Bonn-Gatchina Amplitude Analysis Methods and its Extensions to Electroproduction

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Energy dependent approach

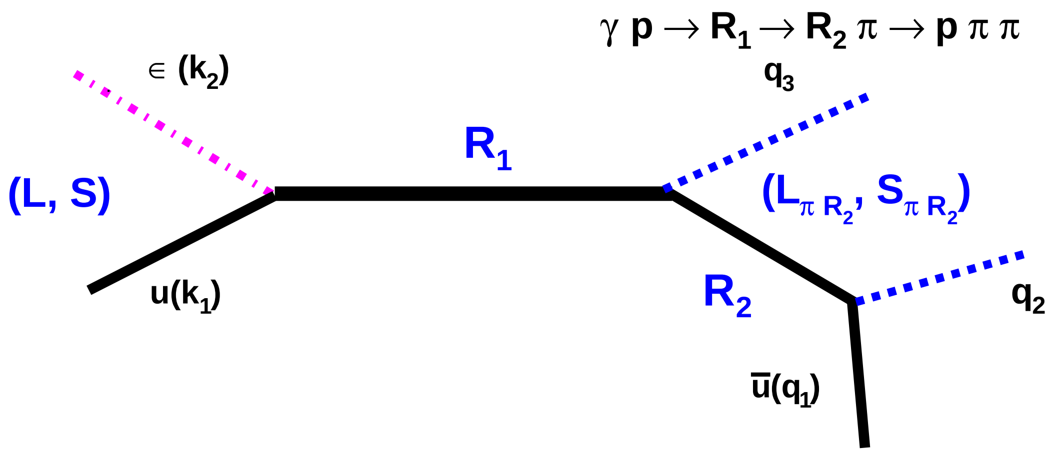
In many cases an unambiguous partial wave decomposition at fixed energies is impossible. Then the energy and angular parts should be analyzed together:

$$A(s,t) \;=\; \sum_{\beta\beta' n} A_n^{\beta\beta'}(s) Q^{(\beta)+}_{\mu_1\ldots\mu_n} F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} Q^{(\beta')}_{\nu_1\ldots\nu_n}$$

1. **Correlations between angular part and energy part are under control.**
2. **Unitarity and analyticity can be introduced from the beginning.**
3. **Parameters can be fixed from a combined fit of many reactions.**

1 C. Zemach, Phys. Rev. 140, B97 (1965); 140, B109 (1965)
2 S.U.Chung, Phys. Rev. D 57, 431 (1998)
3 A. V. Anisovich, V. V. Anisovich, V. N. Markov, M. A. Matveev and A. V. Sarantsev, J. Phys. G G 28, 15 (2002)
4 B. S. Zou and D. V. Bugg, Eur. Phys. J. A 16, 537 (2003)
5 A. Anisovich, E. Klempt, A. Sarantsev and U. Thoma, Eur. Phys. J. A 24, 111 (2005)
6 A. V. Anisovich and A. V. Sarantsev, Eur. Phys. J. A 30, 427 (2006)
7 A. V. Anisovich, V. V. Anisovich, E. Klempt, V. A. Nikonov and A. V. Sarantsev, Eur. Phys. J. A 34, 129 (2007).

Resonance amplitudes for meson photoproduction



General form of the angular dependent part of the amplitude:

$$\bar{u}(q_1)\tilde{N}_{\alpha_1\ldots\alpha_n}(R_2\to\mu N)F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n}(q_1+q_2)\tilde{N}^{(j)\beta_1\ldots\beta_n}_{\gamma_1\ldots\gamma_m}(R_1\to\mu R_2)$$

$$F^{\gamma_1\ldots\gamma_m}_{\xi_1\ldots\xi_m}(P)V^{(i)\mu}_{\xi_1\ldots\xi_m}(R_1\to\gamma N)u(k_1)\varepsilon_\mu$$

$$F^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L}(p)=(m+\hat{p})O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}\frac{L+1}{2L+1}\left(g^{\perp}_{\alpha_1\beta_1}-\frac{L}{L+1}\sigma_{\alpha_1\beta_1}\right)\prod_{i=2}^{L}g_{\alpha_i\beta_i}O^{\beta_1\ldots\beta_L}_{\nu_1\ldots\nu_L}$$

$$\sigma_{\alpha_i\alpha_j}=\frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j}-\gamma_{\alpha_j}\gamma_{\alpha_i})$$

Orbital momentum operator

The angular momentum operator is constructed from momenta of particles k_1, k_2 and metric tensor $g_{\mu\nu}$.

For $L=0$ this operator is a constant: $X^0=1$

The $L=1$ operator is a vector $X^{(1)}_\mu$, constructed from: $k_\mu=\frac{1}{2}(k_{1\mu}-k_{2\mu})$ and $P_\mu=(k_{1\mu}+k_{2\mu})$.

$$X^{(1)}_\mu=k^\perp_\mu=k_\nu g^\perp_{\nu\mu}; \qquad g^\perp_{\nu\mu}=\left(g_{\nu\mu}-\frac{P_\nu P_\nu}{p^2}\right);$$

Recurrent expression for the orbital momentum operators $X^{(n)}_{\mu_1\ldots\mu_n}$

$$X^{(n)}_{\mu_1\ldots\mu_n}=\frac{2n-1}{n^2}\sum_{i=1}^{n}k^\perp_{\mu_i}X^{(n-1)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_n}-\frac{2k^2_\perp}{n^2}\sum_{\substack{i,j=1\\ i<j}}^{n}g_{\mu_i\mu_j}X^{(n-2)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_n}$$

πN interaction

States with $J=L-1/2$ are called '-' states ($1/2^+$, $3/2^-$,$5/2^+$,$\ldots$) and states with $J=L+1/2$ are called '+' states ($1/2^-$, $3/2^+$,$5/2^-$,$\ldots$).

$$\tilde N^+_{\mu_1\ldots\mu_n} = X^{(n)}_{\mu_1\ldots\mu_n} \qquad \tilde N^-_{\mu_1\ldots\mu_n} = i\gamma_\nu\gamma_5 X^{(n+1)}_{\nu\mu_1\ldots\mu_n}$$

$$A = \bar u(k_1) N^{\pm}_{\mu_1\ldots\mu_L} F^{\mu_1\ldots\mu_{L-1}}_{\nu_1\ldots\nu_{L-1}} N^{\pm}_{\nu_1\ldots\nu_L} u(q_1) BW^{\pm}_L(s) \xrightarrow[c.m.s.]{} \omega^* \left[G(s,t) + H(s,t) i(\vec\sigma\vec n)\right]\omega'$$

$$G(s,t) = \sum_L \left[(L{+}1)F^+_L(s) - LF^-_L(s)\right] P_L(z)\ ,$$

$$H(s,t) = \sum_L \left[F^+_L(s) + F^-_L(s)\right] P'_L(z)\ .$$

$$F^+_L \quad = (-1)^{L+1}(|\vec k||\vec q|)^L \sqrt{\chi_i\chi_f}\,\frac{\alpha(L)}{2L{+}1} BW^+_L(s)\ ,$$

$$F^-_L \quad = (-1)^{L}(|\vec k||\vec q|)^L \sqrt{\chi_i\chi_f}\,\frac{\alpha(L)}{L} BW^-_L(s)\ .$$

$$\chi_i = m_i + k_{i0} \qquad\qquad \alpha(L) = \prod_{l=1}^{L} \frac{2l-1}{l} = \frac{(2L-1)!!}{L!}\ .$$

γN interaction

Photon has quantum numbers $J^{PC}=1^{--}$, proton $1/2^+$. Then in S-wave two states can be formed is $1/2^-$ and $3/2^-$.

Then P-wave $1/2^+, 3/2^+$ and $1/2^+, 3/2^+, 5/2^+$.

In general case: $1/2^-$, $1/2^+$ described by two amplitudes and higher states by three amplitudes.

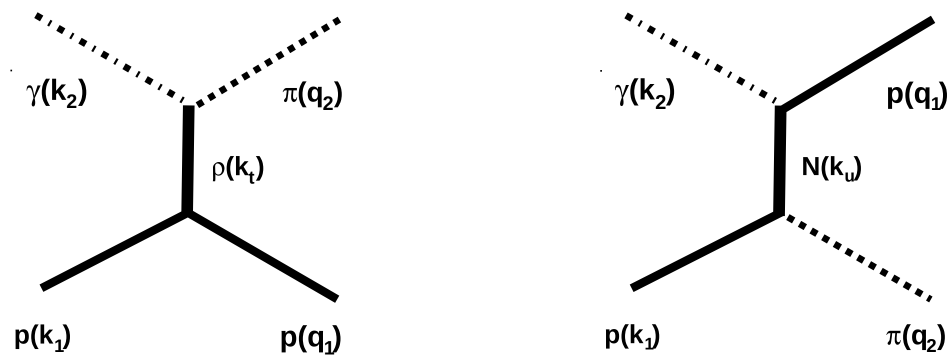
$$
\begin{aligned}
V^{(1+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\mu i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n}\;, &\quad V^{(1-)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\xi\gamma_\mu X^{(n+1)}_{\xi\alpha_1\ldots\alpha_n}\;,\\
V^{(2+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+2)}_{\mu\nu\alpha_1\ldots\alpha_n}\;, &\quad V^{(2-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n+1)}_{\mu\alpha_1\ldots\alpha_n}\;,\\
V^{(3+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+1)}_{\nu\alpha_1\ldots\alpha_n} g^\perp_{\mu\alpha_n}\;, &\quad V^{(3-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n-1)}_{\alpha_2\ldots\alpha_n} g^\perp_{\alpha_1\mu}\;.
\end{aligned}
$$

For the real photons:

$$\varepsilon_\mu V^{(2\pm)\mu}_{\alpha_1\ldots\alpha_n} = C^\pm \varepsilon_\mu V^{(3\pm)\mu}_{\alpha_1\ldots\alpha_n}$$

where $C^\pm$ do not depend on angles.

Reggezied exchanges:



The amplitude for t-channel exchange:

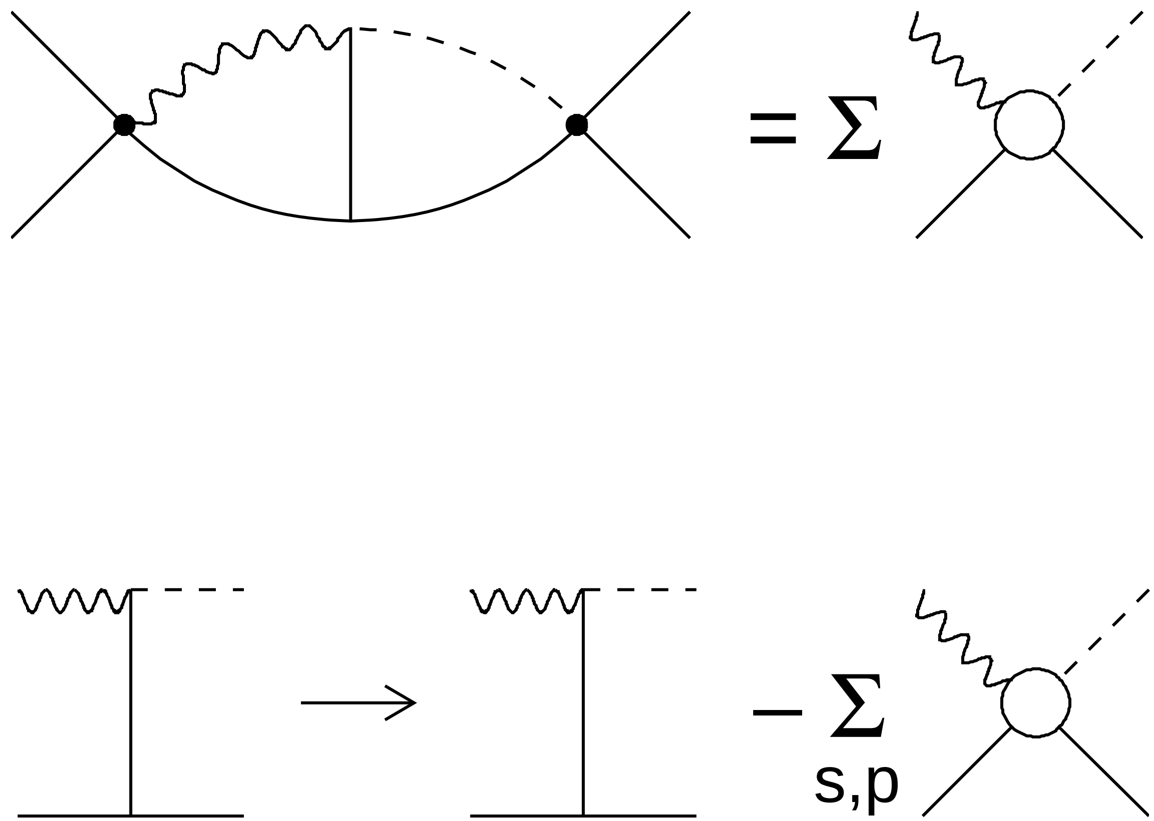
$$A = g_1(t) g_2(t) R(\xi, \nu, t) = g_1(t) g_2(t) \frac{1 + \xi exp(-i\pi\alpha(t))}{\sin(\pi\alpha(t))} \left(\frac{\nu}{\nu_0}\right)^{\alpha(t)} \qquad \nu = \frac{1}{2}(s-u).$$

Here $\alpha(t)$ is the reggion trajectory, and ξ is its signature:

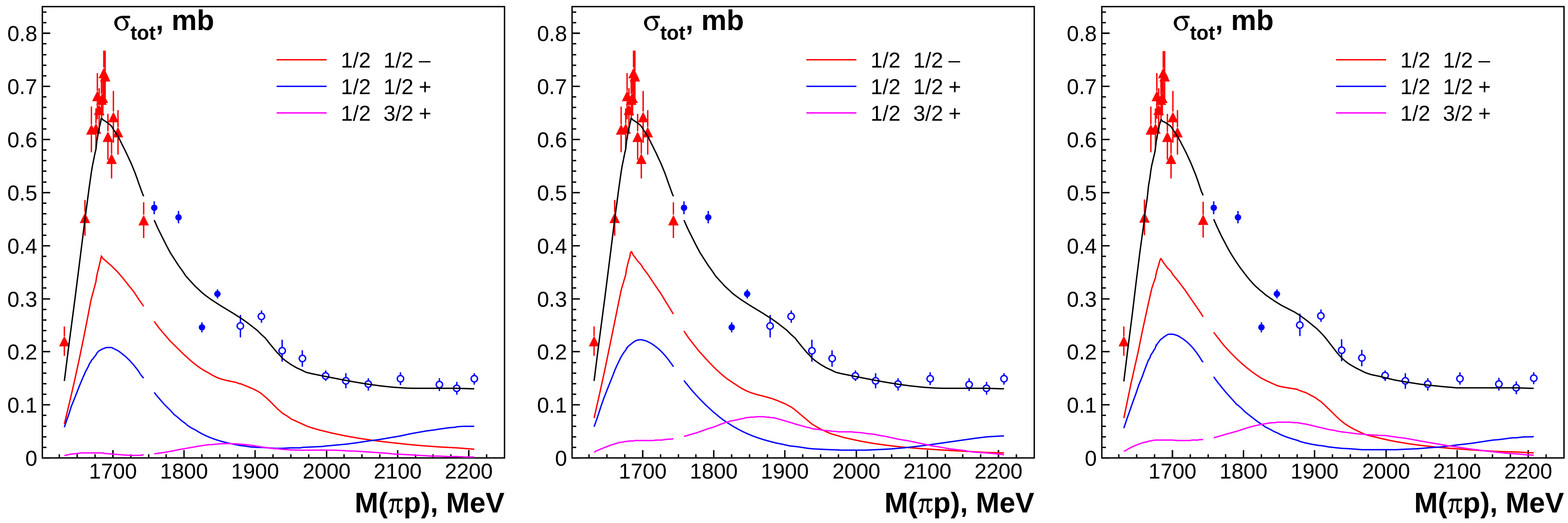
$$R(+,\nu,t) = \frac{e^{-i\frac{\pi}{2}\alpha(t)}}{\sin(\frac{\pi}{2}\alpha(t))\Gamma\left(\frac{\alpha(t)}{2}\right)} \left(\frac{\nu}{\nu_0}\right)^{\alpha(t)},$$

$$R(-,\nu,t) = \frac{ie^{-i\frac{\pi}{2}\alpha(t)}}{\cos(\frac{\pi}{2}\alpha(t))\Gamma\left(\frac{\alpha(t)}{2} + \frac{1}{2}\right)} \left(\frac{\nu}{\nu_0}\right)^{\alpha(t)}.$$

