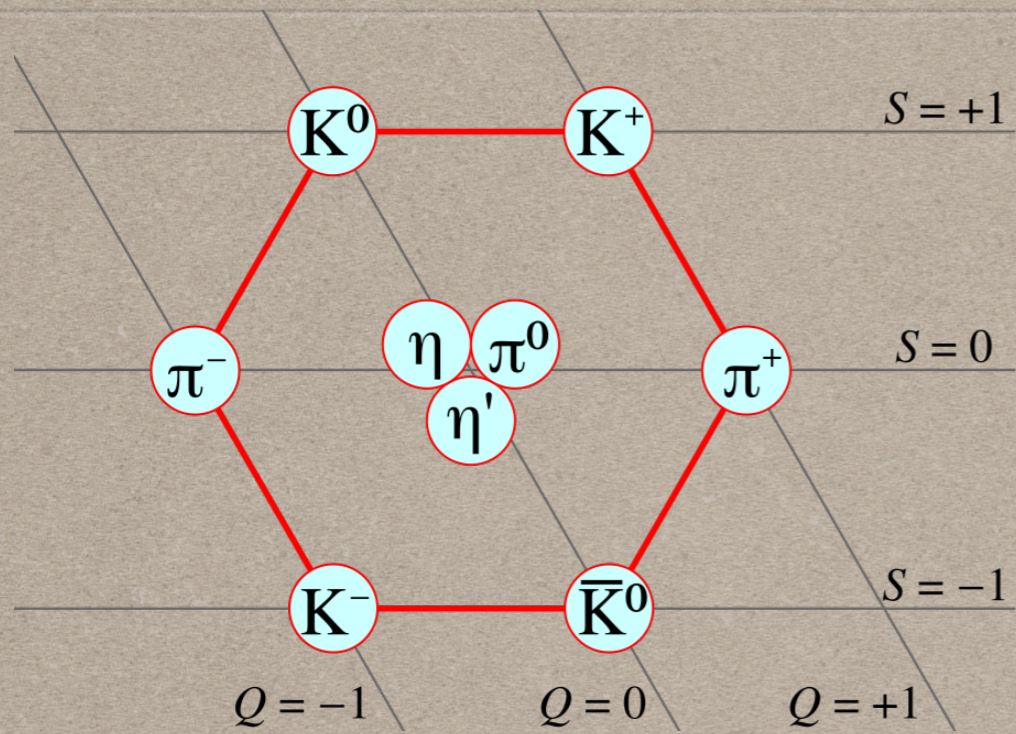
# Meson and Baryon Structure and Confinement Properties

Hadron?

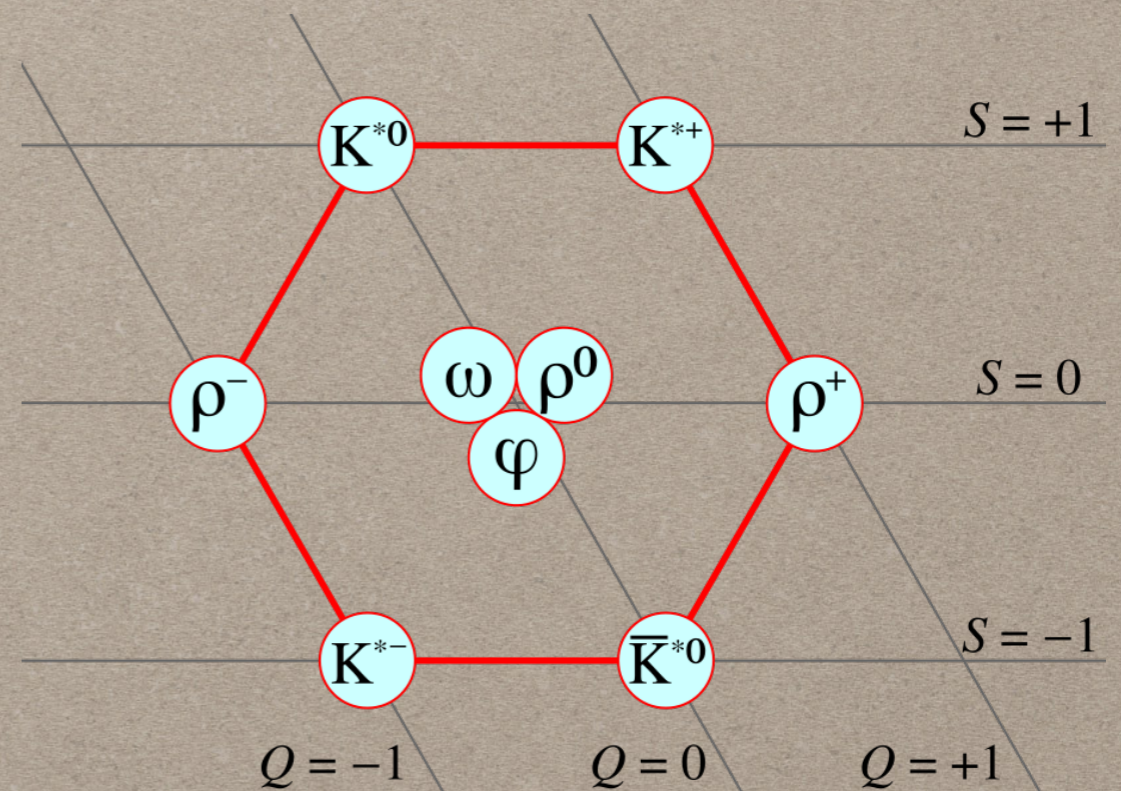
Meson?

Baryon?



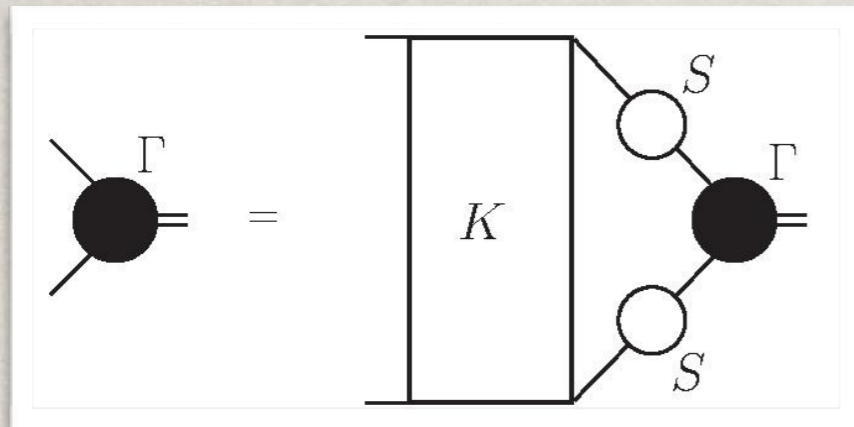


# MESONS





# Bethe-Salpeter Equations for QCD Bound States

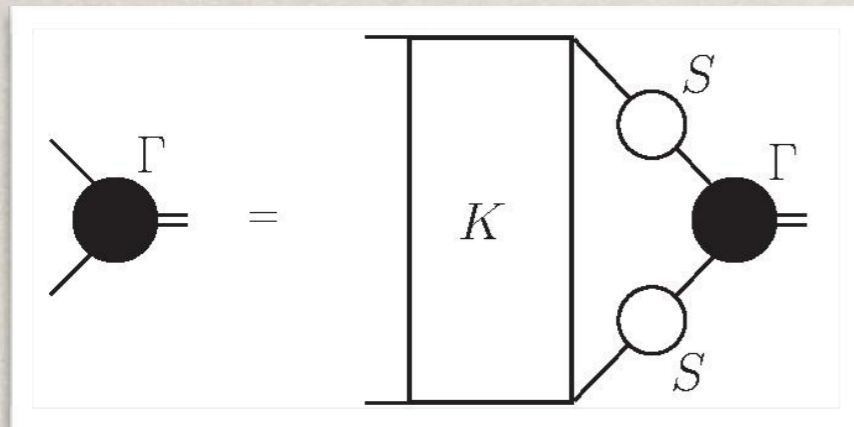


$$\Gamma(P, p) = \int \frac{d^4 k}{(2\pi)^4} K(P, p, k) S(k - \frac{P}{2}) \Gamma(P, k) S(k + \frac{P}{2})$$

Rainbow-Ladder truncation:

$$K(P, p, k) = -\frac{Z_2^2 \mathcal{G}(q^2)}{q^2} \left( \frac{\lambda^a}{2} \gamma_\mu \right) T_{\mu\nu}(q) \left( \frac{\lambda^a}{2} \gamma_\nu \right)$$

# Bethe-Salpeter Equations for QCD Bound States



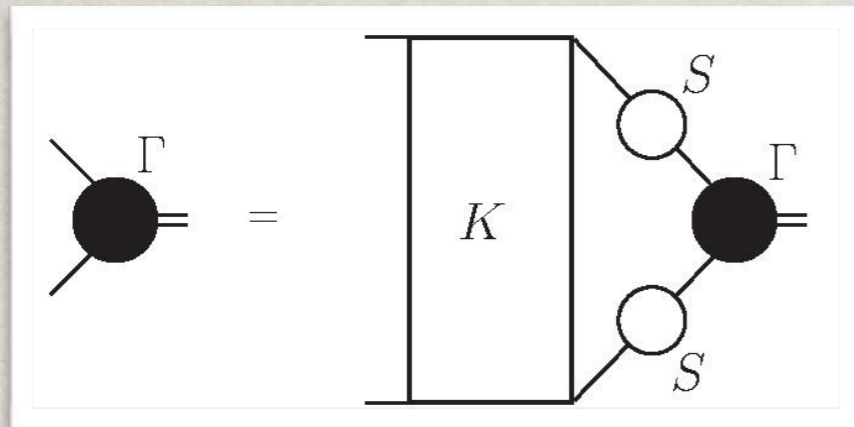
Tough part: how to model nonperturbative QCD interaction beyond rainbow-ladder truncation?

$$\Gamma(P, p) = \int \frac{d^4 k}{(2\pi)^4} K(P, p, k) S(k - \frac{P}{2}) \Gamma(P, k) S(k + \frac{P}{2})$$

Rainbow-Ladder truncation:

$$K(P, p, k) = -\frac{Z_2^2 \mathcal{G}(q^2)}{q^2} \left( \frac{\lambda^a}{2} \gamma_\mu \right) T_{\mu\nu}(q) \left( \frac{\lambda^a}{2} \gamma_\nu \right)$$

# Bethe-Salpeter Equations for QCD Bound States



Tough part: how to model nonperturbative QCD interaction beyond rainbow-ladder truncation?

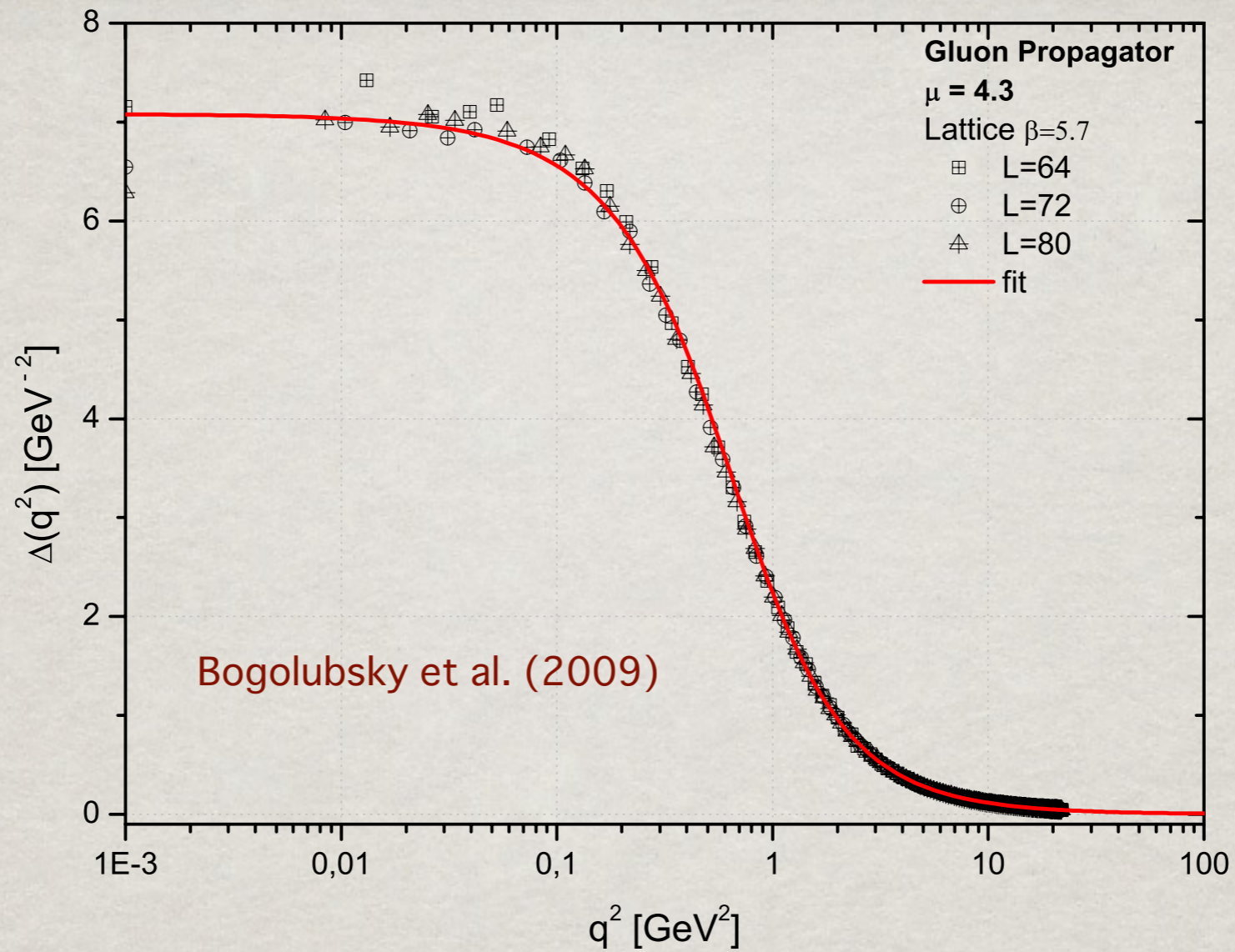
$$\Gamma(P, p) = \int \frac{d^4 k}{(2\pi)^4} K(P, p, k) S(k - \frac{P}{2}) \Gamma(P, k) S(k + \frac{P}{2})$$

Rainbow-Ladder truncation:

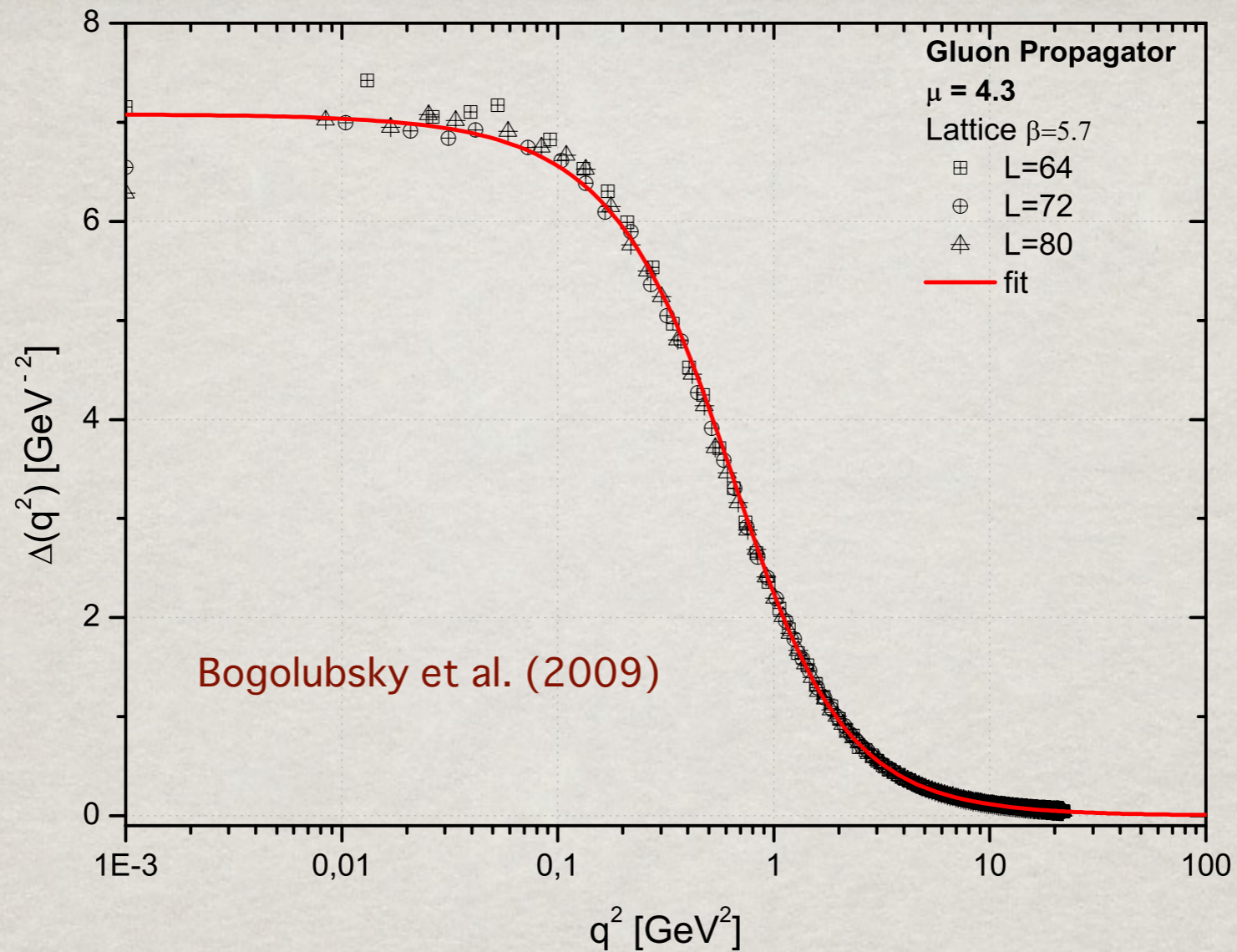
$$K(P, p, k) = -\frac{Z_2^2 \mathcal{G}(q^2)}{q^2} \left( \frac{\lambda^a}{2} \gamma_\mu \right) T_{\mu\nu}(q) \left( \frac{\lambda^a}{2} \gamma_\nu \right)$$

General solution for Poincaré invariant ground- and excited-state amplitudes

$$\Gamma_{P_n}(p, P) = \gamma_5 \left[ i \mathbb{I}_D E_{P_n}(p, P) + \gamma \cdot P F_{P_n}(p, P) + \gamma \cdot p (p \cdot P) G_{P_n}(p, P) + \sigma_{\mu\nu} p_\mu P_\nu H_{P_n}(p, P) \right]$$



Use effective interaction which reproduces Lattice QCD and DSE results for gluon-dressing function: infrared massive fixed point; ultraviolet massless propagator.



Use effective interaction which reproduces Lattice QCD and DSE results for gluon-dressing function: infrared massive fixed point; ultraviolet massless propagator.



Qin, Chang, Liu, Roberts and Wilson 2011

$$\mathcal{G}(s) = \frac{8\pi^2}{\omega^4} D e^{-s/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln \left[ \tau + (1 + s/\Lambda_{\text{QCD}}^2) \right]} \mathcal{F}(s)$$

The Bethe-Salpeter equation is an eigenvalue problem:

$$\lambda(P^2) \Gamma_{P_n}(P, p) = \int \frac{d^4 k}{(2\pi)^4} K(P, p, k) \chi_{P_n}(k, P)$$

$$\chi_{P_n}(k, P) = S(k - \frac{P}{2}) \Gamma(P, k) S(k + \frac{P}{2}) : \text{Bethe-Salpeter wave function}$$

The Bethe-Salpeter equation is an eigenvalue problem:

$$\lambda(P^2) \Gamma_{P_n}(P, p) = \int \frac{d^4 k}{(2\pi)^4} K(P, p, k) \chi_{P_n}(k, P)$$

$$\chi_{P_n}(k, P) = S(k - \frac{P}{2}) \Gamma(P, k) S(k + \frac{P}{2}) : \text{Bethe-Salpeter wave function}$$

The kernel  $\mathcal{K}(P^2)$  has a complete set of real eigenvectors  $\phi_i$  with eigenvalues  $\lambda_i(P^2)$  which are ordered as  $\lambda_0(P^2) > \lambda_1(P^2) > \lambda_2(P^2) > \dots > \lambda_i(P^2)$ .

$$\lambda(P^2) |\Phi\rangle = \mathcal{K}(P^2) |\Phi\rangle \quad \longrightarrow \quad |\Phi\rangle = \sum_{i=1}^{\infty} a_i |\phi_i\rangle$$

The Bethe-Salpeter equation is an eigenvalue problem:

$$\lambda(P^2) \Gamma_{P_n}(P, p) = \int \frac{d^4 k}{(2\pi)^4} K(P, p, k) \chi_{P_n}(k, P)$$

$$\chi_{P_n}(k, P) = S(k - \frac{P}{2}) \Gamma(P, k) S(k + \frac{P}{2}) : \text{Bethe-Salpeter wave function}$$

The kernel  $\mathcal{K}(P^2)$  has a complete set of real eigenvectors  $\phi_i$  with eigenvalues  $\lambda_i(P^2)$  which are ordered as  $\lambda_0(P^2) > \lambda_1(P^2) > \lambda_2(P^2) > \dots > \lambda_i(P^2)$ .

$$\lambda(P^2) |\Phi\rangle = \mathcal{K}(P^2) |\Phi\rangle \quad \longrightarrow \quad |\Phi\rangle = \sum_{i=1}^{\infty} a_i |\phi_i\rangle$$

$$|\phi_n\rangle := \mathcal{K}^n(P^2) |\Phi\rangle = \sum_{i=1}^{\infty} \lambda_i^n a_i |\phi_i\rangle = \lambda_0^n \left[ a_0 |\phi_0\rangle + \sum_{i=1}^{\infty} \left( \frac{\lambda_i}{\lambda_0} \right)^n a_i |\phi_i\rangle \right]$$

$$|\phi_n\rangle \stackrel{n \rightarrow \infty}{\simeq} \lambda_0^n a_0 |\phi_0\rangle \simeq \lambda_0 \mathcal{K}^{n-1}(P^2) |\Phi\rangle$$



- Eigenvalue spectrum is not limited to the ground state.
- Excited states with smaller eigenvalues can be determined with the same iterative methods.
- Usage of Gram-Schmidt orthogonalization process:

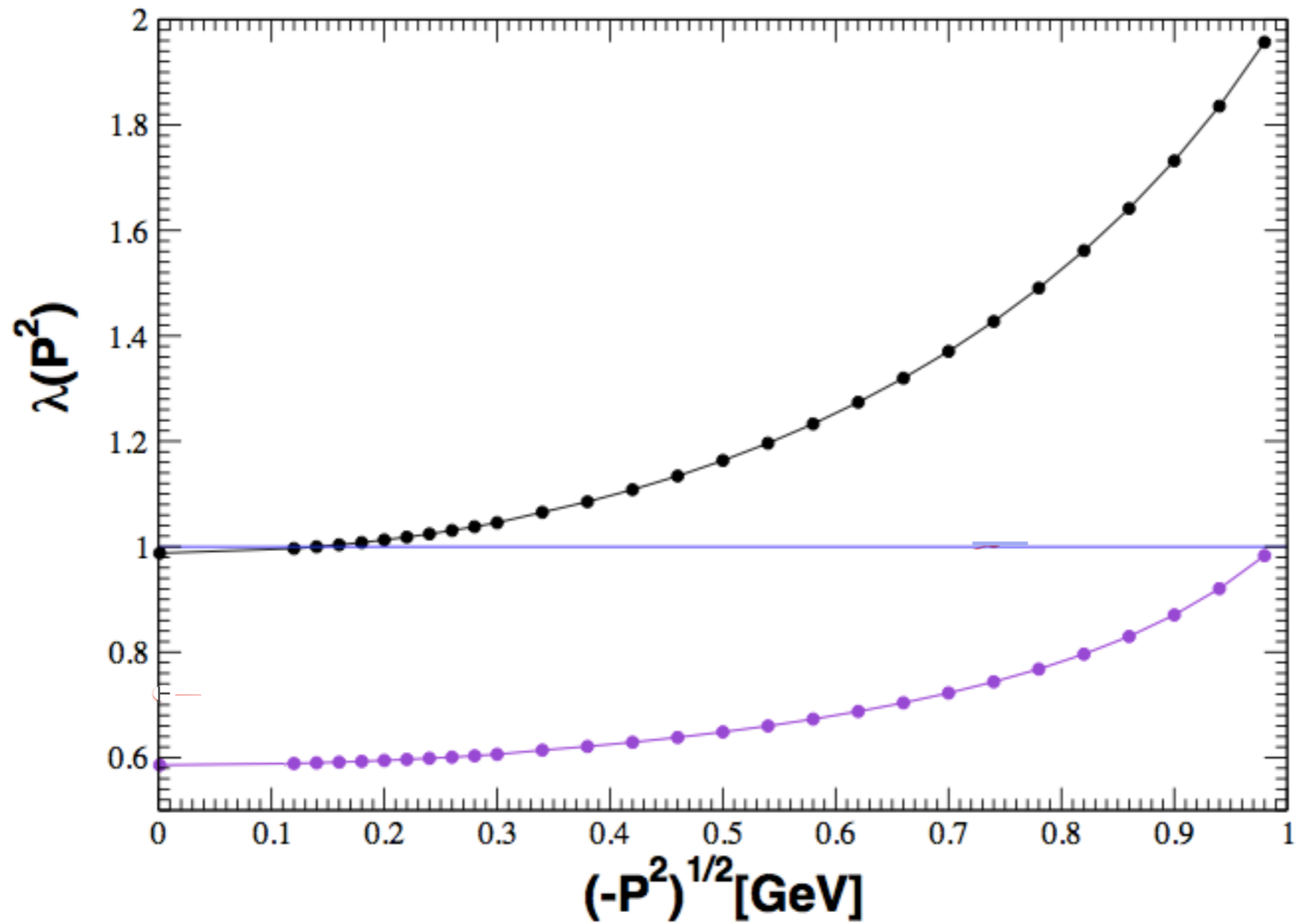
$$|\tilde{\Phi}\rangle = |\Phi\rangle - \frac{\langle\phi_0|\Phi\rangle}{\langle\phi_0|\phi_0\rangle} |\phi_0\rangle$$

- Modern and more efficient approach is the implicitly restarted Arnoldi method (IRAM).
- Based on the **stabilized** Gram-Schmidt orthogonalization in the Krylov subspace obtained by iteration:

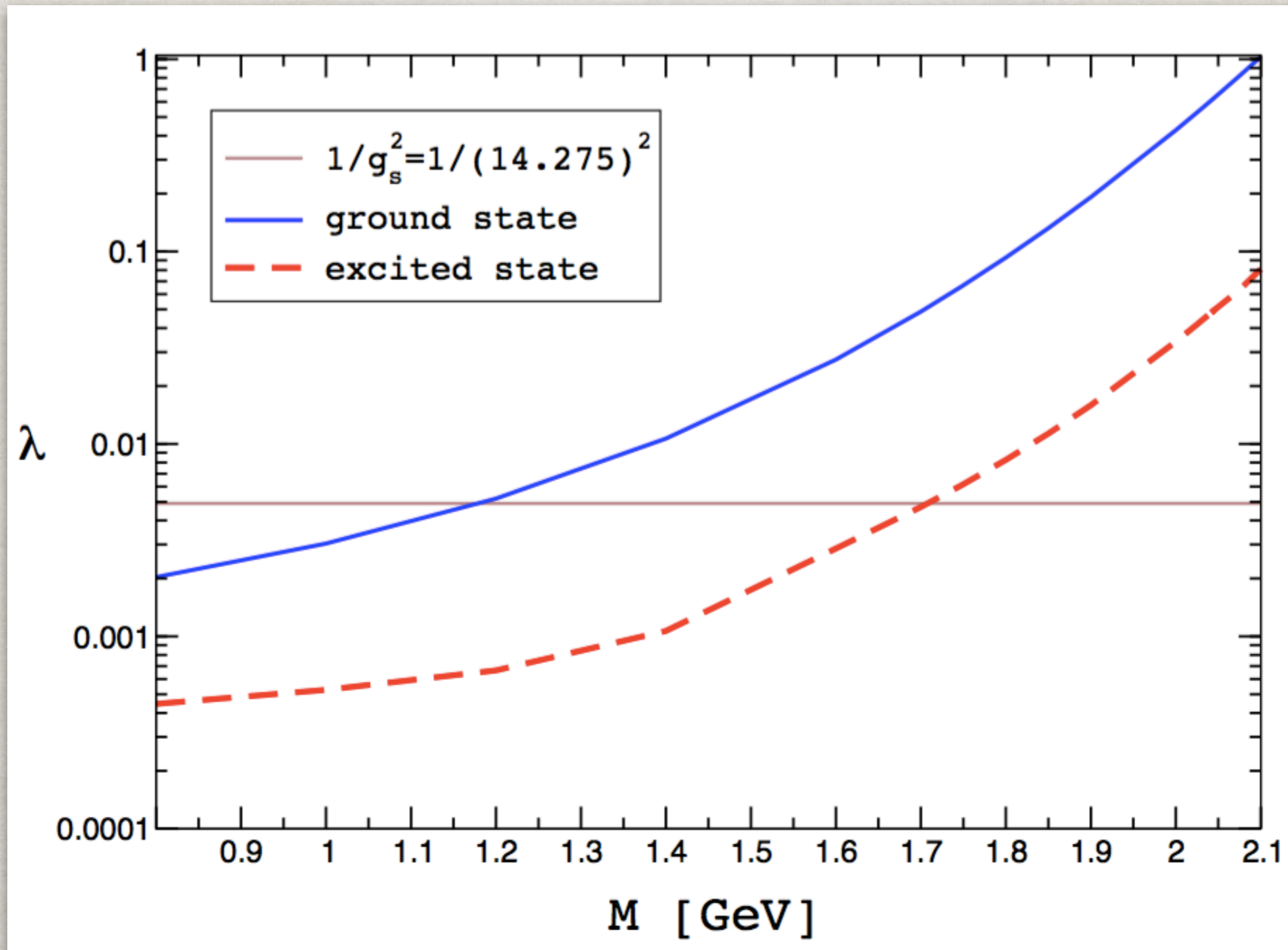
$$\mathcal{S}_r := \{\Phi, K\Phi, K^2\Phi, K^3\Phi, \dots, K^{r-1}\Phi\}$$

- The Arnoldi method generalizes the Gram-Schmidt process by computing the eigenvalues of the orthogonal projection of  $K$  onto the Krylov subspace  $\Rightarrow$  **yields smaller eigenvalues.**

# Examples of eigenvalue spectrum — Pion

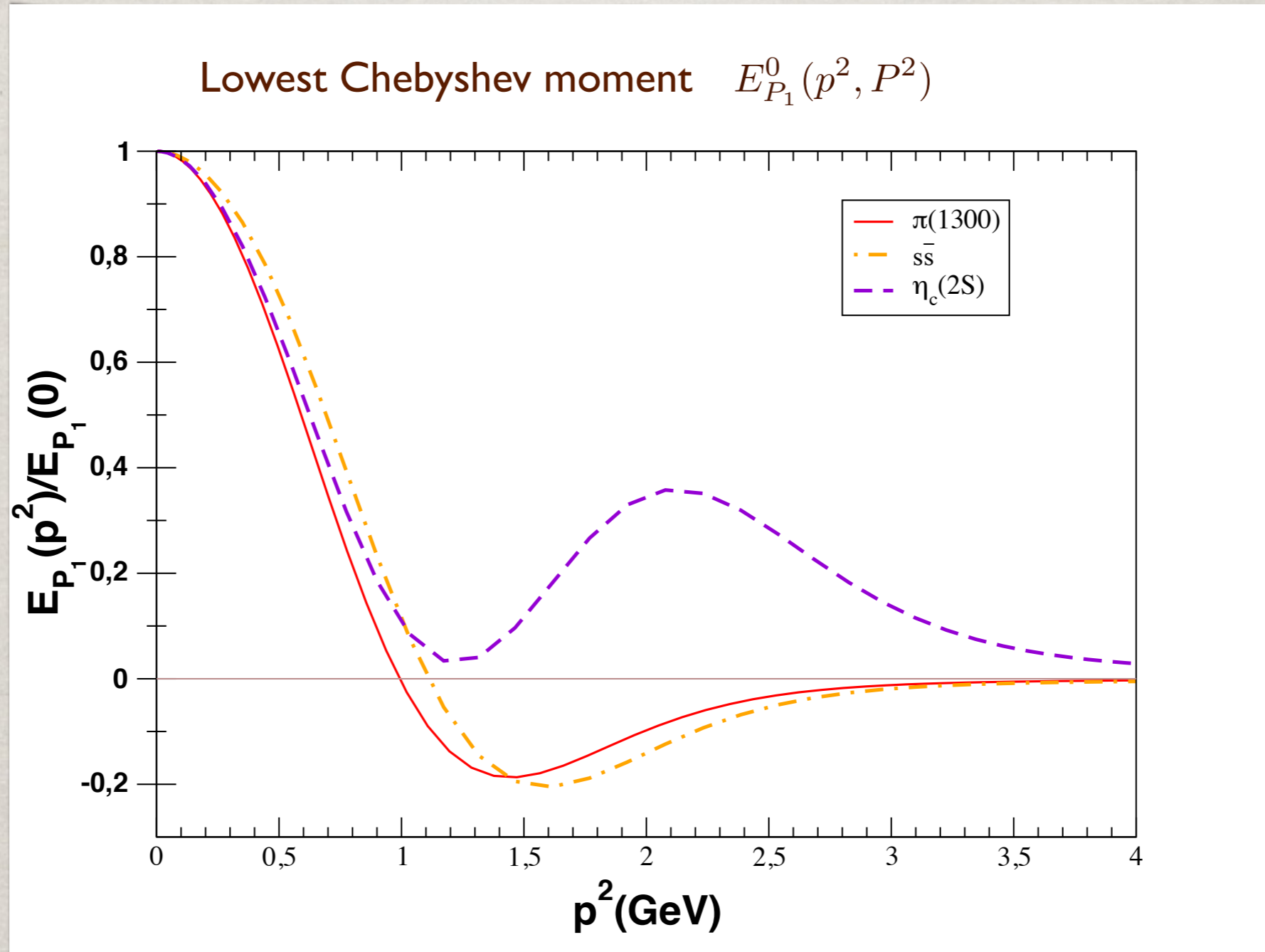


# Examples of eigenvalue spectrum — Nucleon



Chebyshev expansion of 1st excited state:  $E_{P_1}(p, P) = \sum_{m=0}^{\infty} E_{P_1}^m(p, P) U_m(\cos \theta)$

$$\Gamma_{P_n}(p, P) = \gamma_5 [i \mathbb{I}_D E_{P_n}(p, P) + \gamma \cdot P F_{P_n}(p, P) + \gamma \cdot p (p \cdot P) G_{P_n}(p, P) + \sigma_{\mu\nu} p_\mu P_\nu H_{P_n}(p, P)]$$



$$\mathcal{G}(s) = \frac{8\pi^2}{\omega^4} D e^{-s/\omega^2} + \frac{8\pi^2 \gamma_m}{\ln \left[ \tau + (1 + s/\Lambda_{\text{QCD}}^2) \right]} \mathcal{F}(s)$$