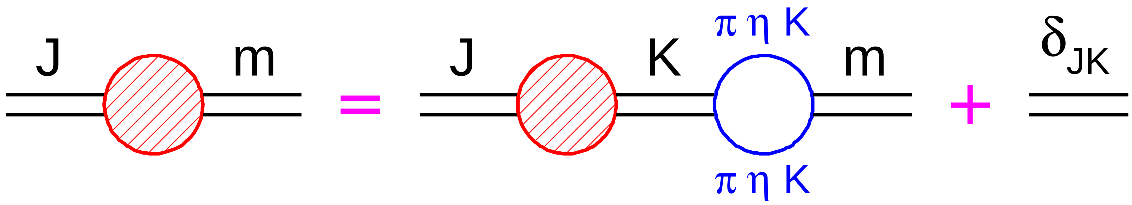
t,u-exchange subtraction procedure



t,u-exchange subtraction procedure



N/D based (D-matrix) analysis of the data



$$D_{jm} = D_{jk} \sum_\alpha B_\alpha^{km}(s) \frac{1}{M_m - s} + \frac{\delta_{jm}}{M_j^2 - s} \qquad \hat{D} = \hat{\kappa}(I - \hat{B}\hat{\kappa})^{-1}$$

$$\hat{\kappa} = diag\left(\frac{1}{M_1^2 - s}, \frac{1}{M_2^2 - s}, \ldots, \frac{1}{M_N^2 - s}, R_1, R_2 \ldots\right)$$

$$\hat{B}_{ij} = \sum_\alpha B_\alpha^{ij} = \sum_\alpha \int \frac{ds'}{\pi} \frac{g_\alpha^{(R)i} \rho_\alpha(s', m_{1\alpha}, m_{2\alpha}) g_\alpha^{(L)j}}{s' - s - i0}$$

In the present fits we calculate the elements of the B^{ij}_α using one subtraction taken at the channel threshold $M_\alpha=(m_{1\alpha}+m_{2\alpha})$:

$$B^{ij}_\alpha(s)=B^{ij}_\alpha(M^2_\alpha)+(s-M^2_\alpha)\int\limits_{m_a^2}^{\infty}\frac{ds'}{\pi}\frac{g^{(R)i}_\alpha\rho_\alpha(s',m_{1\alpha},m_{2\alpha})g^{(L)j}_\alpha}{(s'-s-i0)(s'-M^2_\alpha)}.$$

In this case the expression for elements of the $\hat{B}$ matrix can be rewritten as:

$$B^{ij}_\alpha(s)=g^{(R)i}_a\left(b^\alpha+(s-M^2_\alpha)\int\limits_{m_a^2}^{\infty}\frac{ds'}{\pi}\frac{\rho_\alpha(s',m_{1\alpha},m_{2\alpha})}{(s'-s-i0)(s'-M^2_\alpha)}\right)g^{(L)j}_\beta=g^{(R)i}_a B_\alpha g^{(L)j}_\beta$$

and D-matrix method equivalent to the K-matrix method with loop diagram with real part taken into account:

$$A=\hat{K}(I-\hat{B}\hat{K})^{-1}\qquad\qquad B_{\alpha\beta}=\delta_{\alpha\beta}B_\alpha$$

Minimization methods

1. **The two body final states $\pi N, \gamma N \to \pi N, \eta N, K\Lambda, K\Sigma, \omega N, K^*\Lambda$: χ^2 method.**

   For n measured bins we minimize

$$\chi^2 = \sum_j^n \frac{(\sigma_j(PWA) - \sigma_j(exp))^2}{(\Delta\sigma_j(exp))^2}$$

   Present solution $\chi^2 = 54634$ for 33988 points. $\chi^2/N_F = 1.6$

2. **Reactions with three or more final states are analyzed with logarithm likelihood method. $\pi N, \gamma N \to \pi\pi N, \pi\eta N$. The minimization function:**

$$f = -\sum_j^{N(data)} ln \frac{\sigma_j(PWA)}{\sum\limits_m^{N(rec\,MC)} \sigma_m(PWA)}$$

   This method allows us to take into account all correlations in many dimensional phase space. Above 1 000 000 data events are taken in the fit.

Baryon data base

DATA	BG2013-2014	added in BG2014-2015
$\pi N \to \pi N$ ampl.	SAID or Hoehler energy fixed	
$\gamma p \to \pi N$	$\frac{d\sigma}{d\Omega}, \Sigma, T, P, E, G, H$	E, G, T, P (CB-ELSA, CLAS)
$\gamma n \to \pi N$	$\frac{d\sigma}{d\Omega}, \Sigma, T, P$	$\frac{d\sigma}{d\Omega} (MAMI)$
$\gamma n \to \eta n$	$\frac{d\sigma}{d\Omega}, \Sigma$	$\frac{d\sigma}{d\Omega}$ (MAMI)
$\gamma p \to \eta p$	$\frac{d\sigma}{d\Omega}, \Sigma$	T, P, H, E (CB-ELSA)
$\gamma p \to \eta' p$		$\frac{d\sigma}{d\Omega}, \Sigma$
$\gamma p \to K^+\Lambda$	$\frac{d\sigma}{d\Omega}, \Sigma, P, T, C_x, C_z, O_{x'}, O_{z'}$	Σ, P, T, O_x, O_z (CLAS)
$\gamma p \to K^+\Sigma^0$	$\frac{d\sigma}{d\Omega}, \Sigma, P, C_x, C_z$	Σ, P, T, O_x, O_z (CLAS)
$\gamma p \to K^0\Sigma^+$	$\frac{d\sigma}{d\Omega}, \Sigma, P$	
$\pi^- p \to \eta n$	$\frac{d\sigma}{d\Omega}$	
$\pi^- p \to K^0\Lambda$	$\frac{d\sigma}{d\Omega}, P, \beta$	
$\pi^- p \to K^0\Sigma^0$	$\frac{d\sigma}{d\Omega}, P\ (K^0\Sigma^0)\ \frac{d\sigma}{d\Omega}\ (K^+\Sigma^-)$	
$\pi^+ p \to K^+\Sigma^+$	$\frac{d\sigma}{d\Omega}, P, \beta$	
$\pi^- p \to \pi^0\pi^0 n$	$\frac{d\sigma}{d\Omega}$ (Crystal Ball)	
$\pi^- p \to \pi^+\pi^- n$		$\frac{d\sigma}{d\Omega}$ (HADES)
$\gamma p \to \pi^0\pi^0 p$	$\frac{d\sigma}{d\Omega}, \Sigma, E, I_c, I_s$	
$\gamma p \to \pi^0\eta p$	$\frac{d\sigma}{d\Omega}, \Sigma, I_c, I_s$	
$\gamma p \to \pi^+\pi^- p$		$\frac{d\sigma}{d\Omega}, I_c, I_s$ (CLAS)
$\gamma p \to \omega p$		$\frac{d\sigma}{d\Omega}, \Sigma, \rho^0_{ij}, \rho^1_{ij}, \rho^2_{ij}, E, G$ (CB-ELSA)
$\gamma p \to K^*(890)\Lambda$		$\frac{d\sigma}{d\Omega}, \Sigma, \rho^0_{ij}$ (CLAS)

CBELSA/TAPS: Helicity Asymmetry E for $p\pi^0$

reaction: $\vec{\gamma} + \vec{p} \rightarrow p + \pi^0$

$$E = \frac{\sigma_{1/2} - \sigma_{3/2}}{\sigma_{1/2} + \sigma_{3/2}}$$

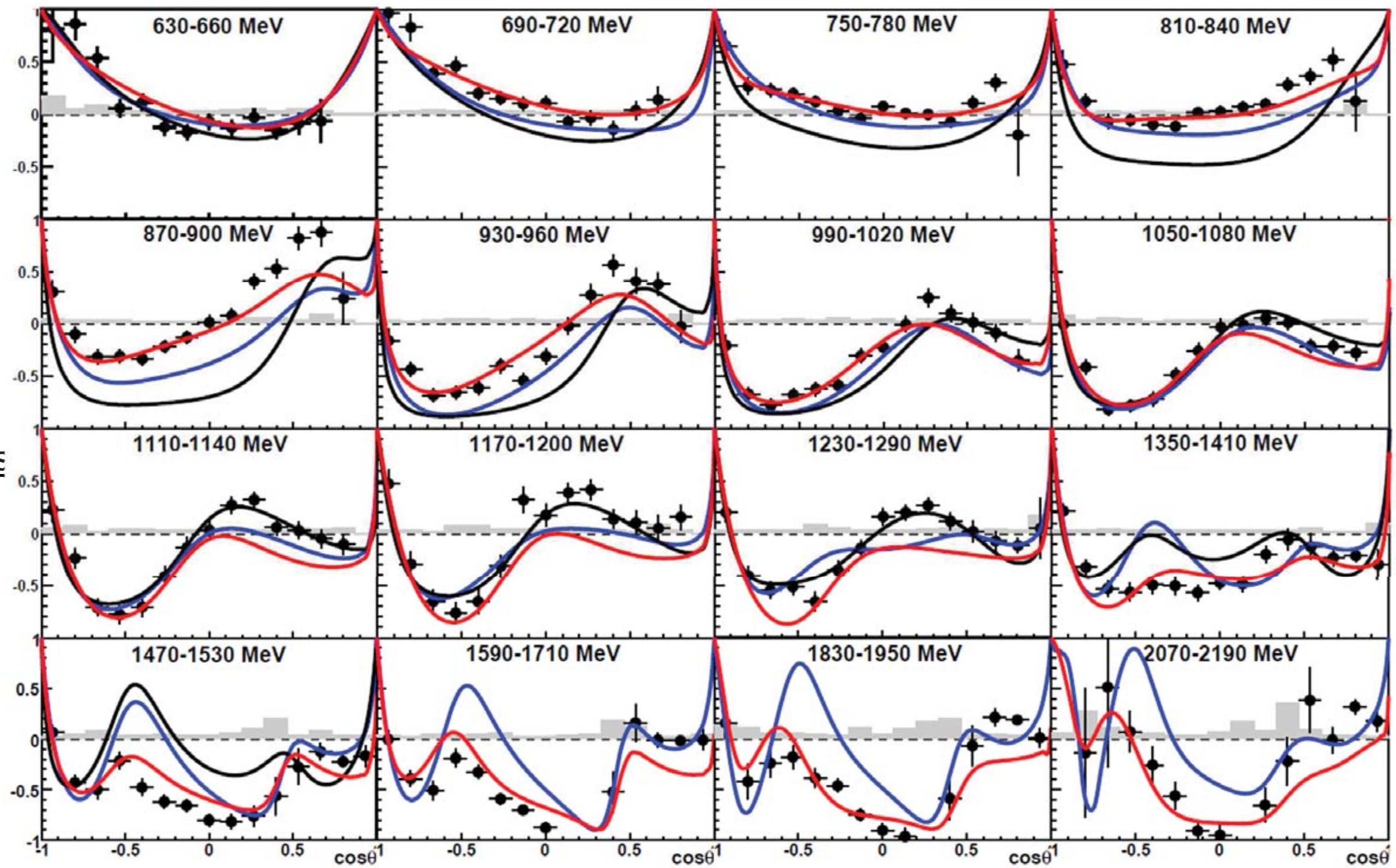
Partial wave analysis prediction:

BnGa

SAID

MAID

CBELSA/TAPS, *M. Gottschall, PRL 112 (2014), 012003*

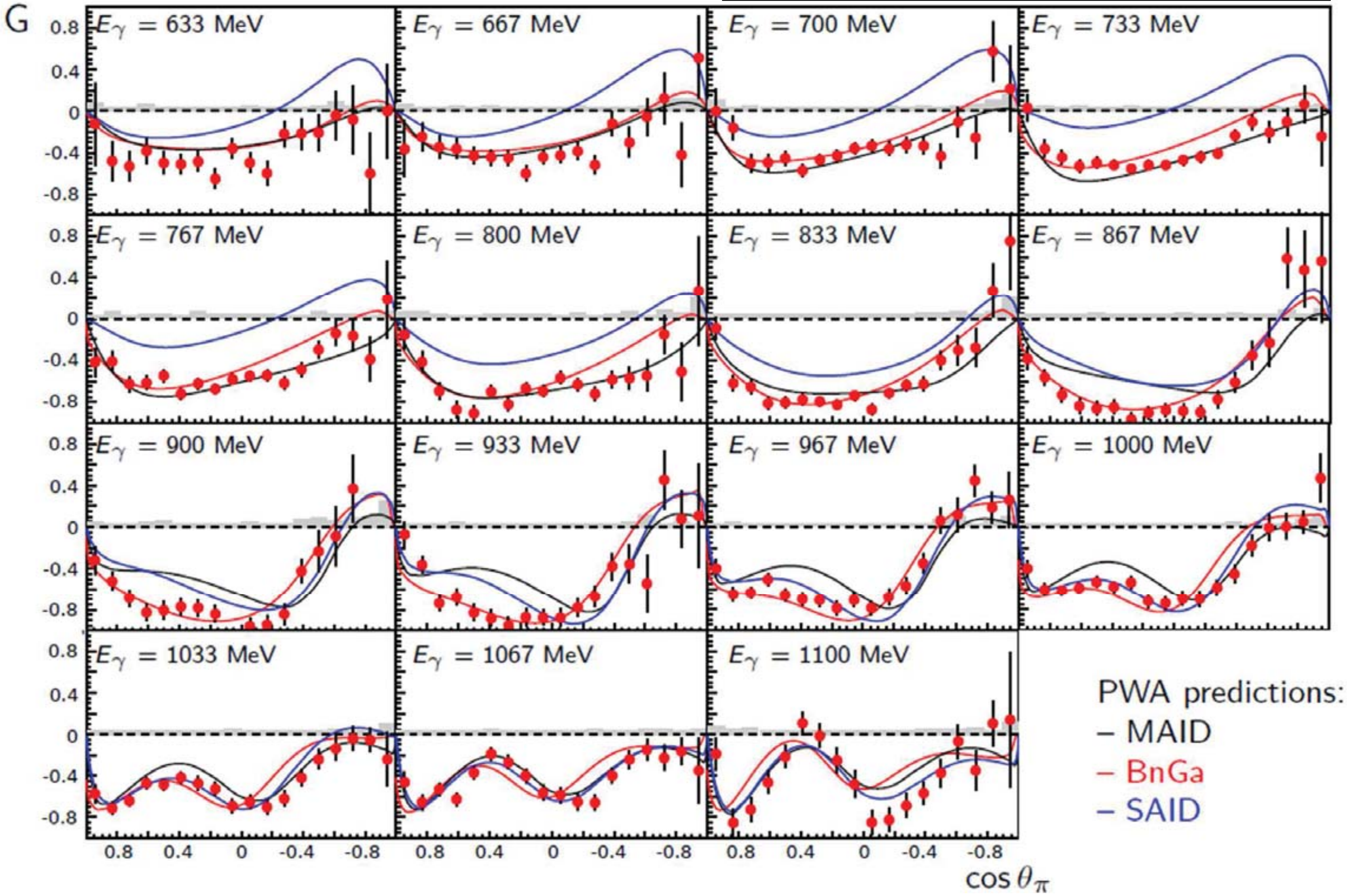


CBELSA/TAPS: Asymmetry G for $p\pi^0$

linearly polarized beam, longitudinally polarized target:

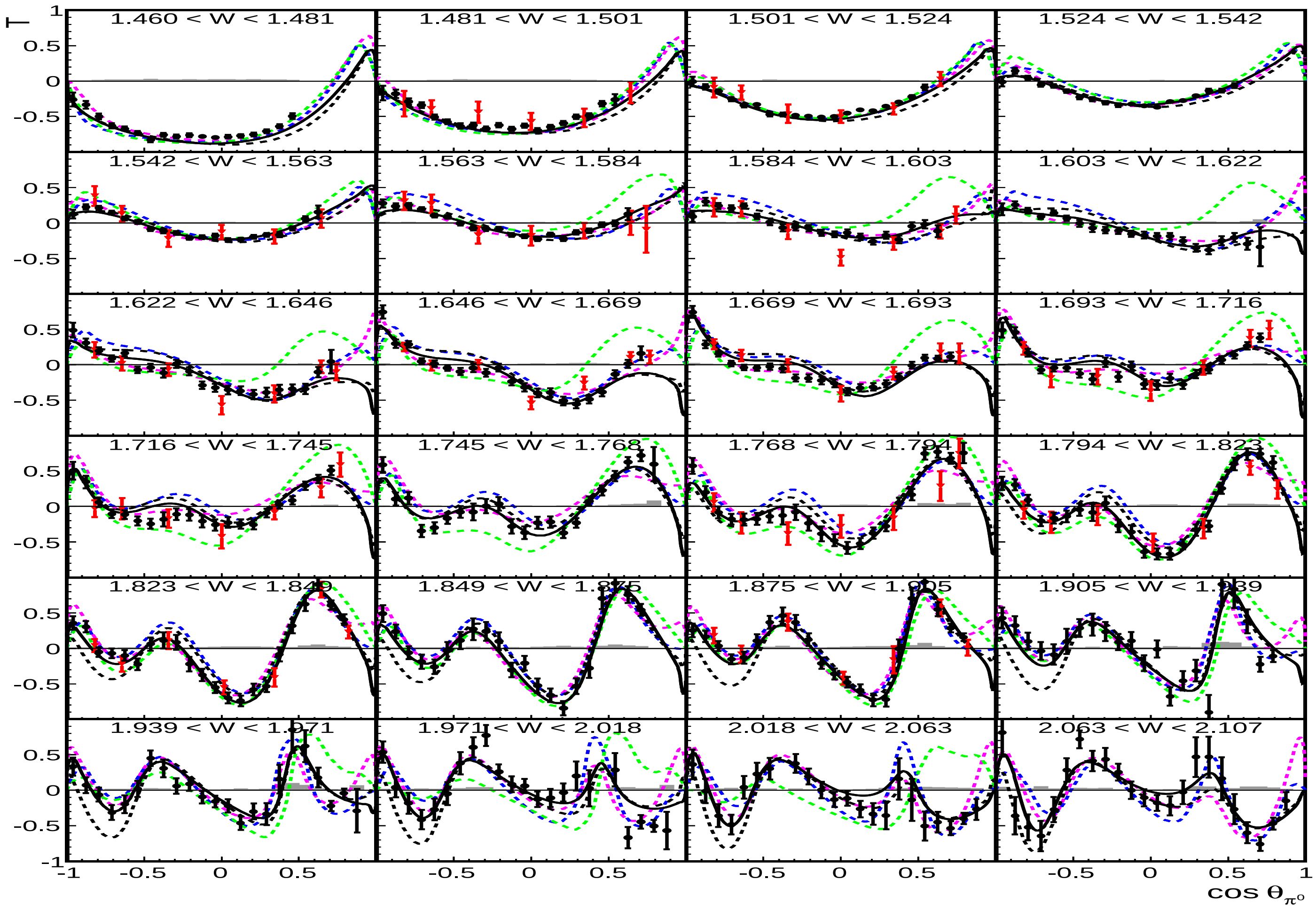
$$\frac{d\sigma}{d\Omega}(\phi) = \frac{d\sigma}{d\Omega}_0 \cdot \left(1 - P_\gamma^{\text{lin}}\,\Sigma\cos(2\phi) + P_\gamma^{\text{lin}} P_z\, G\sin(2\phi)\right)$$

CBELSA/TAPS, *A. Thiel, PRL 109 (2012), 102001*

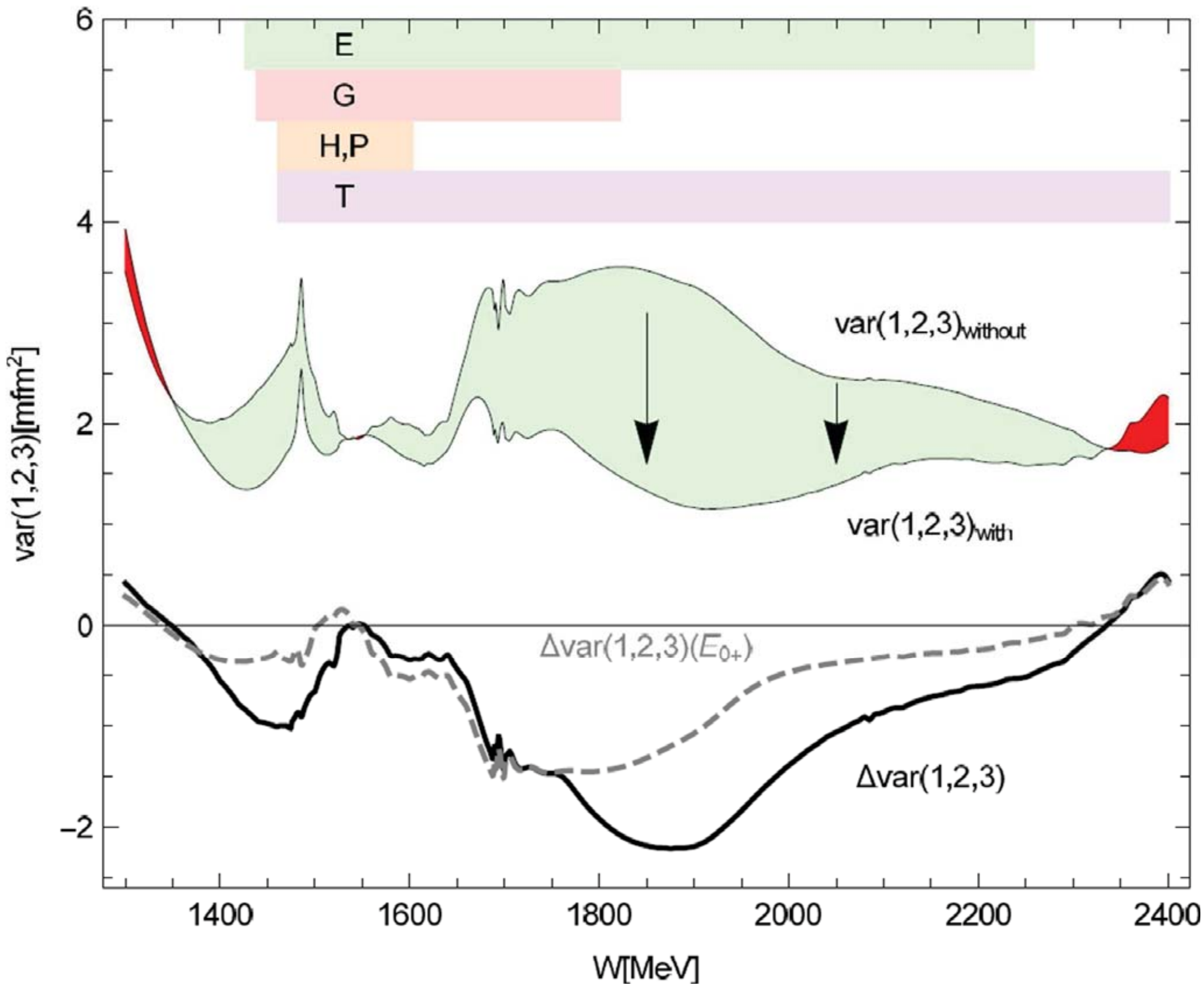


Target asymmetry for $\gamma p \to \pi^0 p$

MAID, SAID, Bonn-Juelich, Bonn-Gatchina



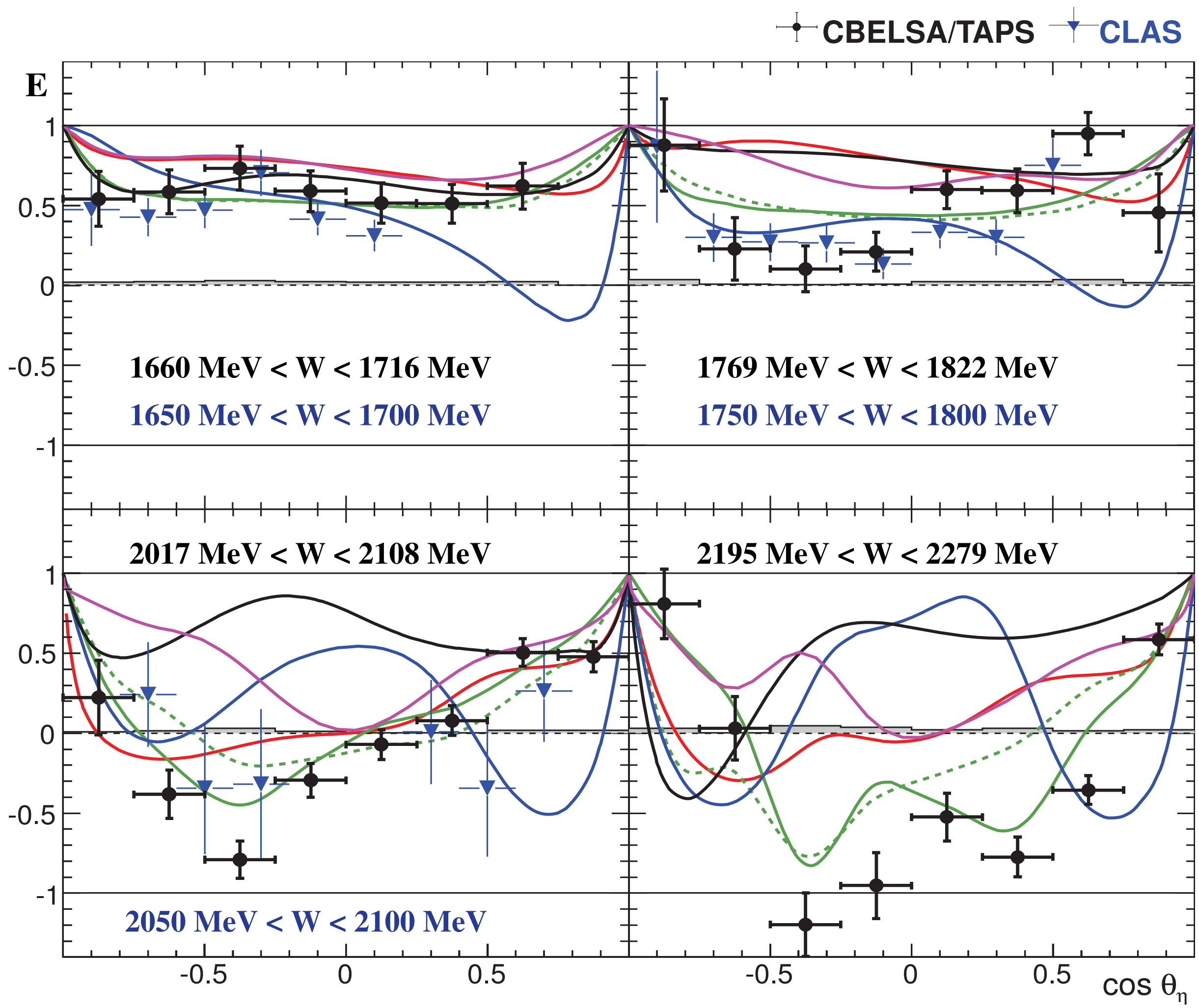
Impact of the new Polarization Data



Clear convergence between different PWA's

Preliminary work:
JüBo: D. Rönchen, M. Döring
U. Meißner
SAID: R. Workmann
BnGa: A. Sarantsev

$\vec{\gamma}\vec{p} \rightarrow p\eta$ - Polarization Observables: E



Circularly polarized photons, longitudinally polarized target

(only a few selected bins shown)

Predictions :

- **Maid ,**
- **Said ,**
- **BoJü,**
- **BnGa 2011**

— new BnGa-fit

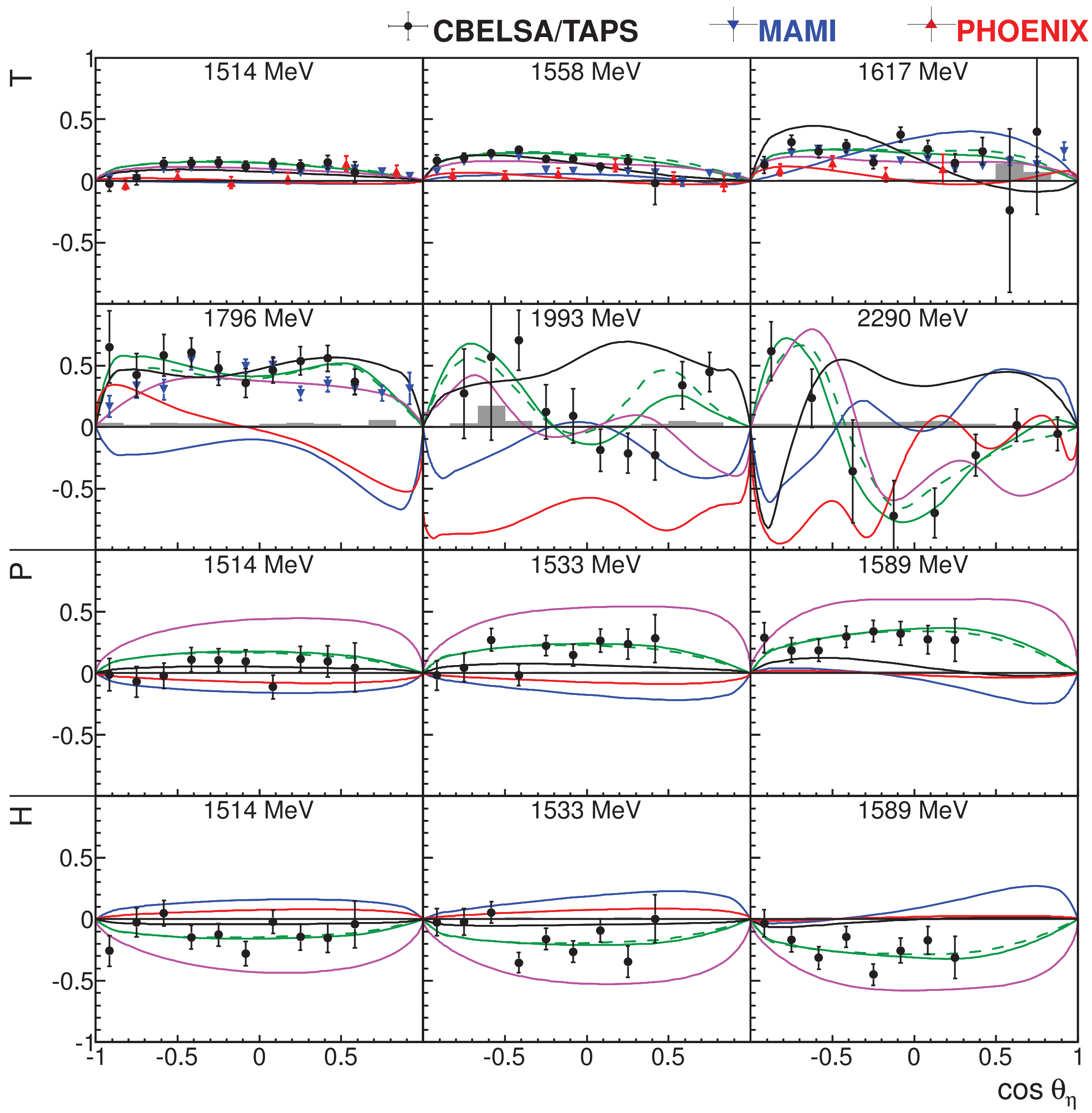
- - - without new $5/2^-$ at 2200 MeV

⇓

⇔ **Large sensitivity!**

⇒ **approaching also the high mass region**

$\vec{\gamma}\vec{p} \to p\eta$ - Polarization Observables: T, P, H



linear pol. photons, transv. pol. target

(only a few selected bins shown)

Predictions :