E. Rojas, B. El-Bennich &amp; J.P.B.C. de Melo (2014)

	Model 1 [GeV]	Model 2 [GeV]	Reference
$m_\pi$	0.138	0.153	0.139 [36]
$f_\pi$	0.139	0.189	0.1304 [36]
$m_{\pi(1300)}$	0.990	1.414	$1.30 \pm 0.10$ [36]
$f_{\pi(1300)}$	$-1.1 \times 10^{-3}$	$-8.3 \times 10^{-4}$	
$m_K$	0.493	0.541	0.493 [36]
$f_K$	0.164	0.214	0.156 [36]
$m_{K(1460)}$	1.158	1.580	1.460 [36]
$f_{K(1460)}$	-0.018	-0.017	
$m_{\bar{s}s}$	1.287	1.702	
$f_{\bar{s}s}$	-0.0214	-0.0216	
$m_{\eta_c(1S)}$	3.065	3.210	2.984 [36]
$f_{\eta_c(1S)}$	0.389	0.464	0.395 [37]
$m_{\eta_c(2S)}$	3.402	3.784	3.639 [36]
$f_{\eta_c(2S)}$	0.089	0.105	

$$f_{P_n}^0(\mu) \equiv 0, \quad n \geq 1$$

The two models correspond to different parametrizations of the gluon-dressing function neither model reproduces equally well ground and excited states.

⇒ Must go beyond rainbow-ladder truncation in DSE and BSE !

# Open-Charm Mesons

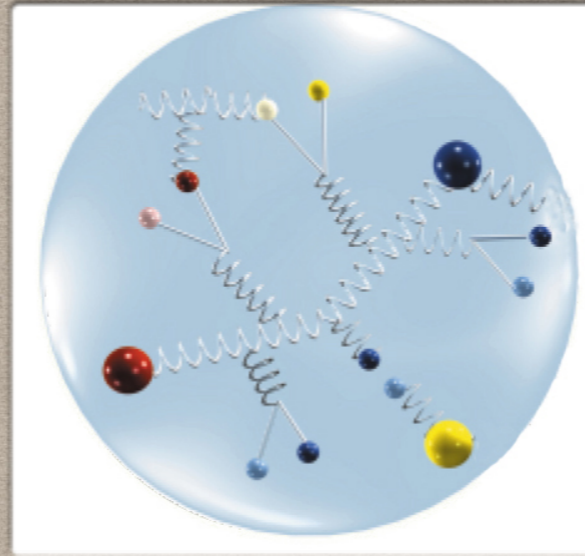
- So far, we have first results for the heavy-light systems:  $D$  mesons

	Model	Experiment [63]
$m_D$	2.115	1.869
$f_D$	0.204	$0.2067 \pm 0.0085 \pm 0.0025$
$m_{D_s}$	2.130	1.968
$f_{D_s}$	0.249	$0.260 \pm 0.004$

E. Rojas, B. El-Bennich & J.P.B.C. de Melo (2014)

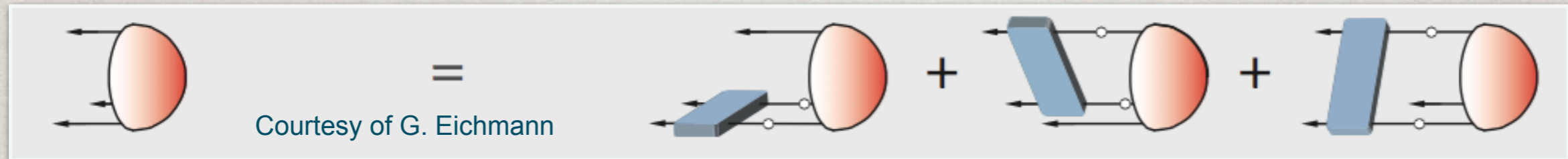
However, masses too large and mass difference too small.

- This was expected, strong mass asymmetry doesn't allow for simple quark-gluon vertex and rainbow-ladder truncation.



# NUCLEONS

# Covariant Fadeev Equation



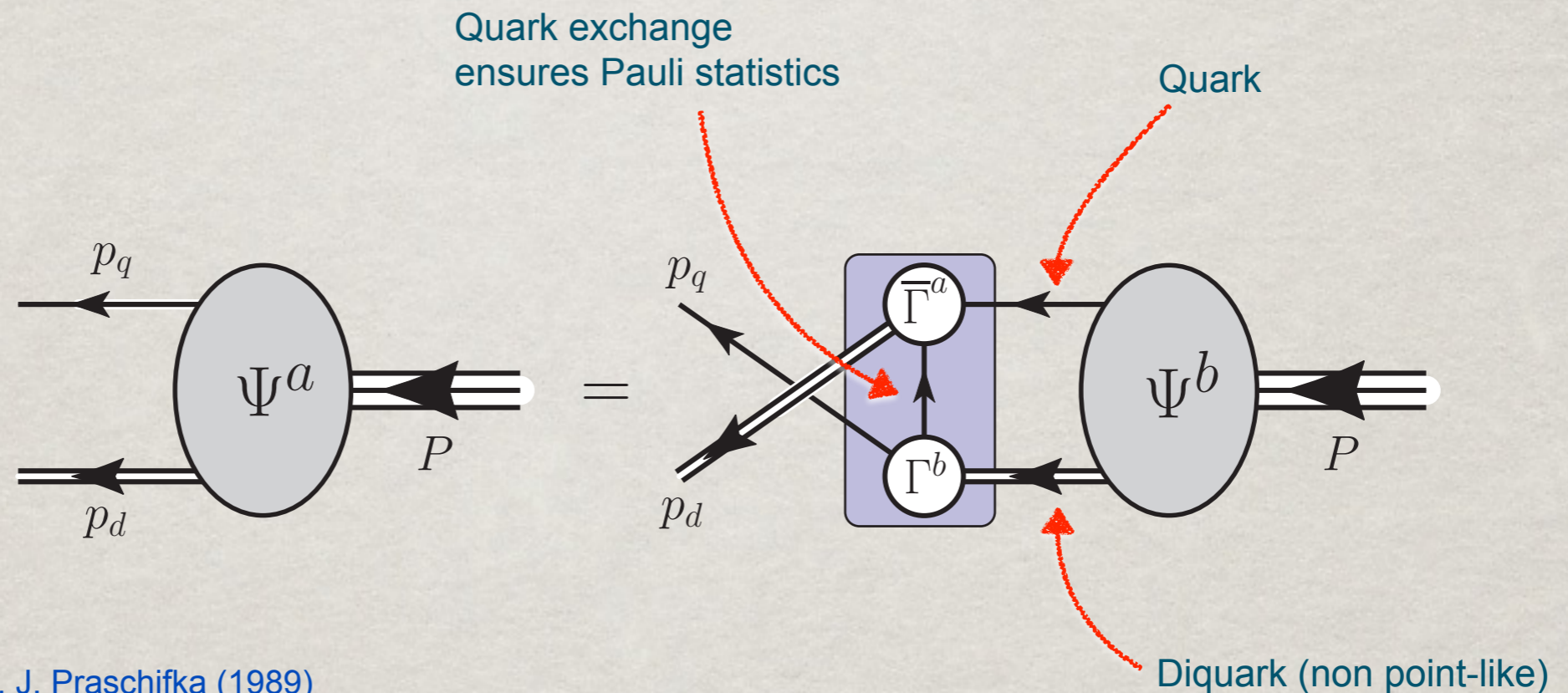
R.T. Cahill, C.D. Roberts, J. Praschifka (1989)

M. Oettel, L. von Smekal, R. Alkofer (2001)

G. Eichmann, R. Alkofer, A. Krassnigg, D. Nicmorus (2010)



# Covariant Faddeev Equation



R.T. Cahill, C.D. Roberts, J. Praschifka (1989)

M. Oettel, L. von Smekal, R. Alkofer (2001)

G. Eichmann, R. Alkofer, A. Krassnigg, D. Nicmorus (2010)

Linear homogeneous matrix equation yields Poincaré covariant Faddeev amplitude (wave function) that describes relative motion of quark-diquark within nucleon.

## Diquark-Quark Description

- Tractable Faddeev equation is based on the observation that an interaction which describes color-singlet mesons also generates non point-like quark-quark (diquark) correlations in the color anti-triplet channel.
- Diquark correlations are a dynamical consequence of strong-coupling in **QCD**: scalar & axial-vector diquarks.
- The same mechanism that produces an almost massless pion from two dynamically-massive quarks (**DCSB**) forces a strong correlation between two quarks in color anti-triplet channels within a baryon.
- Diquark correlations employed in Faddeev equation are **not** point-like.
- Typically,  $r_{0+} \sim r_{\pi}$  &  $r_{1+} \sim r_{\rho}$  (actually 10% larger).
- They have soft form factors.