— Maid ,

— Said ,

— BoJü,

— BnGa 2011

— new BnGa-fit

- - - without new $5/2^-$ at 2200 MeV

$\vec{\gamma}\vec{p} \to p\eta$ - Results including new data on E, G, T, P, H

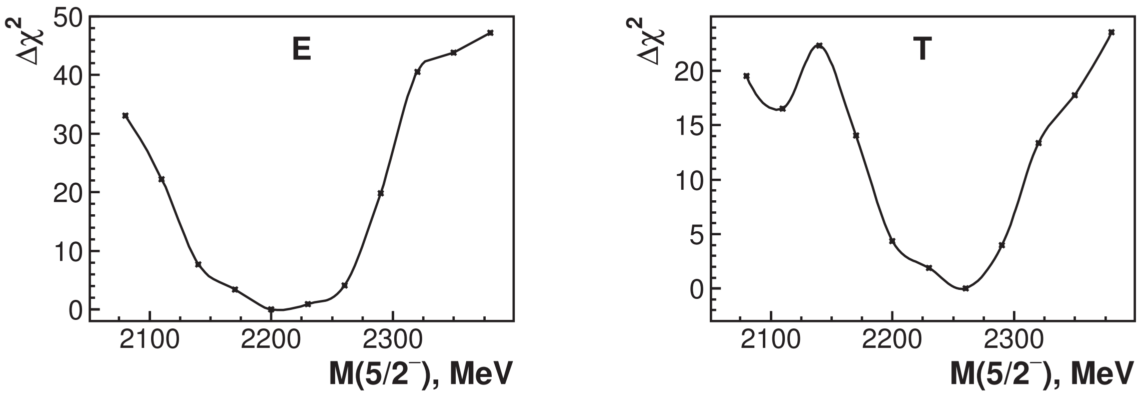
Determination of $p\eta$-branching ratios for various resonances, e.g. :

	$N(1535)1/2^-$	$N(1650)1/2^-$	$N(1710)1/2^+$	$N(1720)3/2^+$
BnGa	0.42±0.04	0.32±0.04	0.27±0.09	0.03±0.02
PDG	0.42±0.10	0.05 - 0.15	0.10 - 0.30	0.021±0.014

⇓

large and heavily discussed difference in the $p\eta$-branching ratio of N(1535)1/2$^-$ and N(1650)1/2$^-$ now significantly reduced

⇒ Hints for a new resonance around 2200 MeV with J^P=5/2$^-$



Parity doublets of N and Δ resonances at high mass region

Parity doublets must not interact by pion emission

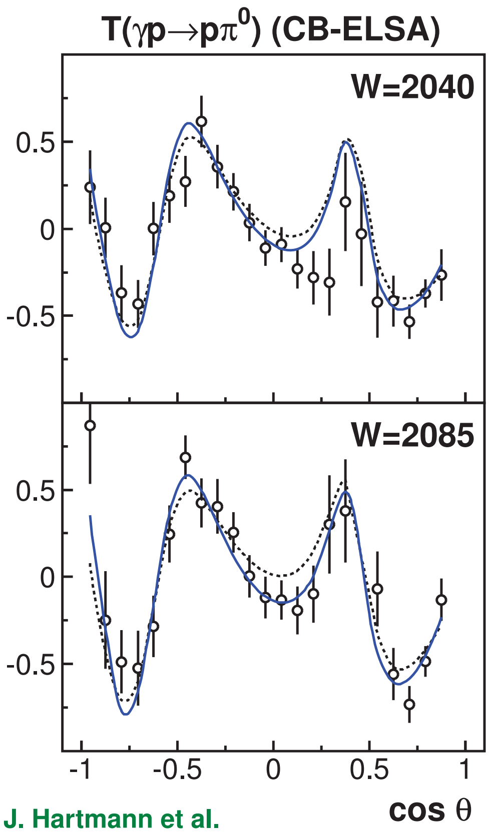
and could have a small coupling to πN.

$J=\frac{1}{2}$	$N_{1/2+}(1880)$	**	$N_{1/2-}(1890)$	**	$\Delta_{1/2+}(1910)$	****	$\Delta_{1/2-}(1900)$	**
$J=\frac{3}{2}$	$N_{3/2+}(1900)$	***	$N_{3/2-}(1875)$	**	$\Delta_{3/2+}(1940)$	***	$\Delta_{3/2-}(1990)$	**
$J=\frac{5}{2}$	$N_{5/2+}(1880)$	**	$N_{5/2-}(2060)$	**	$\Delta_{5/2+}(1940)$	****	$\Delta_{5/2-}(1930)$	***
$J=\frac{7}{2}$	$N_{7/2+}(1980)$	**	$N_{7/2-}(2170)$	****	$\Delta_{7/2+}(1920)$	****	$\Delta_{7/2-}(2200)$	*
$J=\frac{9}{2}$	$N_{9/2+}(2220)$	****	$N_{9/2-}(2250)$	****	$\Delta_{9/2+}(2300)$	**	$\Delta_{9/2-}(2400)$	**

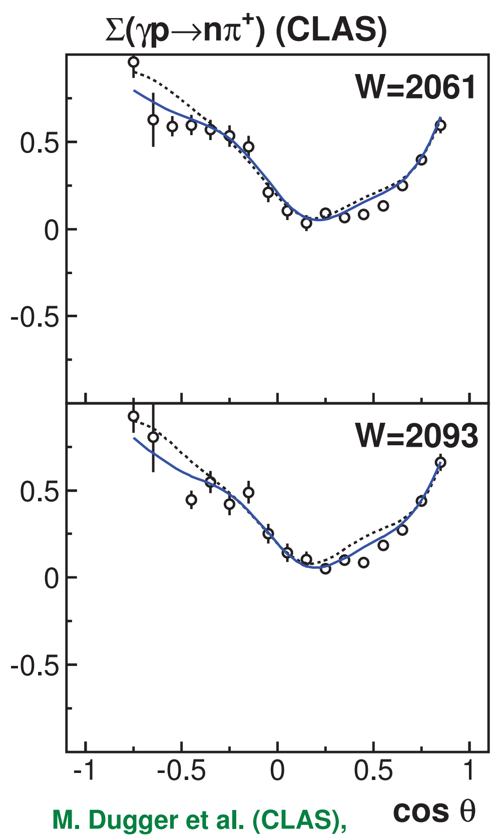
$J=\frac{5}{2}$	$N_{5/2+}(2090)$	**	$N_{5/2-}(2060)$	**	$\Delta_{5/2+}(1940)$	****	$\Delta_{5/2-}(1930)$	***
$J=\frac{7}{2}$	$N_{7/2+}(2100)$	**	$N_{7/2-}(2150)$	****	$\Delta_{7/2+}(1950)$	****	$\Delta_{7/2-}(2200)$	*
$J=\frac{9}{2}$	$N_{9/2+}(2220)$	****	$N_{9/2-}(2250)$	****	$\Delta_{9/2+}(2300)$	**	$\Delta_{9/2-}(2400)^a$	**

Precise Measurements of Polarisation Observables

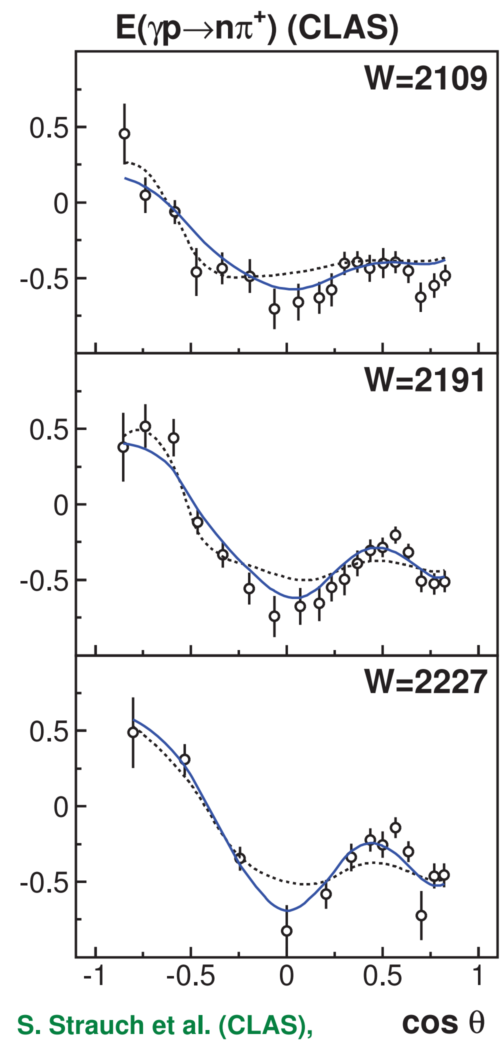
CBELSA/TAPS, CLAS-data (only a few of the measured bins shown:)



J. Hartmann et al. (CBELSA/TAPS), PLB 748, 212 (2015)



M. Dugger et al. (CLAS), PRC 88, 065203 (2013)



S. Strauch et al. (CLAS), arXiv:1503.05163 (2015)

data included in the multi-channel BnGa-PWA:
fit with (——) / without (- - - -) $\Delta(2200)7/2^-$

Search for Parity doublets

Idea (L. Glozman): chiral symmetry restoration in highly excited baryon states.

$\Leftrightarrow$ **Mass-gaps due to spontaneous chiral symmetry breaking like:**
ρ(770) $\leftrightarrow$ a_1(1260) or N(940)$1/2^+$ $\leftrightarrow$ N(1535)$1/2^-$
no longer present in highly excited baryon states

$\Rightarrow$ **ALL high mass states should have a parity partner!**

$\Downarrow$

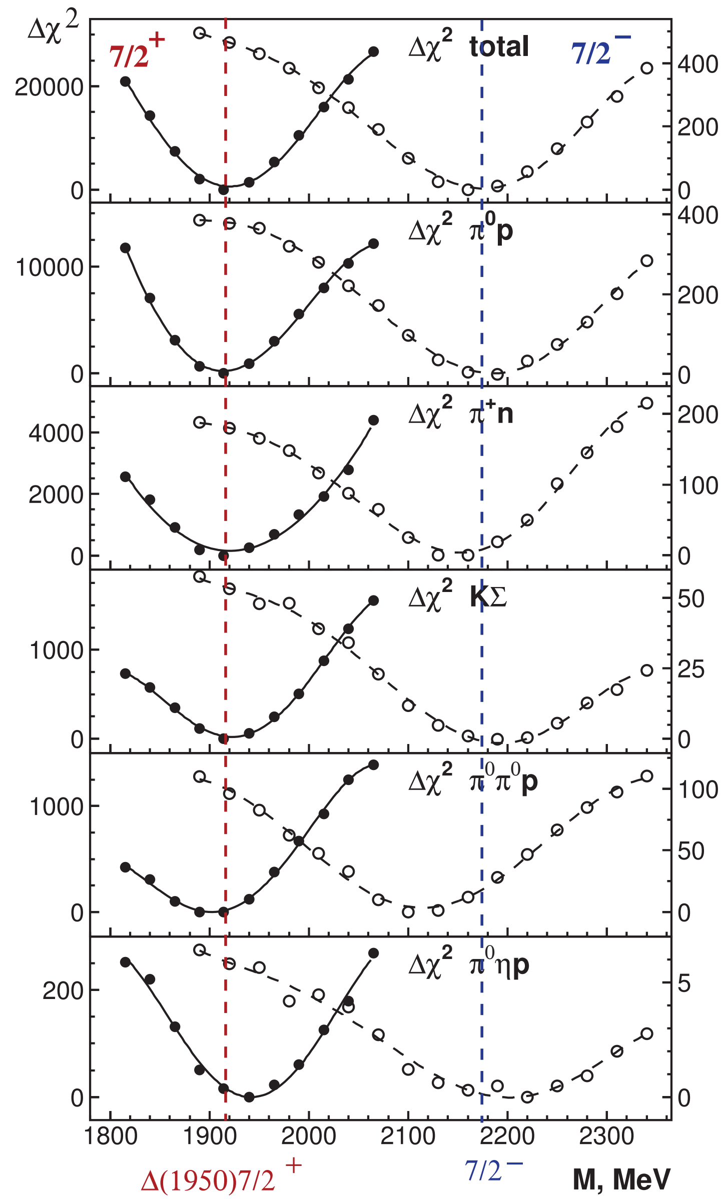
Δ(1910)$1/2^+$ Δ(1920)$3/2^+$ Δ(1905)$5/2^+$ Δ(1950)$7/2^+$
Δ(1900)$1/2^-$ Δ(1940)$3/2^-$ Δ(1930)$5/2^-$??? $7/2^-$

Search for the parity partner of the well known Δ(1950)$7/2^+$ (4*) $\Longrightarrow$

$\Rightarrow$ **J^P = 7/2$^-$-state found at a significantly higher mass: m = 2200 MeV**

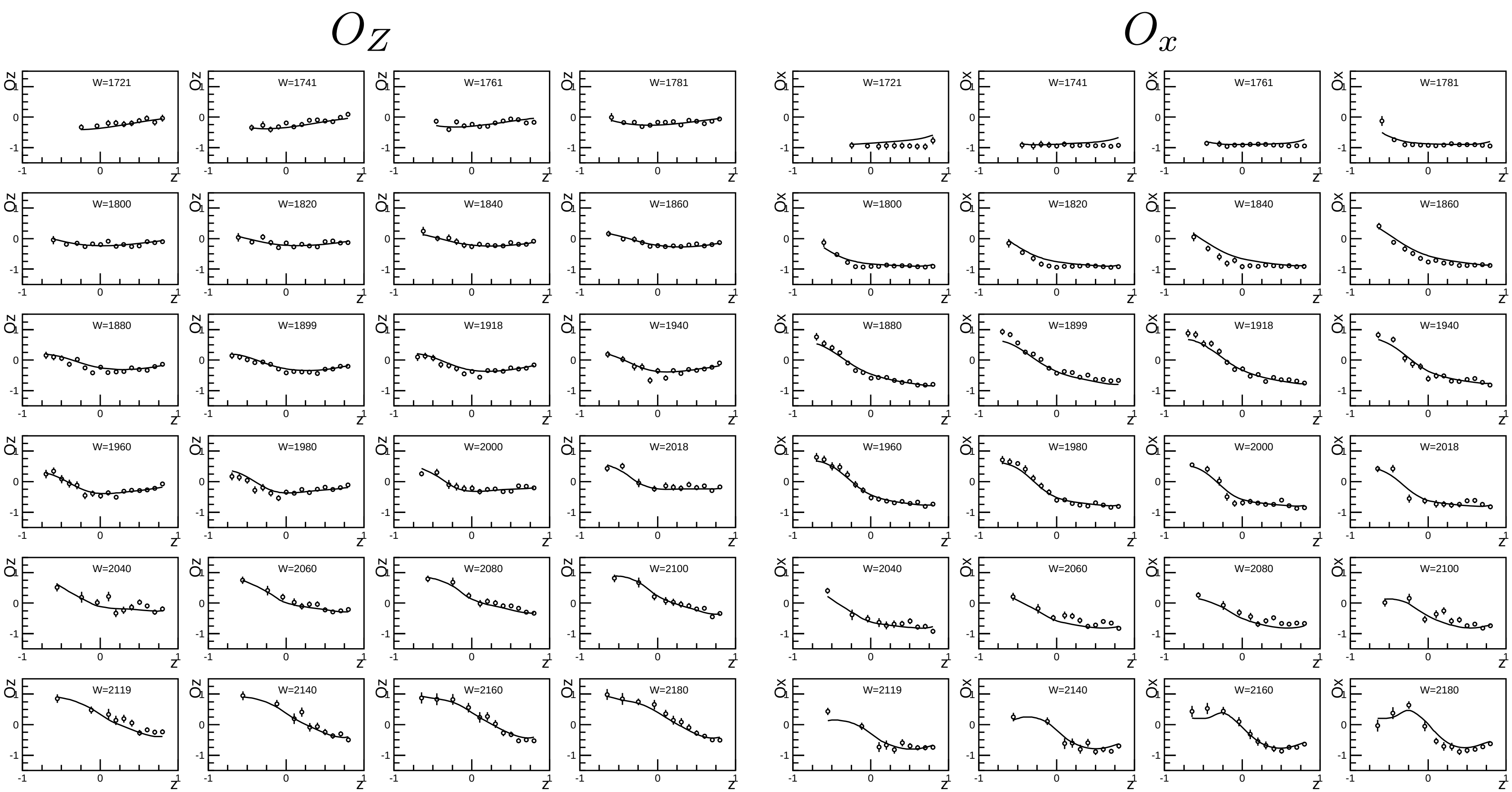
(7/2$^-$ (2200) - (1*)-resonance (PDG) confirmed)