$$\text{SU}(3): 3 \otimes 3 = \bar{3} \oplus 6$$

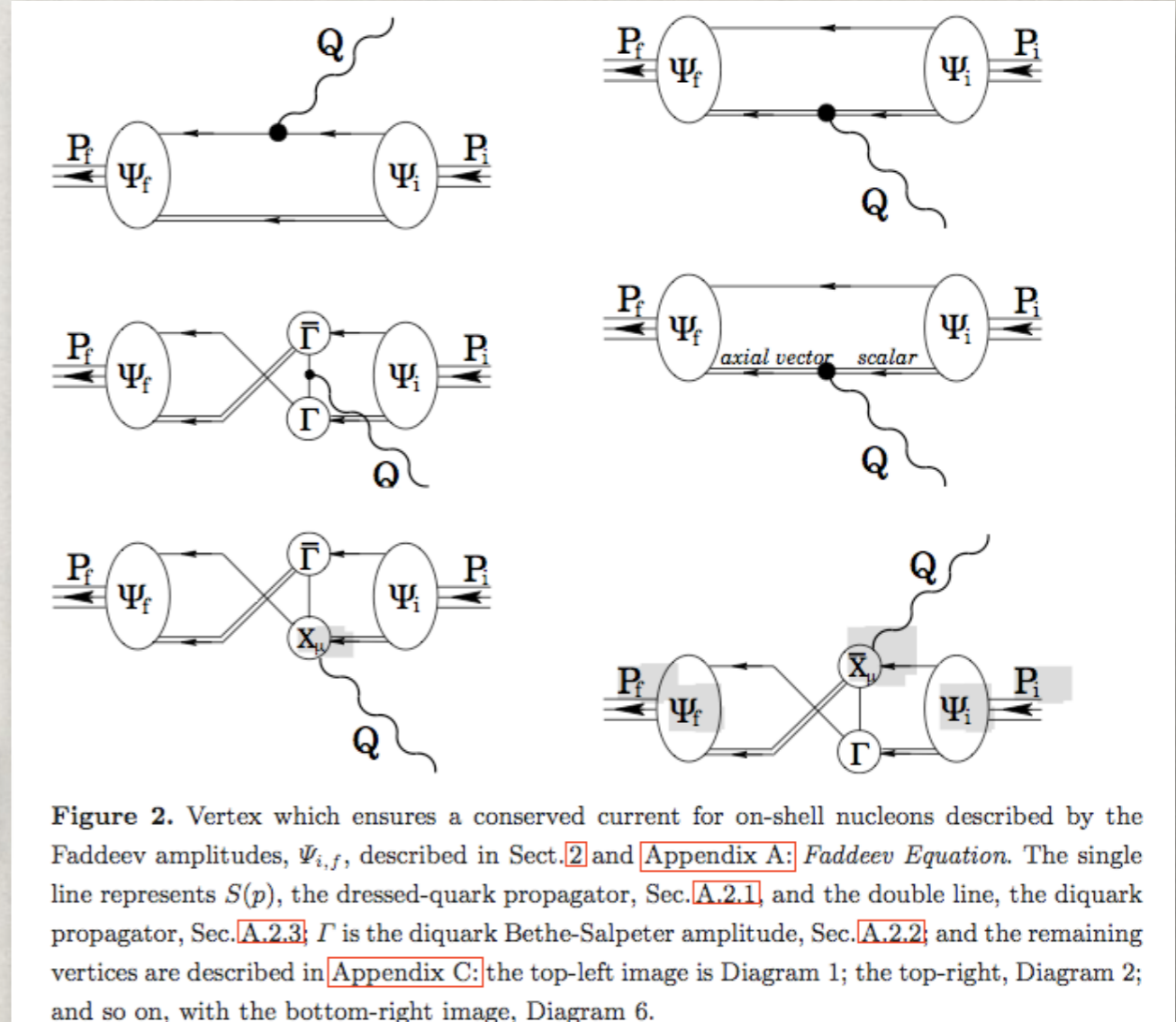
# Nucleon Electromagnetic Form Factors

Composite nucleon must interact with photon via nontrivial current constrained by Ward-Takahashi identities!

$$\begin{aligned}
 J_\mu(P', P) &= ie \bar{u}(P') \Lambda_\mu(q, P) u(P), \\
 &= ie \bar{u}(P') \left( \gamma_\mu F_1(Q^2) + \frac{1}{2M} \sigma_{\mu\nu} Q_\nu F_2(Q^2) \right) u(P).
 \end{aligned}$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

$$\mu_n = \kappa_n = G_M^n(0), \quad \mu_p = 1 + \kappa_p = G_M^p(0)$$



# Nucleon Electromagnetic Form Factors

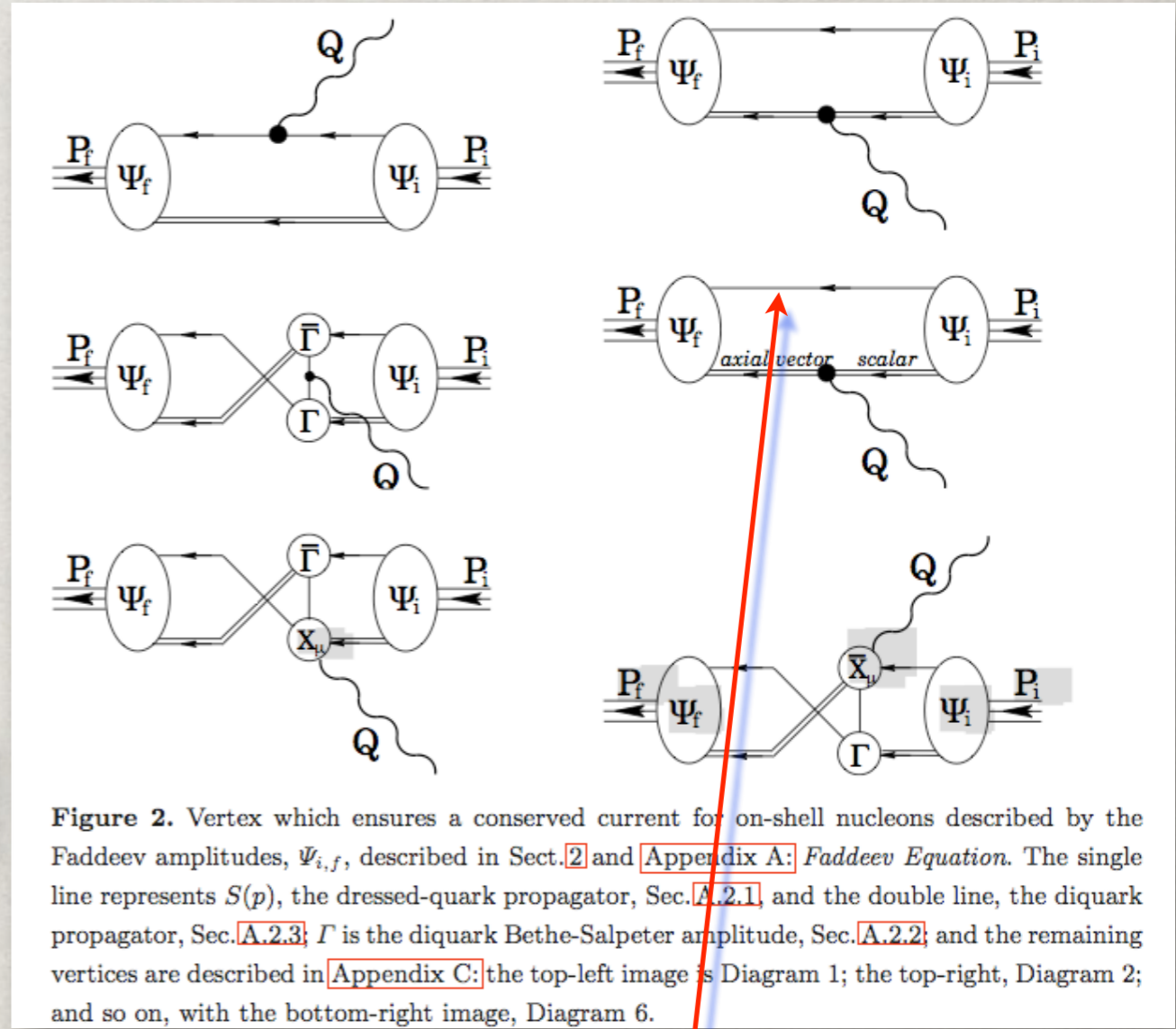
Composite nucleon must interact with photon via nontrivial current constrained by Ward-Takahashi identities!

$$J_\mu(P', P) = ie \bar{u}(P') \Lambda_\mu(q, P) u(P),$$

$$= ie \bar{u}(P') \left( \gamma_\mu F_1(Q^2) + \frac{1}{2M} \sigma_{\mu\nu} Q_\nu F_2(Q^2) \right) u(P).$$

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

$$\mu_n = \kappa_n = G_M^n(0), \quad \mu_p = 1 + \kappa_p = G_M^p(0)$$

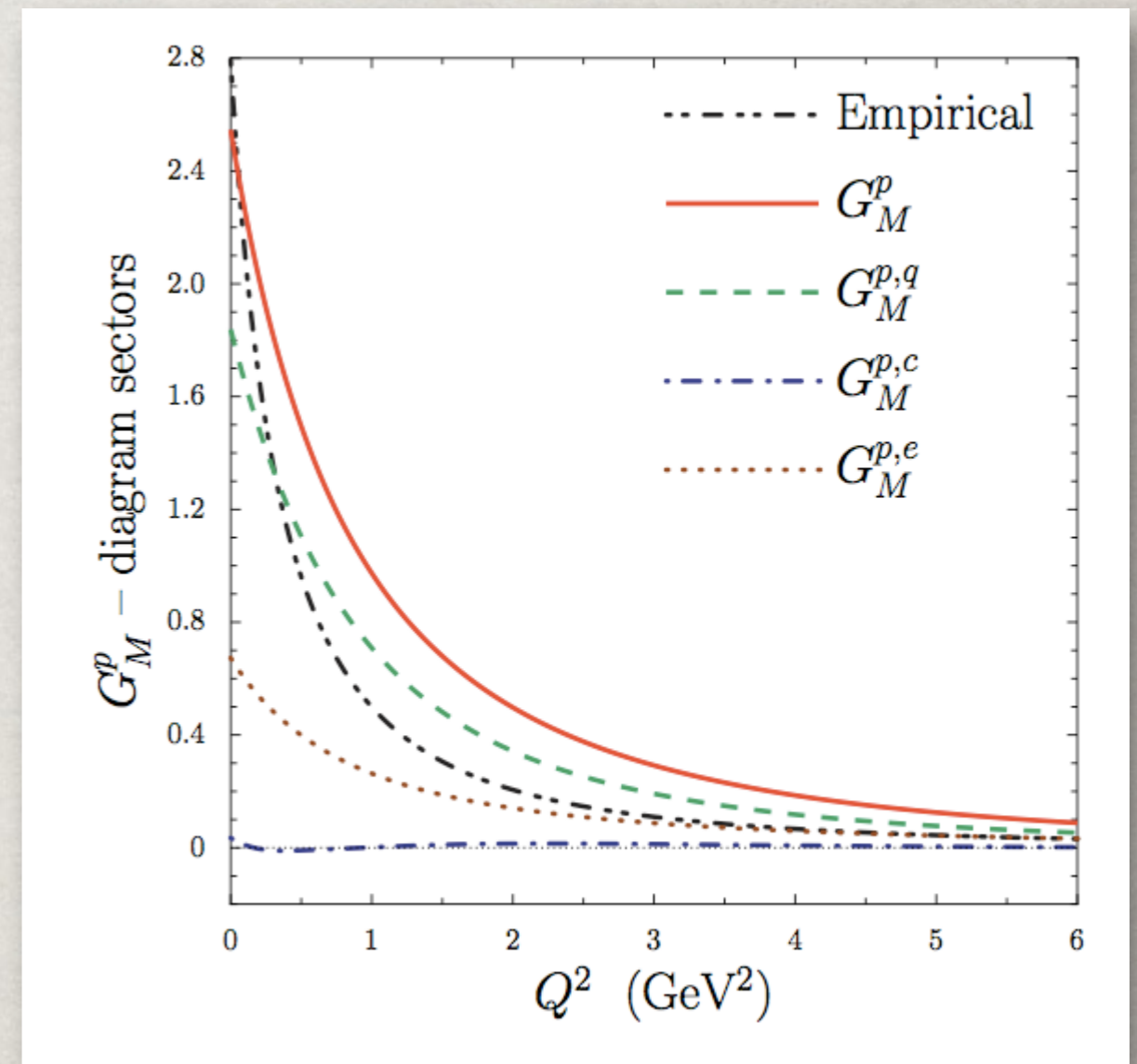
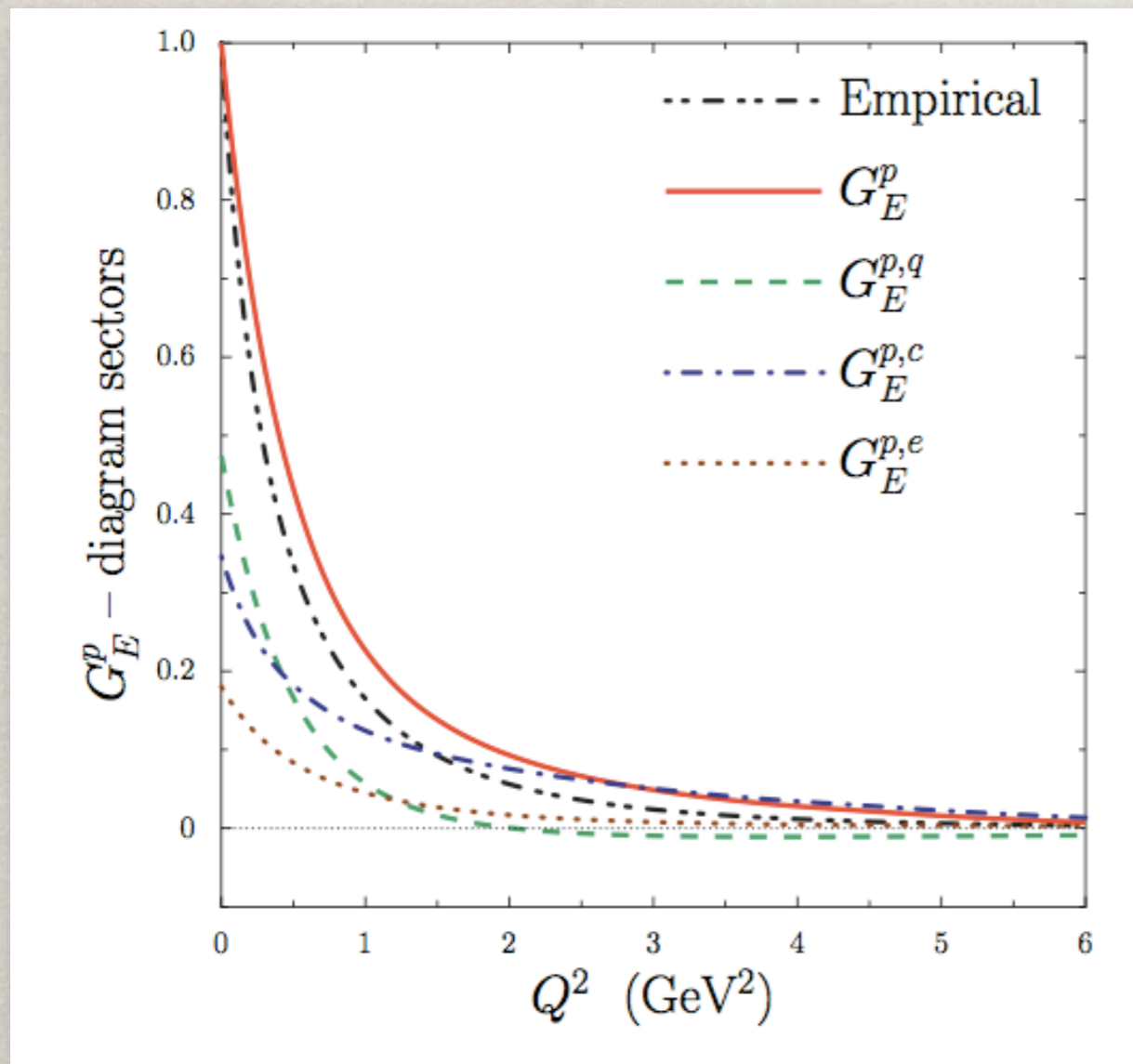