$\Leftrightarrow$ **No parity-partner found**



V. Anisovich et al. (BnGa-PWA), arXiv:1503.05774 (2015)

Fit of the new polarization data on $\gamma p \to K\Lambda$ (CLAS Preliminary, courtesy of D. Ireland)



The best improvement is also from D_{15} state:

M∼2260 MeV, $\Gamma \sim 300$ MeV, $A^{\frac{1}{2}}/A^{\frac{3}{2}}$ ∼-1.0

Photoproduction of vector mesons. Spin density matrices

$$\frac{d\sigma}{d\Omega_\omega\, d\Omega_{dec}} = \frac{d\sigma}{d\Omega_\omega} W(\cos\Theta_{dec}, \Phi_{dec})$$

$$\gamma p \to p\omega(\pi^+\pi^-\pi^0)$$

$$W(\cos\Theta,\Phi) = \frac{3}{4\pi}\Bigg(\frac{1}{2}(1-\rho_{00}) + \frac{1}{2}(3\rho_{00}-1)\cos^2\Theta - \sqrt{2}Re\rho_{10}\sin 2\Theta\cos\Phi - \rho_{1-1}\sin^2\Theta\cos 2\Phi\Bigg)\,.$$

$\cos\Theta, \Phi$ **direction of the vector** $n = \varepsilon_{ijkm} p_j^{\pi^+} p_k^{\pi^-} p_m^{\pi^0}$ **in the** ω **rest frame.**

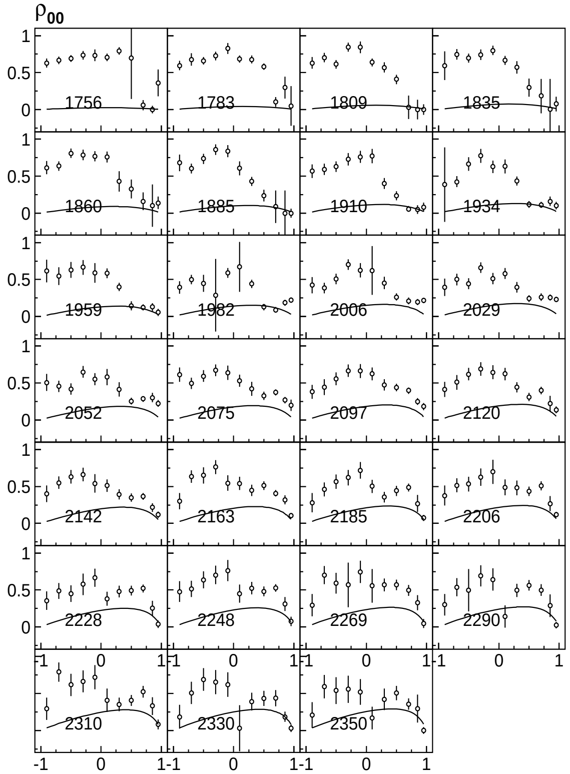
$$\gamma p \to p\omega(\gamma\pi^0)$$

$$W(\cos\Theta,\Phi) = \frac{3}{8\pi}\Bigg(\frac{1}{2}(1+\cos^2\Theta) + \frac{1}{2}(1-3\cos^2\Theta)\rho_{00} + \sqrt{2}Re\rho_{10}\sin(2\Theta)\cos\Phi + \rho_{1-1}\sin^2\Theta\cos 2\Phi\Bigg)\,.$$

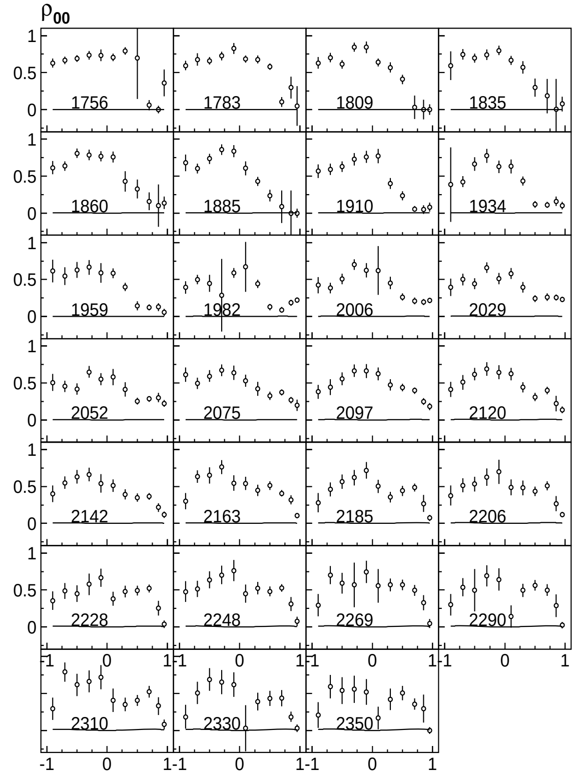
$\cos\Theta, \Phi$ **angles of photon from** ω **decay in the** ω **rest frame**

$$\gamma p \to p\omega$$

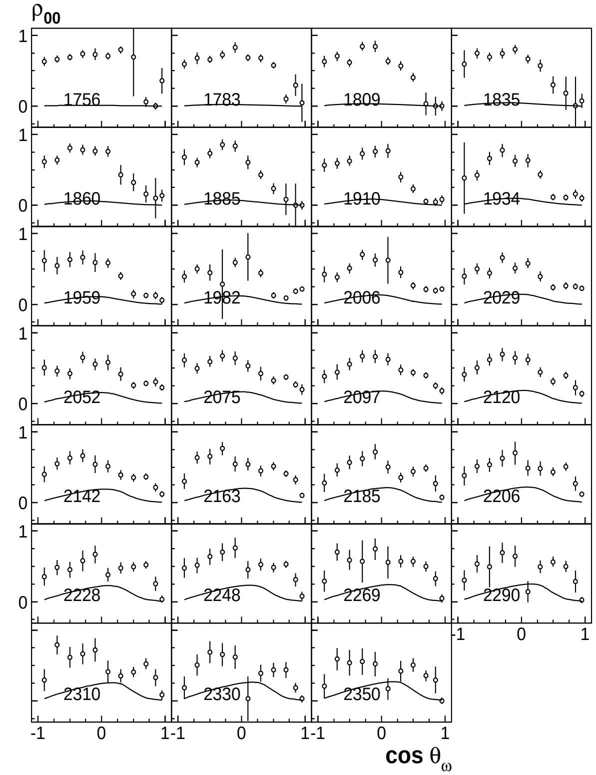
Pion exchange



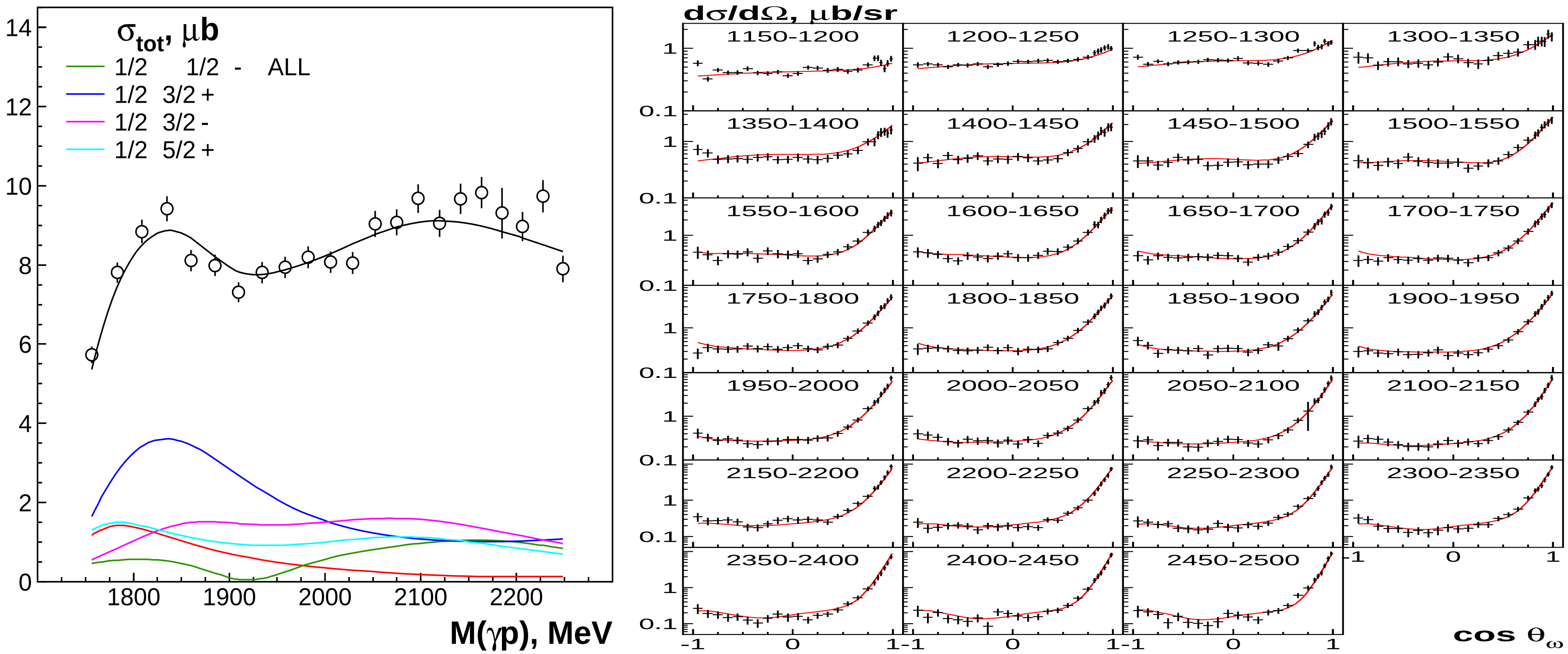
Pomeron exchange



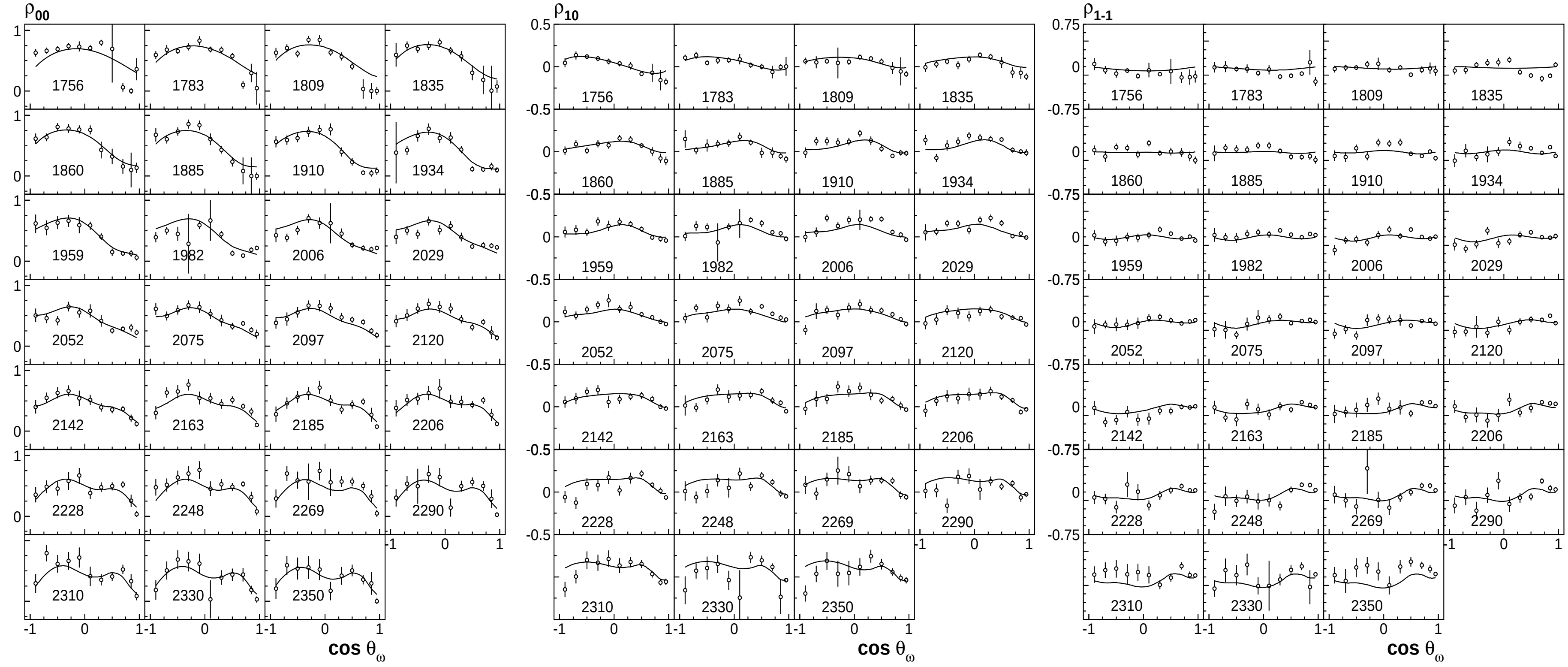
Pion+Pomeron



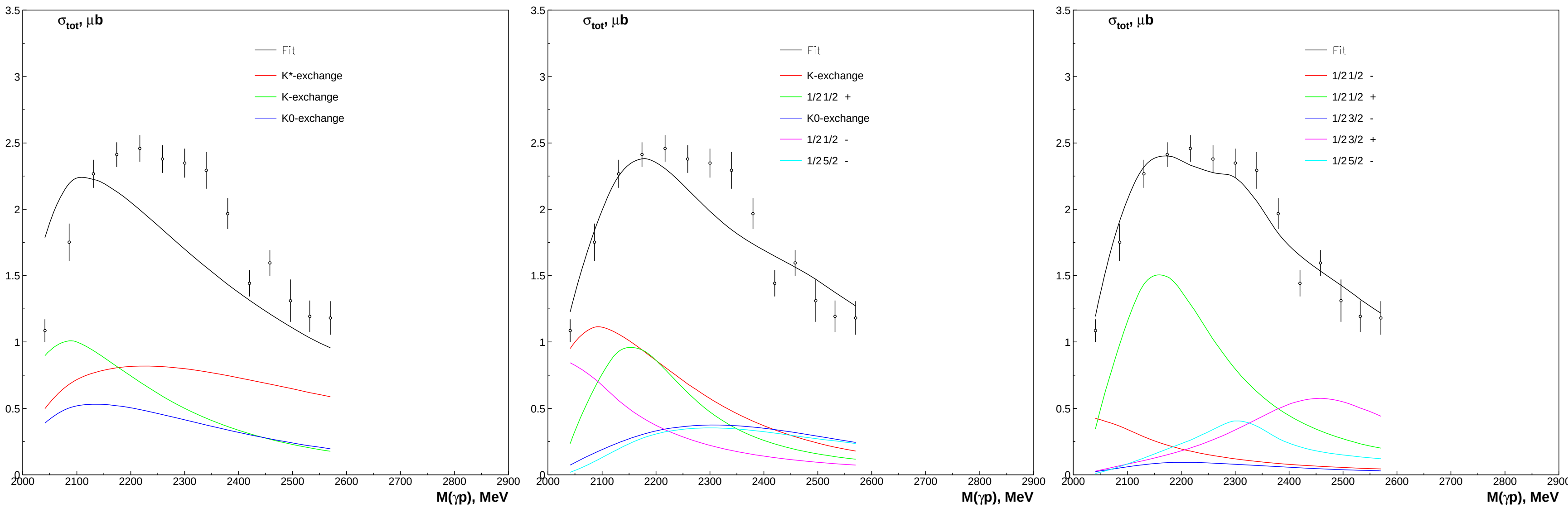
$\gamma p \to p\omega$ Fit of the Crystal Barrel data



$\gamma p \to p\omega$ Fit of the Crystal Barrel data

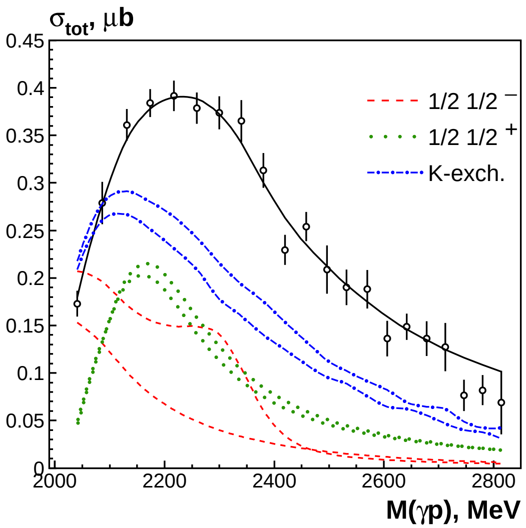


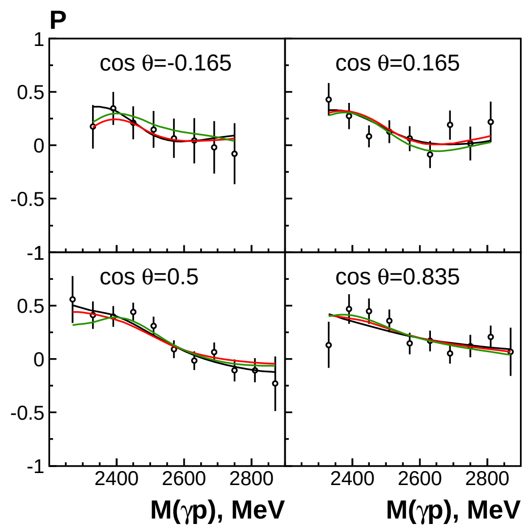
The analysis of $\gamma p \to K^*\Lambda$ (CLAS).

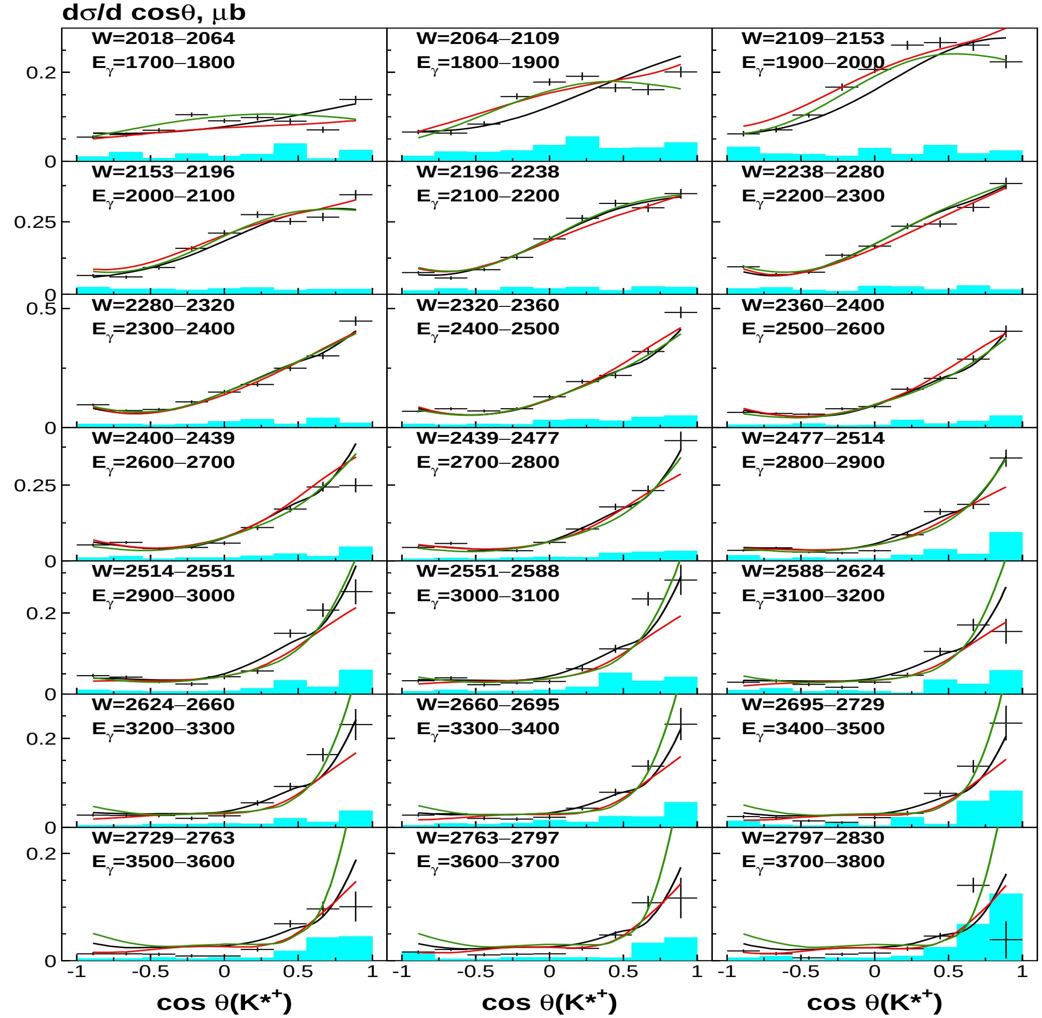


D_{15}: M∼2280 MeV, $\Gamma \sim 170$ MeV, $A^{\frac{1}{2}}/A^{\frac{3}{2}}$ ∼-0.8

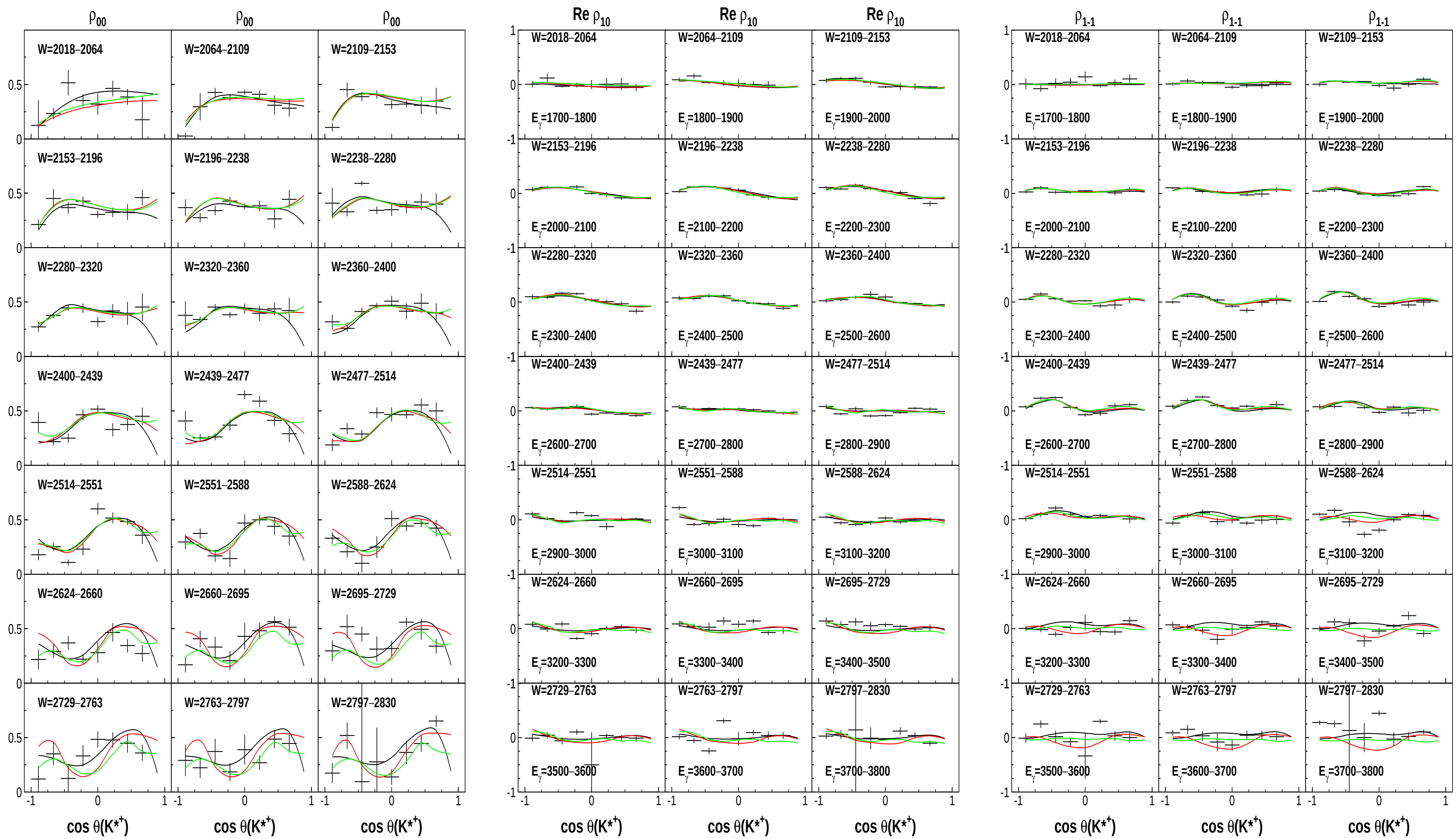
But there are three solutions: $S_{11}+D_{15}$, $D_{13}+D_{15}$ and $S_{11}+D_{13}$







Density matrix elements $\gamma p \to K^* \Lambda$ (CLAS, Preliminary)



The third shell 30 N^*'s and 15 Δ^*'s expected in a large number of multiplets:

$(70,3^-)$; $(56,3^-)$; $(20,3^-)$; $(70,2^-)$; $(70,1^-)$; $(70,1^-)$; $(56,1^-)$; $(20,1^-)$

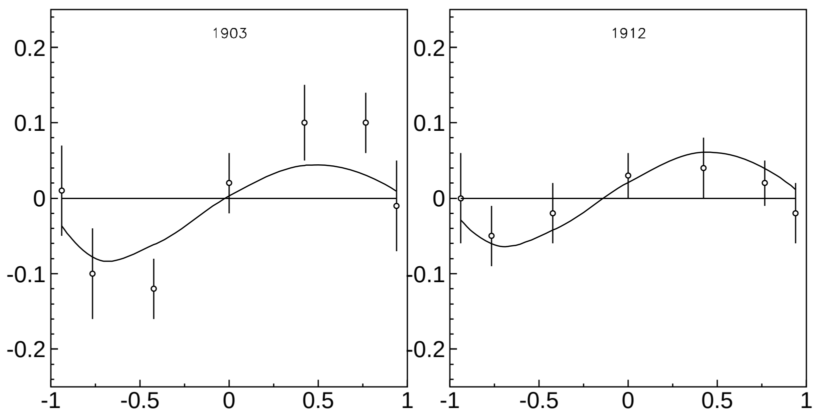
$(56,1^-)$:	$\Delta(1900)1/2^-$	$\Delta(1940)3/2^-$	$\Delta(1930)5/2^-$
	$N(1895)1/2^-$	$N(1875)3/2^-$	

$(70,3^-)$:		$\Delta(2223)5/2^-$	$\Delta(2200)7/2^-$	
	$N(2150)3/2^-$	$N(2280)5/2^-$?	$N(2190)7/2^-$	$N(2250)9/2^-$
		$N(2060)5/2^-$	missing	

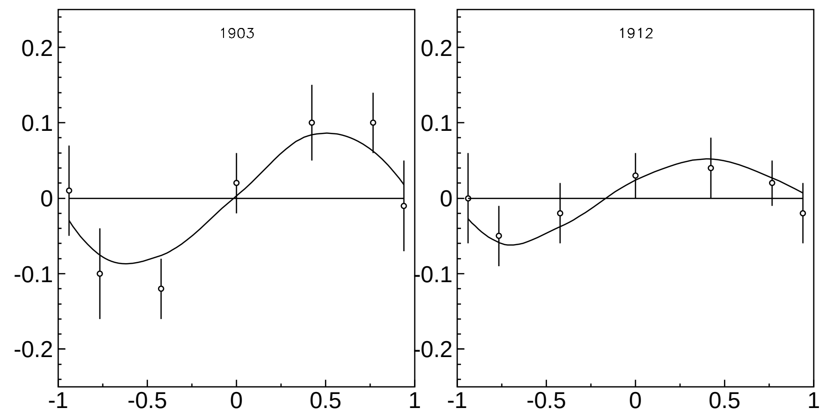
Do we have a proof for the resonances in the region 1.9 GeV from the $\gamma p \to \eta' p$ data?

The description of the GRAAL beam asymmetry.

With CLASS differential cross section

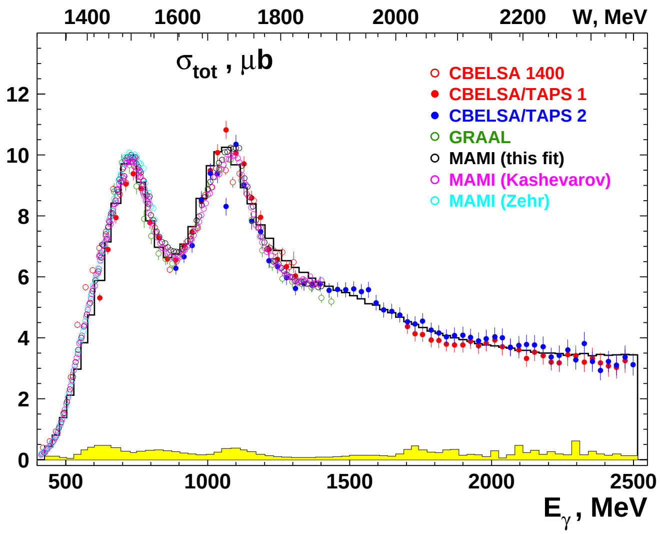


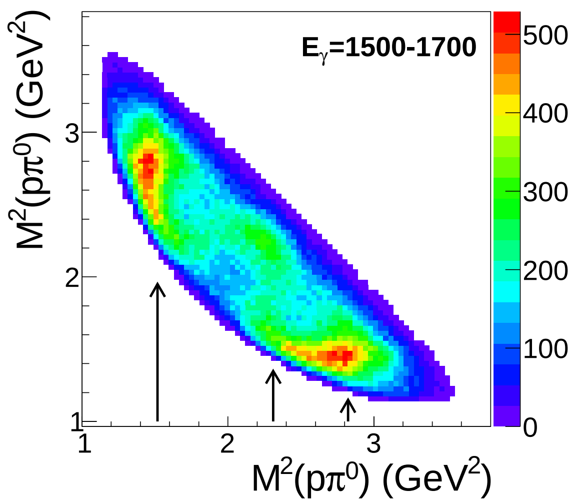
With CB-ELSA differential cross section

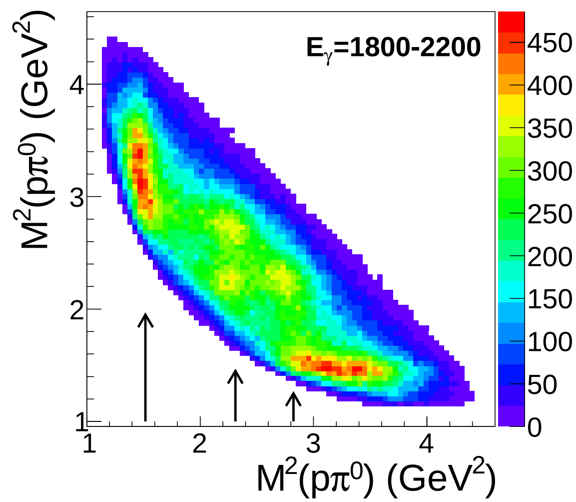


colorblue Interference between $N(1895)1/2^-$ **and** $N(1875)3/2^-$**.**

The data on $\gamma p \to \pi^0\pi^0 p$ and $\gamma p \to \pi^0\eta p$







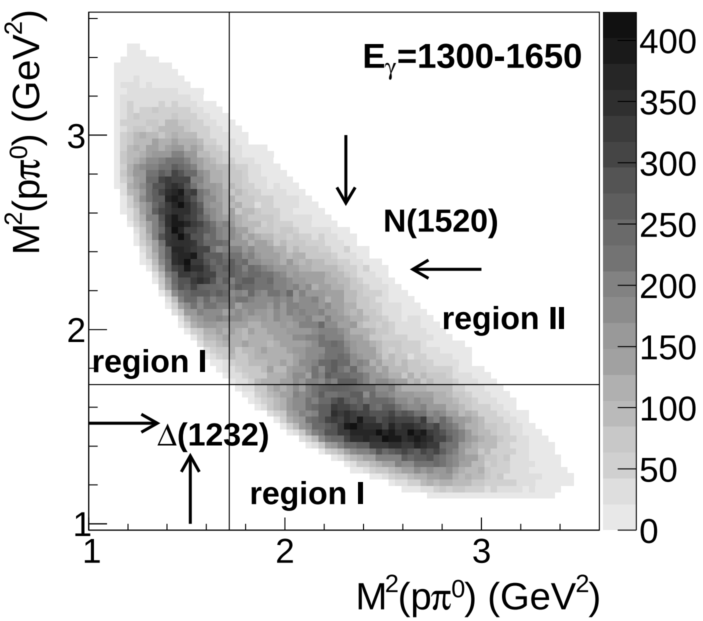
The $\gamma p \to \pi^+\pi^- p$ data should define the decay amplitudes of the resonances into $\rho(770) - N$ and practically saturate the unitarity condition in the region up to W=1.8 GeV. We include in our data base the data on:

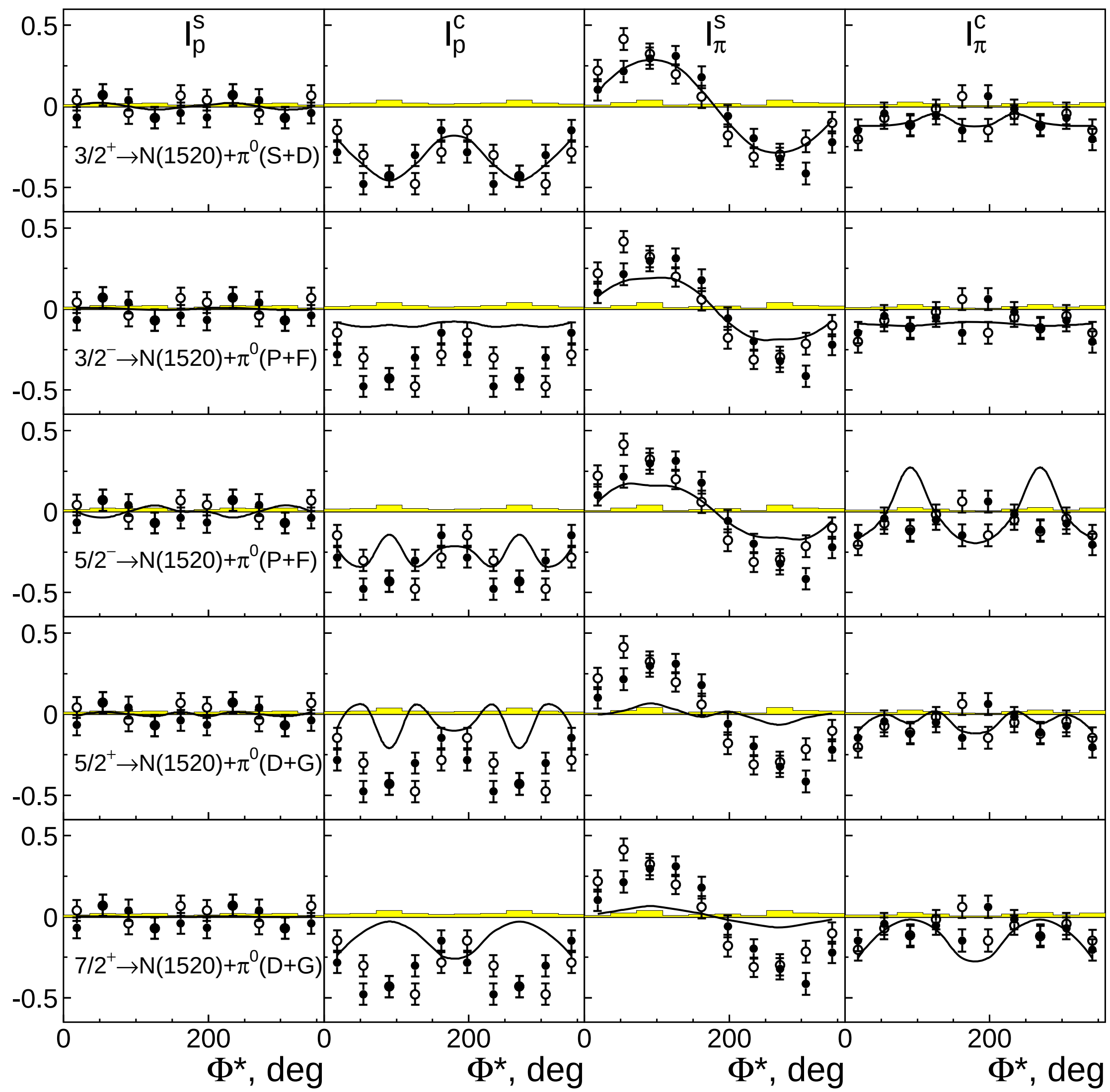
1) $\gamma p \to \pi^+\pi^- p$ differential cross section (SAPHIR, CLAS)

2) $\gamma p \to \pi^+\pi^- p$, I_c, I_s (CLAS)

3) New HADES data on $\pi^- p \to \pi^+\pi^- n$.

I_c and I_s polarization data are very important for the partial wave analysis

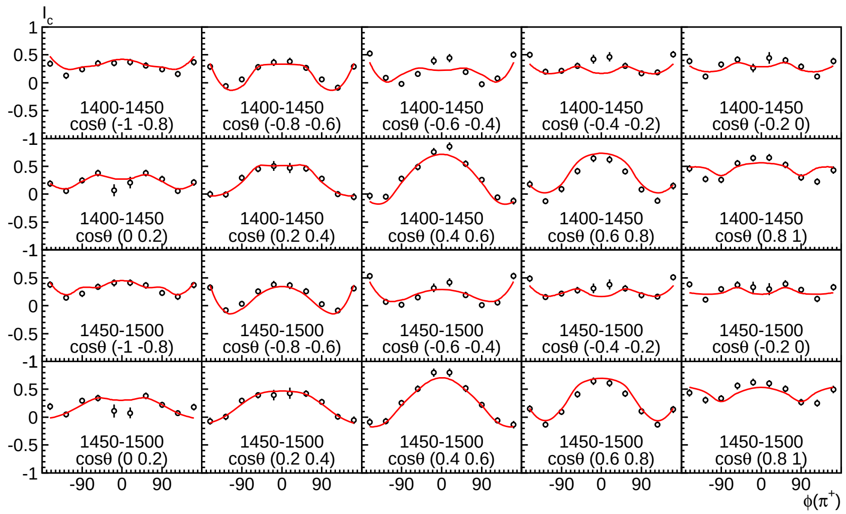




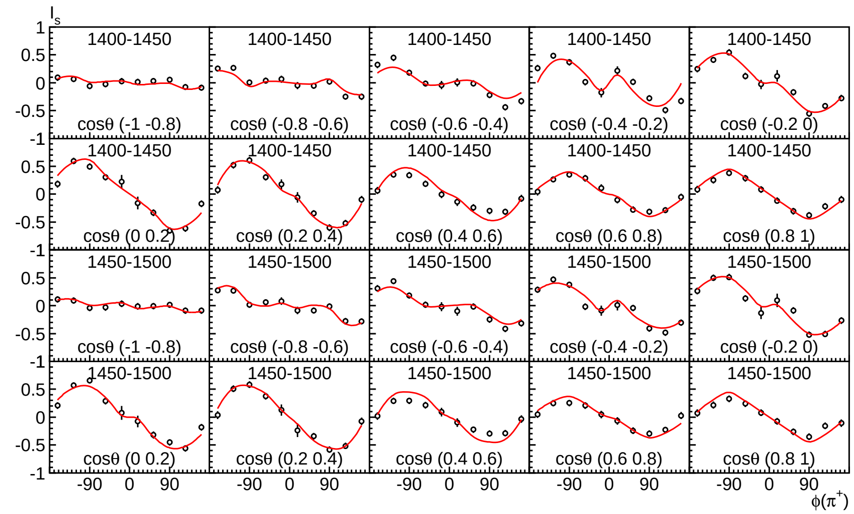
I_c and I_s for $\gamma p \to \pi^+\pi^- p$ from CLAS (Preliminary)

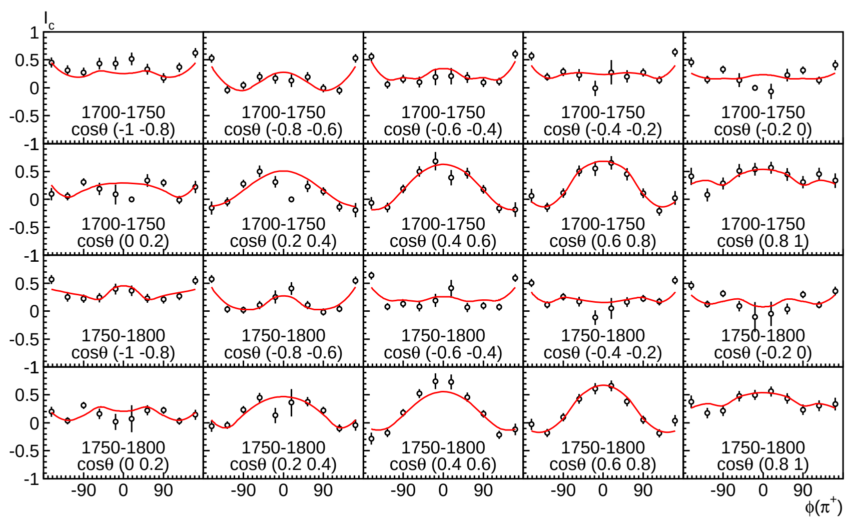
Courtesy of V. Crede, Florida State U

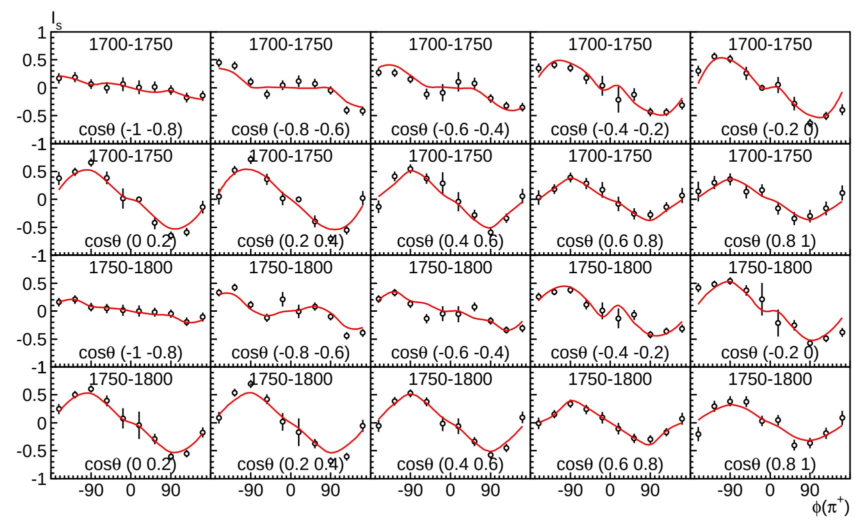
I_c



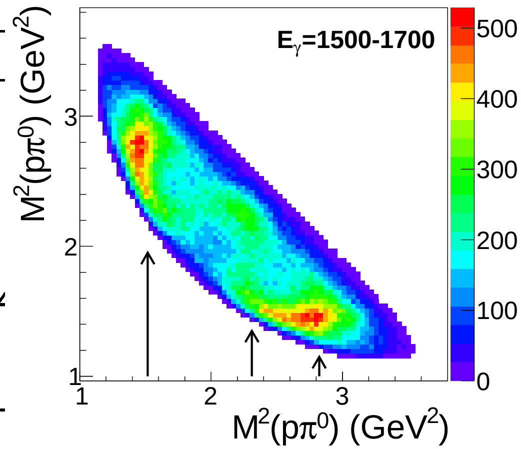
I_s

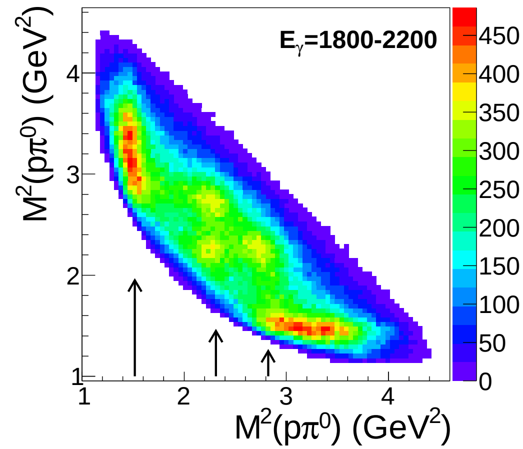






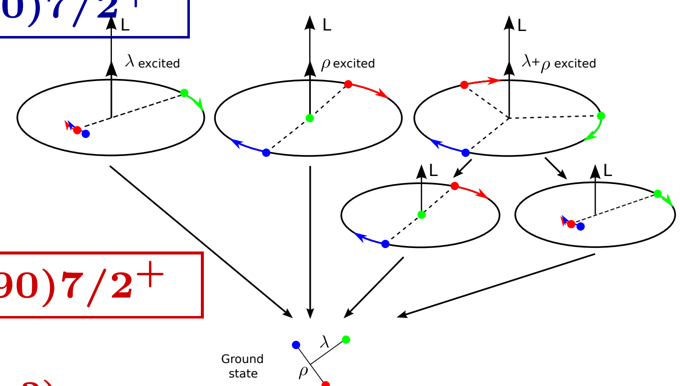
$\mathcal{S}$	space spin isospin
$\mathcal{S}_1$	$\mathcal{SSS}$
$\mathcal{S}_2$	$\mathcal{S}(\mathcal{M_S M_S}+\mathcal{M_A M_A})$
$\mathcal{S}_3$	$(\mathcal{M_S M_S}+\mathcal{M_A M_A})\mathcal{S}$
$\mathcal{S}_4$	$(\mathcal{M_A M_A}-\mathcal{M_S M_S})\mathcal{M_S}$
	$+(\mathcal{M_S M_A}+\mathcal{M_A M_S})\mathcal{M_A}$
$\mathcal{S}_5$	$(\mathcal{M_S S M_S}+\mathcal{M_A S M_A})$
$\mathcal{S}_6$	$\mathcal{A}(\mathcal{M_A M_S}-\mathcal{M_S M_A})$





$\Delta(1910)1/2^+$ $\Delta(1920)3/2^+$ $\Delta(1905)5/2^+$ $\Delta(1950)7/2^+$

$$\mathcal{S}=\frac{1}{\sqrt{2}}\left\{\left[\phi_{0s}(\vec\rho)\times\phi_{0d}(\vec\lambda)\right]+\left[\phi_{0d}(\vec\rho)\times\phi_{0s}(\vec\lambda)\right]\right\}^{(L=2)}$$



$N(1880)1/2^+$ $N(1900)3/2^+$ $N(2000)5/2^+$ $N(1990)7/2^+$

$$\mathcal{M_S}=\frac{1}{\sqrt{2}}\left\{\left[\phi_{0s}(\vec\rho)\times\phi_{0d}(\vec\lambda)\right]-\left[\phi_{0d}(\vec\rho)\times\phi_{0s}(\vec\lambda)\right]\right\}^{(L=2)}$$

$$\mathcal{M_A}=\left[\phi_{0p}(\vec\rho)\times\phi_{0p}(\vec\lambda)\right]^{(L=2)}\ .$$

γN interaction

Photon has quantum numbers $J^{PC}=1^{--}$, proton $1/2^+$. Then in S-wave two states can be formed is $1/2^-$ and $3/2^-$.

Then P-wave $1/2^+, 3/2^+$ and $1/2^+, 3/2^+, 5/2^+$.