Dressed quark propagator solutions of QCD's Dyson-Schwinger equations.

$$S(p) = -i\gamma \cdot p \sigma_V(p^2, \zeta^2) + \sigma_S(p^2, \zeta^2) = \frac{1}{i\gamma \cdot p A(p^2, \zeta^2) + B(p^2, \zeta^2)} = \frac{Z(p^2, \zeta^2)}{i\gamma \cdot p + M(p^2)}$$

⇒ momentum dependence !

# Proton's Sachs Electric and Magnetic Form Factors

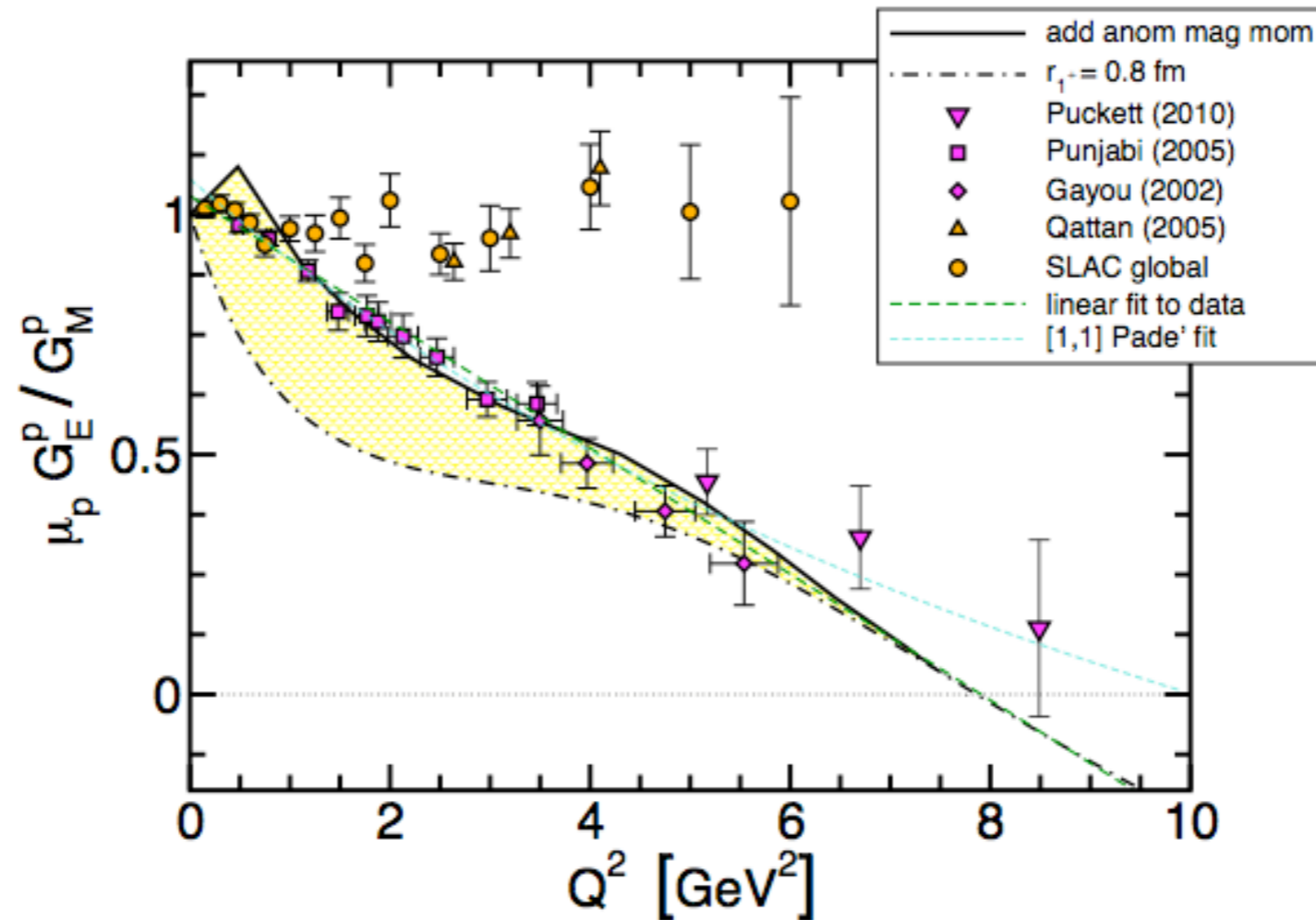


I.C. Cloët, G. Eichmann, B. El-Bennich, T. Klähn and C.D. Roberts, Few Body Syst. 46 (2009)

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2} F_2(Q^2), \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2).$$

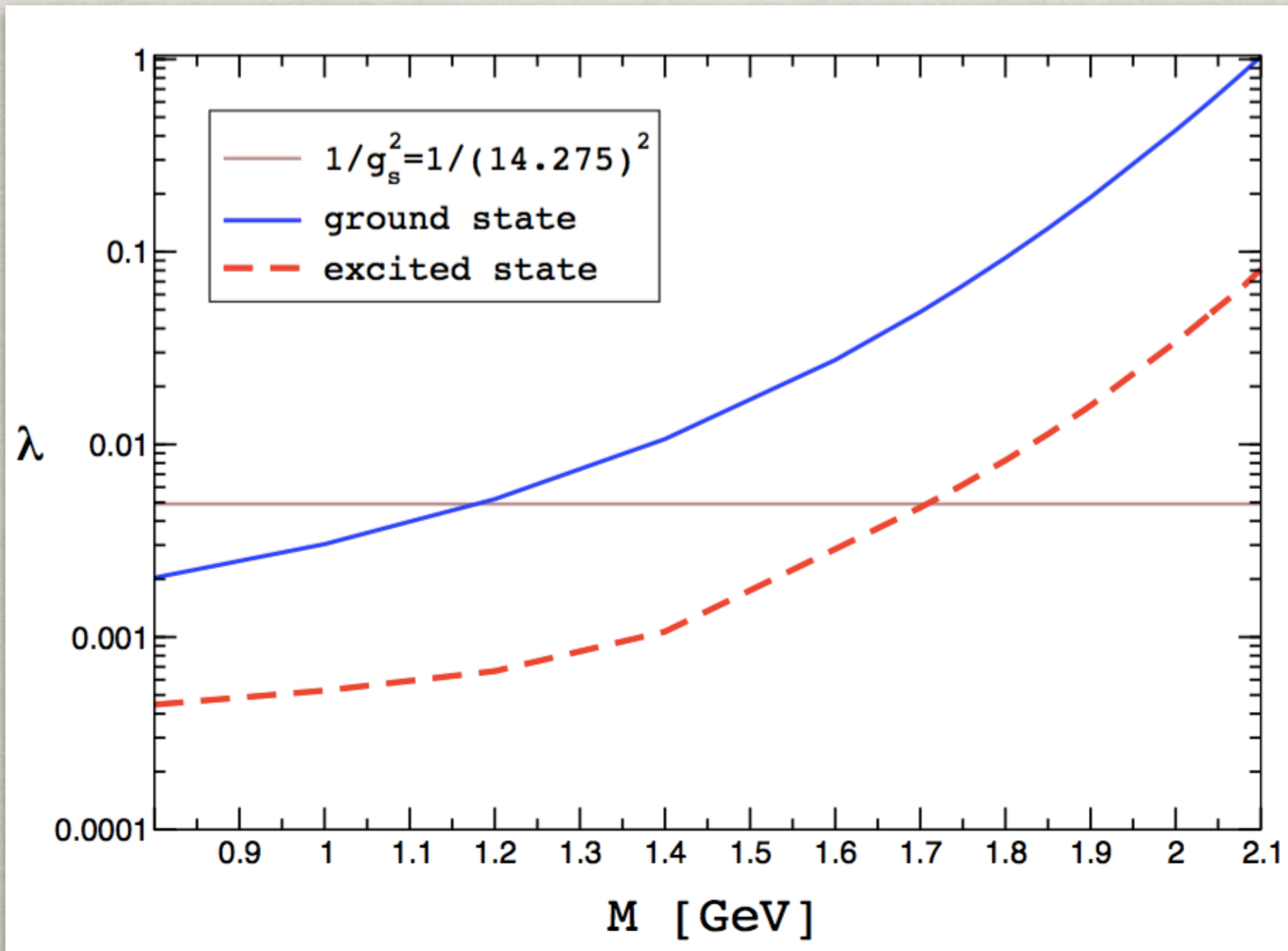
$$\mu_n = \kappa_n = G_M^n(0), \quad \mu_p = 1 + \kappa_p = G_M^p(0)$$

# Exposing the dressed mass function



**FIGURE 1.** Proton electric-to-magnetic form factor ratio. The *dot-dashed curve* is the result in Ref. [12], whereas the *solid curve* is obtained by repeating that calculation with inclusion of a momentum-dependent dressed-quark anomalous magnetic moment that is characterised by a  $Q^2 = 0$  strength  $\eta_{em} = 0.4$ . Data: diamonds – [27]; squares – [28]; up-triangles – [29]; circles [30]; and down-triangles [31]. *Dashed curve*: [1, 1]-Padé fit to available JLab data; and *dotted curve*, a linear fit.

# Ground and Radially Excited States of the Nucleon



## Roper Quark-Core Mass

	$R_{\text{core}}^{\text{DSE}}$	$R_{\text{core}}^{\text{Contact}}$	$R_{\text{bare}}^{\text{DCCM}}$
Mass	1.73	1.72	1.76

**DSE** : Faddeev amplitude of 1st excited state with dressed quark propagators

J. Segovia, B. El-Bennich, E. Rojas, I.C. Cloët, C.D. Roberts, S.-S. Xu, H.-S. Zhong, Phys. Rev. Lett. (2015)

**Contact** : Faddeev amplitude of 1st excited state with contact interaction gap equation

D.J. Wilson, I. C. Cloët, L. Chang, C.D. Roberts, Phys. Rev. C (2012)

**DCCM** : Dynamical Coupled Channel Model

N. Suzuki, B. Julio-Díaz, H. Kamano, T.-S. H. Lee, A. Matsuyama, T. Sato, Phys. Rev. Lett. (2010)

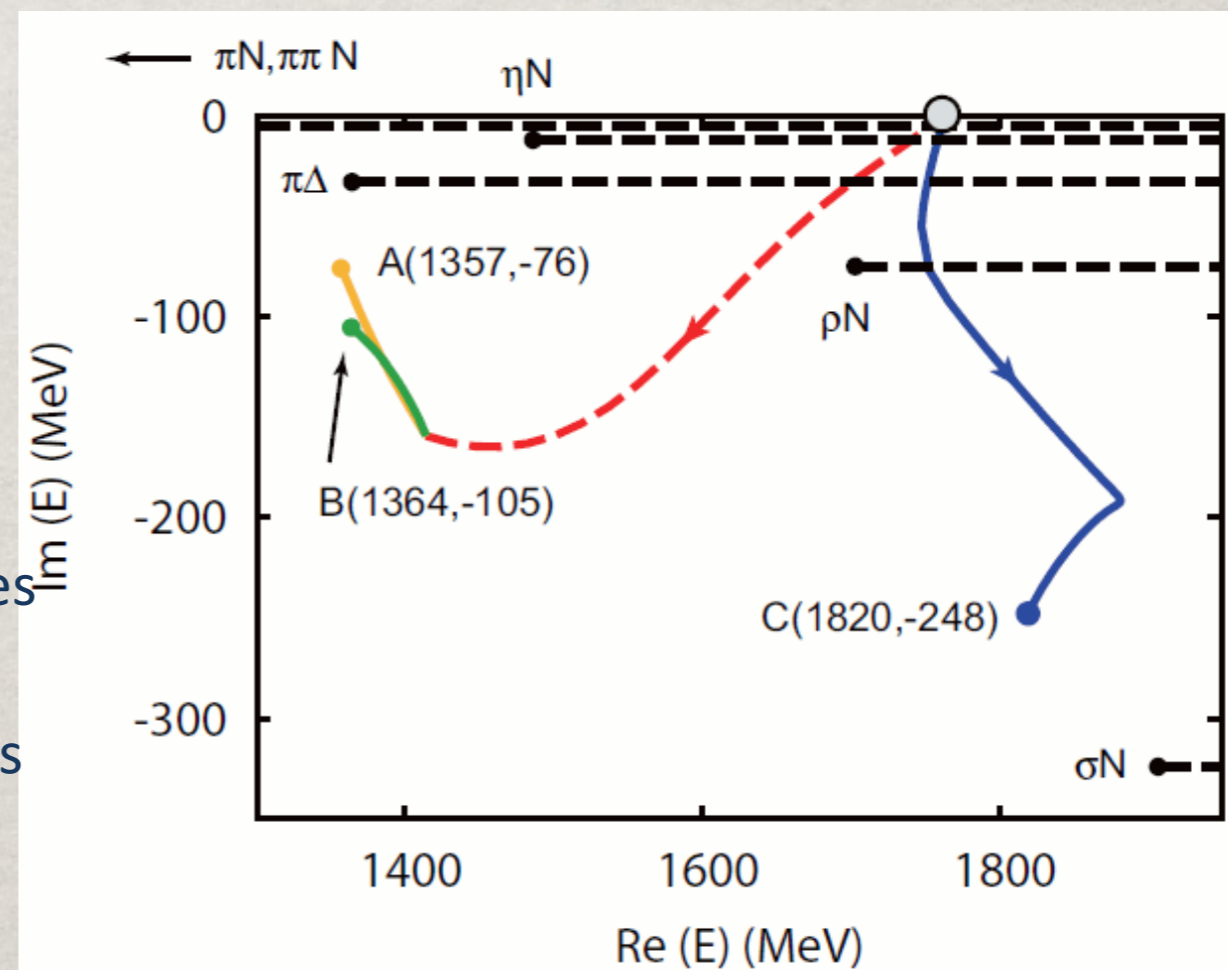


# Roper Quark-Core Mass

N. Suzuki et al., [Phys.Rev.Lett. 104 \(2010\) 042302](#)

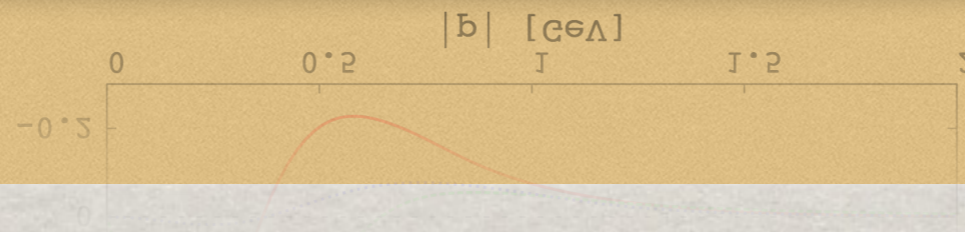
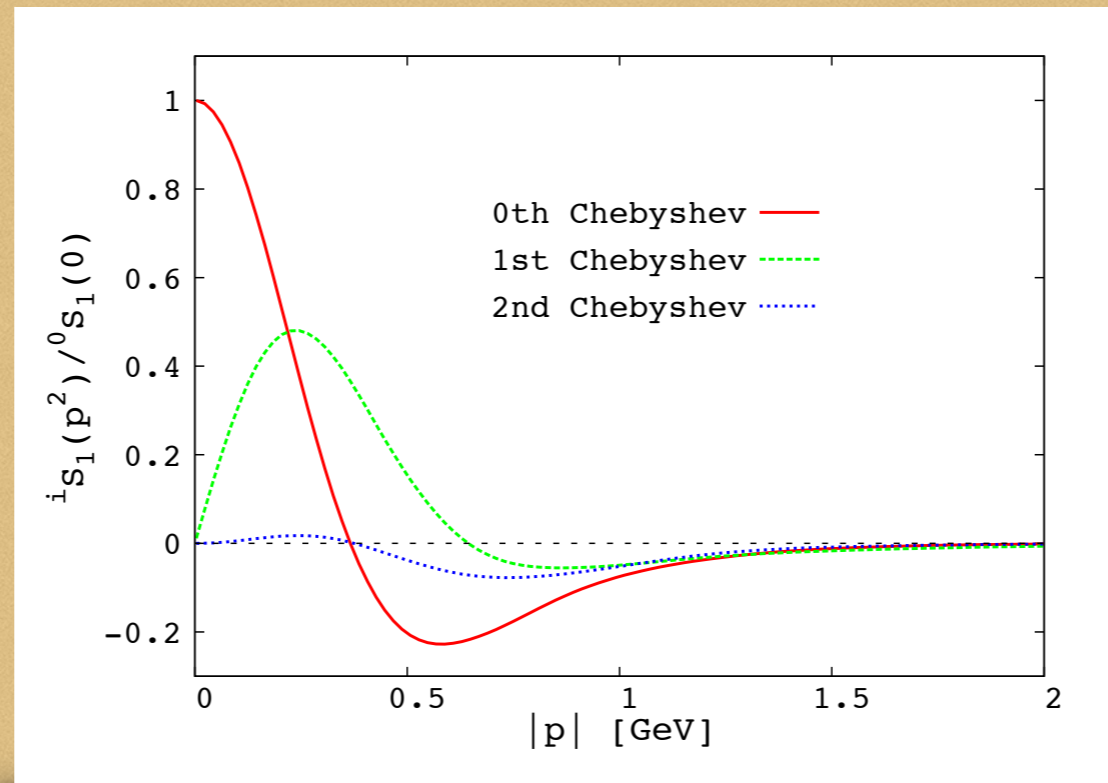
- EBAC examined the dynamical origins of the two poles associated with the Roper resonance
- Both of them, together with the next higher resonance in the  $P_{11}$  partial wave were found to have the same originating bare state
- Coupling to the meson-baryon continuum induces multiple observed resonances from the same bare state.
- All PDG identified resonances consist of a core state and meson-baryon components.

## EBAC & the Roper resonance



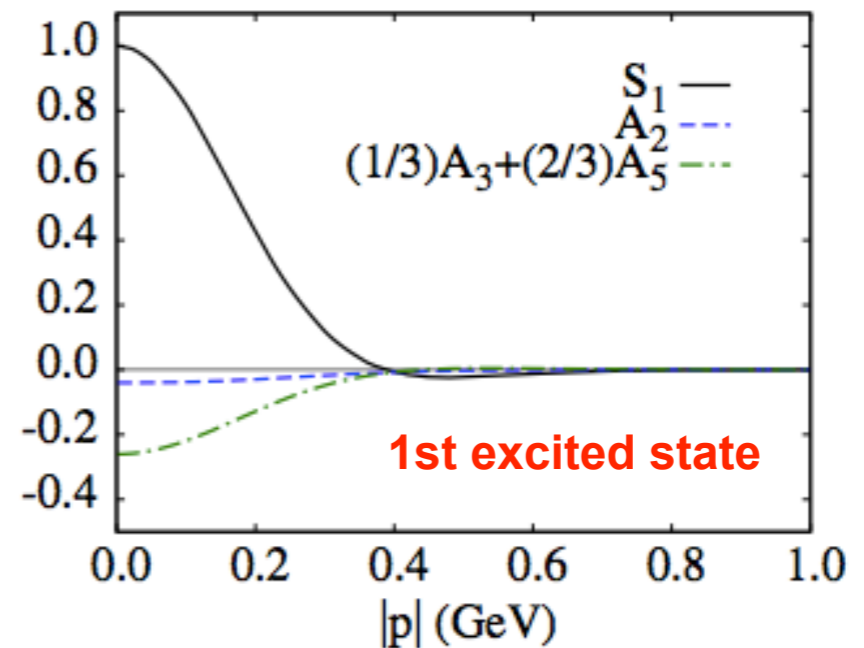
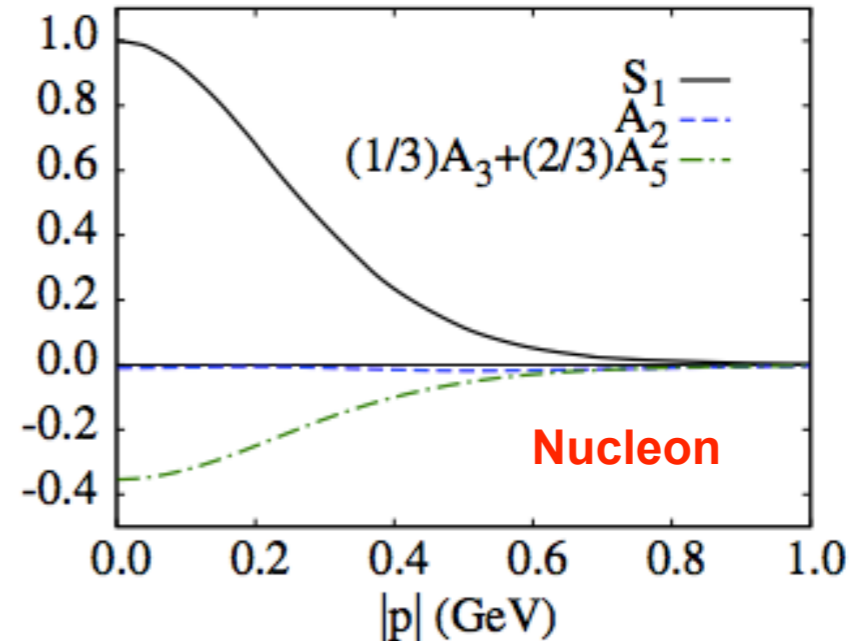
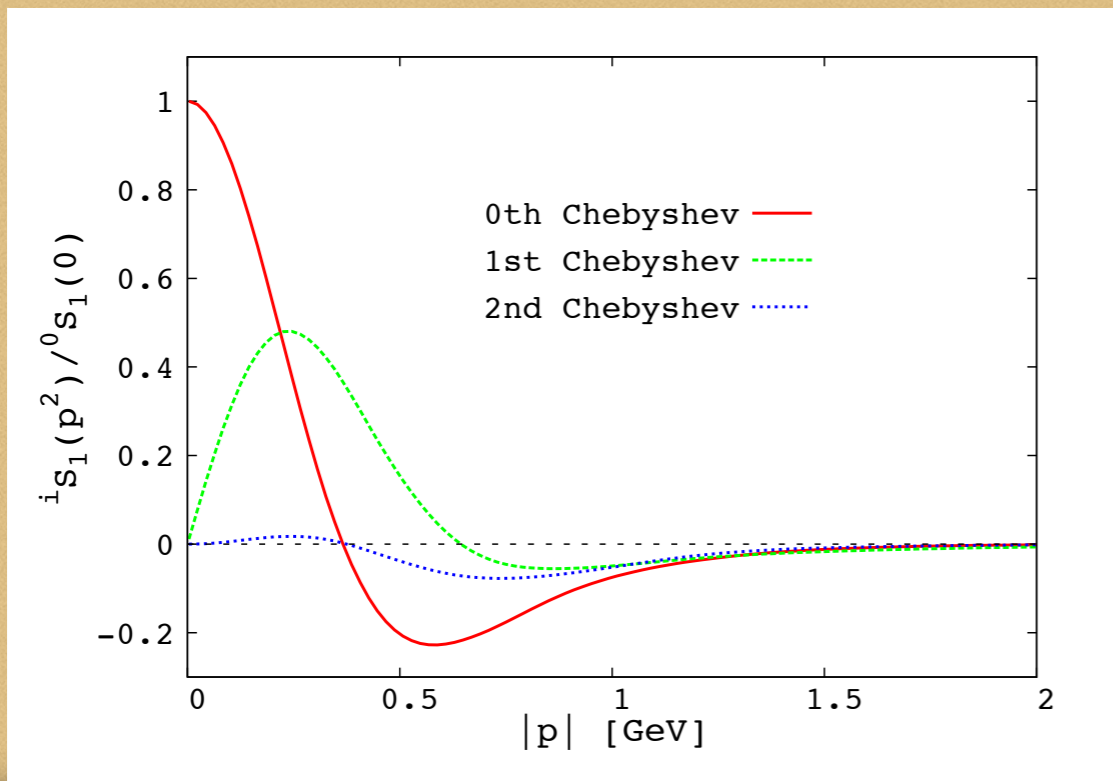
# Chebyshev Moments

First three Chebyshev moments of leading  $S_1$  component of 1st excited state's Faddeev amplitude



# Chebyshev Moments

First three Chebyshev moments of leading  $S_1$  component of 1st excited state's Faddeev amplitude



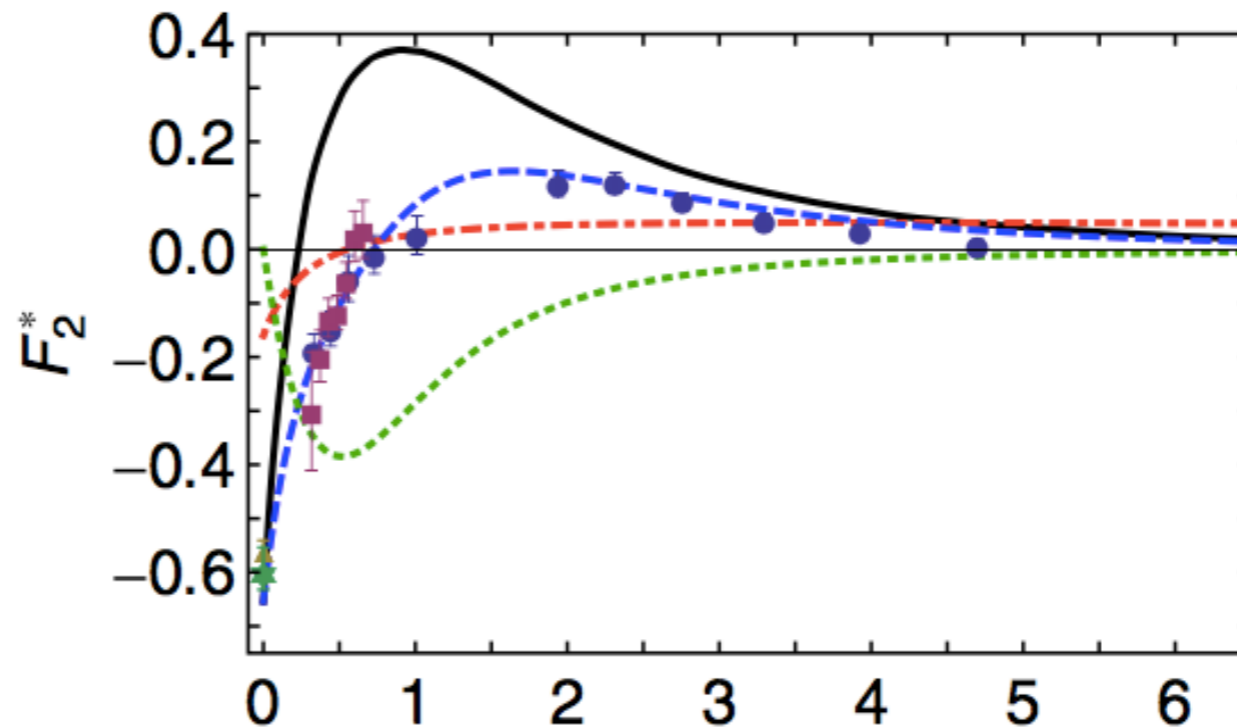
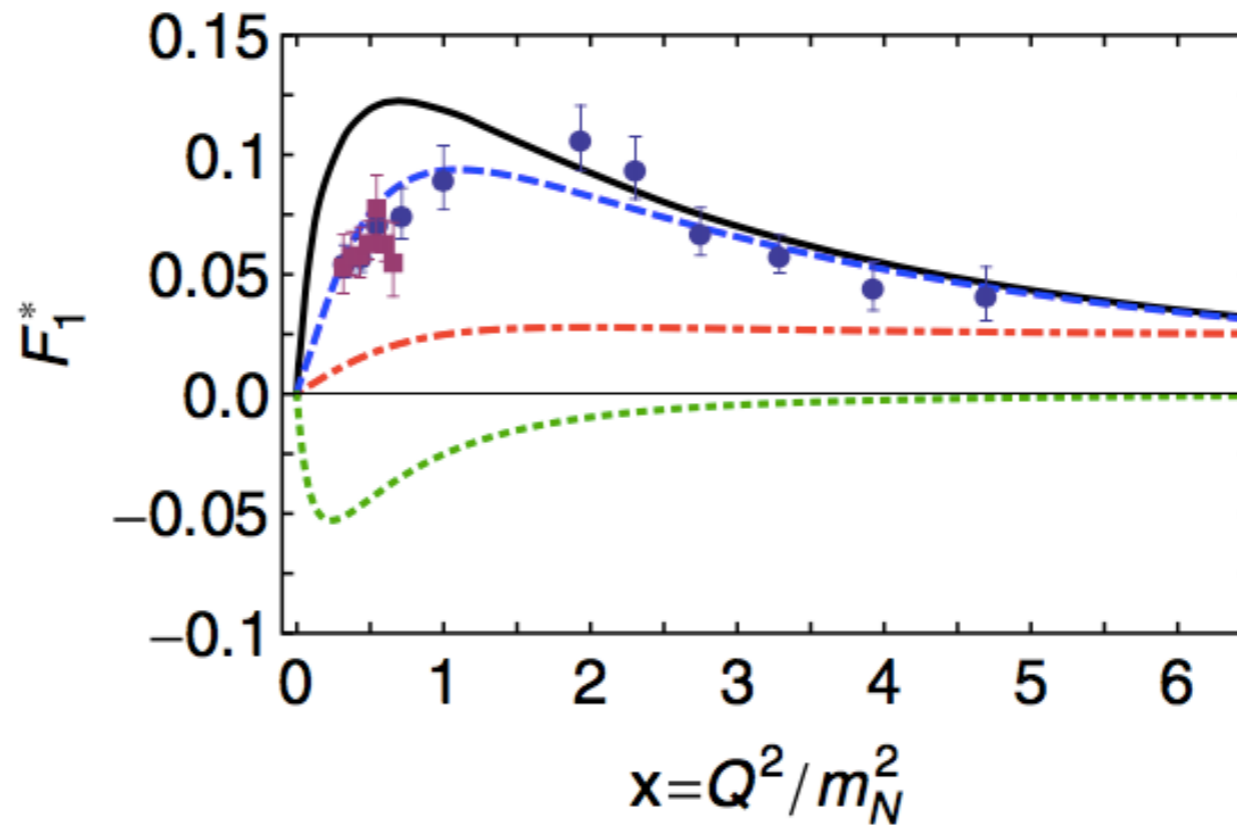
Zeroth Chebyshev moments of all  $S$ -wave components in the Faddeev wave function.  $S_1$  is associated with the baryon's scalar diquark;  $A_2$ ,  $A_3$ ,  $A_5$  associated with axialvector correlation.

# Dirac and Pauli Transition Form Factors

DSE-Faddeev solution

Contact interaction

Meson Cloud Correction



## Conclusive Remarks

- Computed spectrum of 1st radial excitations for pseudoscalar (un)flavored mesons based on a rainbow-ladder kernel.
- The meson spectrum obtained clearly indicates that the ladder approximation is neither appropriate for radial excitations of light mesons nor for heavy-light (charmed) mesons.
- Along similar lines we show that the first radial excitation of the 3-quark nucleon core using a quark-diquark Faddeev kernel.
- The mass found for this excited nucleon agrees very well with that of the bare unclothed quark core.