In general case: $1/2^-$, $1/2^+$ are described by two amplitudes and higher states by three vertices.

$$
\begin{aligned}
V^{(1+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\mu i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n}\,, &\quad V^{(1-)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\xi\gamma_\mu X^{(n+1)}_{\xi\alpha_1\ldots\alpha_n}\,,\\
V^{(2+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+2)}_{\mu\nu\alpha_1\ldots\alpha_n}\,, &\quad V^{(2-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n+1)}_{\mu\alpha_1\ldots\alpha_n}\,,\\
V^{(3+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+1)}_{\nu\alpha_1\ldots\alpha_n} g^\perp_{\mu\alpha_n}\,, &\quad V^{(3-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n-1)}_{\alpha_2\ldots\alpha_n} g^\perp_{\alpha_1\mu}\,.
\end{aligned}
$$

$$
X^0=1 \qquad X^{(1)}_\mu = k^\perp_\mu = k_\nu g^\perp_{\nu\mu}; \qquad g^\perp_{\nu\mu} = \left(g_{\nu\mu} - \frac{P_\nu P_\nu}{p^2}\right)\;;
$$

$$
X^{(n)}_{\mu_1\ldots\mu_n} = \frac{2n-1}{n^2}\sum_{i=1}^{n} k^\perp_{\mu_i} X^{(n-1)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_n} - \frac{2k^2_\perp}{n^2}\sum_{\substack{i,j=1\\ i<j}}^{n} g_{\mu_i\mu_j} X^{(n-2)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_n}
$$

General structure of the single–meson electro-production amplitude in c.m.s. of the reaction is given by

$$J_\mu = i\mathcal{F}_1 \tilde{\sigma}_\mu + \mathcal{F}_2 (\vec{\sigma}\vec{q}) \frac{\varepsilon_{\mu ij}\sigma_i k_j}{|\vec{k}||\vec{q}|} + i\mathcal{F}_3 \frac{(\vec{\sigma}\vec{k})}{|\vec{k}||\vec{q}|} \tilde{q}_\mu + i\mathcal{F}_4 \frac{(\vec{\sigma}\vec{q})}{\vec{q}^2} \tilde{q}_\mu + i\mathcal{F}_5 \frac{(\vec{\sigma}\vec{k})}{|\vec{k}|^2} k_\mu + i\mathcal{F}_6 \frac{(\vec{\sigma}\vec{q})}{|\vec{q}||\vec{k}|} k_\mu \, ,$$

where $\vec{q}$ is the momentum of the nucleon in the πN channel and $\vec{k}$ the momentum of the nucleon in the γN channel calculated in the c.m.s. of the reaction. The σ_i are Pauli matrices.

$$\tilde{\sigma}_\mu = \sigma_\mu - \frac{\vec{\sigma}\vec{k}}{|\vec{k}|^2} k_\mu \qquad \mu = 1, 2, 3$$

$$\tilde{q}_\mu = q_\mu - \frac{\vec{q}\vec{k}}{|\vec{k}||\vec{q}|} k_\mu = q_\mu - z\, k_\mu$$

The functions $\mathcal{F}_i$ have the following angular dependence:

$$\begin{aligned}
\mathcal{F}_1(z) &= \sum_{L=0}^{\infty} \; [LM_L^+ + E_L^+]P'_{L+1}(z) + [(L+1)M_L^- + E_L^-]P'_{L-1}(z), \\
\mathcal{F}_2(z) &= \sum_{L=1}^{\infty} \; [(L+1)M_L^+ + LM_L^-]P'_L(z)\;, \\
\mathcal{F}_3(z) &= \sum_{L=1}^{\infty} \; [E_L^+ - M_L^+]P''_{L+1}(z) + [E_L^- + M_L^-]P''_{L-1}(z)\;, \\
\mathcal{F}_4(z) &= \sum_{L=2}^{\infty} \; [M_L^+ - E_L^+ - M_L^- - E_L^-]P''_L(z)\;, \\
\mathcal{F}_5(z) &= \sum_{L=0}^{\infty} \; [(L+1)S_L^+ P'_{L+1}(z) - LS_L^- P'_{L-1}(z)]\;, \\
\mathcal{F}_6(z) &= \sum_{L=1}^{\infty} \; [LS_L^- - (L+1)S_L^+]P'_L(z)
\end{aligned}$$

Here L corresponds to the orbital angular momentum in the πN system, $P'_L(z)$, $P''_L(z)$ are derivatives of Legendre polynomials $z = (\vec{k}\vec{q})/(|\vec{k}||\vec{q}|)$.

For the positive states $J = L + 1/2 \quad (L\!=\!n)$:

$$A^{i+}_{\mu} \quad = \quad \bar u(q_N) X^{(n)}_{\alpha_1\ldots\alpha_n}(q^\perp) F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n} V^{(i+)\mu}_{\beta_1\ldots\beta_n}(k^\perp) u(k_N)$$

$$\begin{aligned}
\mathcal{F}^{1+}_1 &= \lambda_n\, P'_{n+1} \\
\mathcal{F}^{1+}_2 &= \lambda_n\, P'_{n} \\
\mathcal{F}^{1+}_3 &= 0 \\
\mathcal{F}^{1+}_4 &= 0 \\
\mathcal{F}^{1+}_5 &= +\lambda_n\, P'_{n+1} \\
\mathcal{F}^{1+}_6 &= -\lambda_n\, P'_{n}
\end{aligned}$$

where

$$\lambda_n = \frac{\alpha_n}{2n+1} (|\vec k||\vec q|)^n \chi_i \chi_f \qquad \chi_{i,f} = \sqrt{m_{i,f} + k_{0i,f}}$$

Therefore

$$E^{1+}_n = M^{1+}_n = S^{1+}_n = \frac{\lambda_n}{n+1}$$

The correspondence of the vertices and multipoles ($J = n + \frac{1}{2}$):

	E	M	S
V_n^{1+}	$\frac{\lambda_n}{n+1}$	$\frac{\lambda_n}{n+1}$	$\frac{\lambda_n}{n+1}$
V_n^{2+}	$\frac{\lambda_n}{n+1}$	$-\frac{\lambda_n}{n(n+1)}$	$\frac{\lambda_n}{n+1}$
V_n^{3+}	ξ^n	**0**	$-\xi_n \frac{n+2}{n+1}$
V_n^{1-}	$-\frac{\zeta_{n+1}}{n+1}$	$\frac{\zeta_{n+1}}{n+1}$	$-\frac{\zeta_{n+1}}{n+1}$
V_n^{2-}	$-\Delta_n$	**0**	$-\Delta_n \frac{2n^2}{n+1}$
V_n^{3-}	$-\varrho_{n-1}$	**0**	$\varrho_{n-1} \frac{n-1}{n}$

$$\lambda_n = \frac{\alpha_n}{2n+1}(|\vec{k}||\vec{q}|)^n \chi_i \chi_f \qquad \Delta_n = \frac{\alpha_n}{n(n+1)^2}(|\vec{k}||\vec{q}|)^{n+1} \chi_i \chi_f$$

$$\zeta_n = \frac{\alpha_n}{n}(|\vec{k}||\vec{q}|)^n \chi_i \chi_f \qquad \varrho_n = \frac{\alpha_n}{(n+1)(n+2)}|\vec{k}|^n |\vec{q}|^{n+2} \chi_i \chi_f$$

$$\xi_n = \frac{\alpha_n}{(n+2)(n+1)}|\vec{k}|^{n+2} |\vec{q}|^n \chi_i \chi_f$$

The Reggezied t and u-exchanges are treated with the prescription from

M. Guidal, J-M. Laget and M. Vanderhaeghen Nucl.Phys. A627,(645) 1997.

However it can be wrong....

SUMMARY

- **The number of new photoproduction data sets are included in the fit and successfully described.**
- **The new precise data on π and η photoproduction provide a strong constrain on the partial wave amplitude decomposition.**
- **The analysis of photoproduction of vector mesons like ωN and $K^*(890)\Lambda$ provides an important constraint on the branching ratios and reveals signals from resonances above 2 GeV.**
- **The fit of the $\pi^0\pi^0$ and $\pi^+\pi^-$ final state should provide an important information about resonance properties and almost saturate the unitarity condition up to invariant masses 1.8 GeV**
- **The decay properties of the resonances via cascade decays can provide an important information for systematization and classification of observed states.**
- **The formalism for the analysis of the electro-production data is almost developed and encoded (but not tested yet).**

1 Boson projection operators

In momentum representation:

$$P^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n} = (-1)^n O^{\mu_1\mu_2\ldots\mu_n}_{\nu_1\nu_2\ldots\nu_n} = \sum_{i=1}^{2n+1} u^{(i)}_{\mu_1\mu_2\ldots\mu_n} u^{(i)*}_{\nu_1\nu_2\ldots\nu_n}$$

The projection operator can depends only on the total momentum and the metric tensor.

For spin 0 it is a unit operator. For spin 1 the only possible combination is:

$$O^{\mu}_{\nu} = g^{\perp}_{\mu\nu} = g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}$$

The propagator for the particle with spin $S > 2$ must be constructed from the tensors $g^{\perp}_{\mu\nu}$: this is the only combination which satisfies:

$$p_\mu g^{\perp}_{\mu\nu} = 0.$$

Then for spin 2 state we obtain:

$$O^{\mu_1\mu_2}_{\nu_1\nu_2} = \frac{1}{2}(g^{\perp}_{\mu_1\nu_1} g^{\perp}_{\mu_2\nu_2} + g^{\perp}_{\mu_1\nu_2} g^{\perp}_{\mu_2\nu_1}) - \frac{1}{3} g^{\perp}_{\mu_1\mu_2} g^{\perp}_{\nu_1\nu_2}$$

Recurrent expression for the boson projector operator

$$O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} = \frac{1}{L^2}\left(\sum_{i,j=1}^{L} g^{\perp}_{\mu_i\nu_j} O^{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_L}_{\nu_1\ldots\nu_{j-1}\nu j+1\ldots\nu_L} - \right.$$

$$\left.\frac{4}{(2L-1)(2L-3)}\sum_{i<j,k<m}^{L} g^{\perp}_{\mu_i\mu_j} g^{\perp}_{\nu_k\nu_m} O^{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_L}_{\nu_1\ldots\nu_{k-1}\nu_{k+1}\ldots\nu_{m-1}\nu_{m+1}\ldots\nu_L}\right)$$

Normalization condition:

$$O^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L} O^{\nu_1\ldots\nu_L}_{\alpha_1\ldots\alpha_L} = O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}$$

Orbital momentum operator

The angular momentum operator is constructed from momenta of particles k_1, k_2 and metric tensor $g_{\mu\nu}$.

For $L=0$ this operator is a constant: $X^0=1$

The $L=1$ operator is a vector $X^{(1)}_\mu$, constructed from: $k_\mu=\frac{1}{2}(k_{1\mu}-k_{2\mu})$ and $P_\mu=(k_{1\mu}+k_{2\mu})$. Orthogonality:

$$\int \frac{d^4k}{4\pi} X^{(1)}_{\mu_1} X^{(0)} = \int \frac{d^4k}{4\pi} X^{(n)}_{\mu_1\ldots\mu_n} X^{(n-1)}_{\mu_2\ldots\mu_n} = \xi P_{\mu_1} = 0$$

Then:

$$X^{(1)}_\mu P_\mu = 0 \qquad X^{(n)}_{\mu_1\ldots\mu_n} P_{\mu_j} = 0$$

and:

$$X^{(1)}_\mu = k^\perp_\mu = k_\nu g^\perp_{\nu\mu}; \qquad g^\perp_{\nu\mu} = \left(g_{\nu\mu} - \frac{P_\nu P_\nu}{p^2} \right) ;$$

in c.m.s $k^\perp = (0, \vec{k})$

Recurrent expression for the orbital momentum operators $X^{(n)}_{\mu_1\ldots\mu_n}$

$$X^{(n)}_{\mu_1\ldots\mu_n}=\frac{2n\!-\!1}{n^2}\sum_{i=1}^{n}k^{\perp}_{\mu_i}X^{(n-1)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_n}-\frac{2k^2_{\perp}}{n^2}\sum_{\substack{i,j=1\\ i<j}}^{n}g_{\mu_i\mu_j}X^{(n-2)}_{\mu_1\ldots\mu_{i-1}\mu_{i+1}\ldots\mu_{j-1}\mu_{j+1}\ldots\mu_n}$$

Taking into account the traceless property of $X^{(n)}$ we have:

$$X^{(n)}_{\mu_1\ldots\mu_n}X^{(n)}_{\mu_1\ldots\mu_n}=\alpha(n)(k^2_{\perp})^n \qquad \alpha(n)=\prod_{i=1}^{n}\frac{2i-1}{i}=\frac{(2n-1)!!}{n!}.$$

From the recursive procedure one can get the following expression for the operator $X^{(n)}$:

$$X^{(n)}_{\mu_1\ldots\mu_n}=\alpha(n)\bigg[k^{\perp}_{\mu_1}k^{\perp}_{\mu_2}\ldots k^{\perp}_{\mu_n}-\frac{k^2_{\perp}}{2n-1}\Big(g^{\perp}_{\mu_1\mu_2}k^{\perp}_{\mu_3}\ldots k^{\perp}_{\mu_n}+\cdots\Big)+$$
$$\frac{k^4_{\perp}}{(2n-1)(2n-3)}\Big(g^{\perp}_{\mu_1\mu_2}g^{\perp}_{\mu_3\mu_4}k^{\perp}_{\mu_5}\cdots k_{\mu_4}+\cdots\Big)+\cdots\bigg].$$

Scattering of two spinless particles

Denote relative momenta of particles before and after interaction as q and k, correspondingly. The structure of partial–wave amplitude with orbital momentum $L = J$ is determined by convolution of operators $X^{(L)}(k)$ and $X^{(L)}(q)$:

$$A_L = BW_L(s) X^{(L)}_{\mu_1 \ldots \mu_L}(k) O^{\mu_1 \ldots \mu_L}_{\nu_1 \ldots \nu_L} X^{(L)}_{\nu_1 \ldots \nu_L}(q) = BW_L(s) X^{(L)}_{\mu_1 \ldots \mu_L}(k) X^{(L)}_{\mu_1 \ldots \mu_L}(q)$$

$BW_L(s)$ **depends on the total energy squared only.**

The convolution $X^{(L)}_{\mu_1 \ldots \mu_L}(k) X^{(L)}_{\mu_1 \ldots \mu_L}(q)$ can be written in terms of Legendre polynomials $P_L(z)$:

$$X^{(L)}_{\mu_1 \ldots \mu_L}(k) X^{(L)}_{\mu_1 \ldots \mu_L}(q) = \alpha(L) \left(\sqrt{k_\perp^2} \sqrt{q_\perp^2} \right)^L P_L(z) \; ,$$

$$z = \frac{(k^\perp q^\perp)}{\sqrt{k_\perp^2} \sqrt{q_\perp^2}} \qquad\qquad \alpha(L) = \prod_{n=1}^{L} \frac{2n-1}{n}$$

πN interaction

States with $J=L-1/2$ are called '-' states ($1/2^+$, $3/2^-$,$5/2^+$,...) and states with $J=L+1/2$ are called '+' states ($1/2^-$, $3/2^+$,$5/2^-$,...).

$$\tilde{N}^+_{\mu_1\ldots\mu_n}=X^{(n)}_{\mu_1\ldots\mu_n}\qquad \tilde{N}^-_{\mu_1\ldots\mu_n}=i\gamma_\nu\gamma_5 X^{(n+1)}_{\nu\mu_1\ldots\mu_n}$$

$$A=\bar{u}(k_1)N^{\pm}_{\mu_1\ldots\mu_L}F^{\mu_1\ldots\mu_{L-1}}_{\nu_1\ldots\nu_{L-1}}N^{\pm}_{\nu_1\ldots\nu_L}u(q_1)BW^{\pm}_L(s)\xrightarrow[c.m.s.]{}\omega^*\left[G(s,t)+H(s,t)i(\vec{\sigma}\vec{n})\right]\omega'$$

$$G(s,t)=\sum_L\left[(L{+}1)F^+_L(s)-LF^-_L(s)\right]P_L(z)\ ,$$

$$H(s,t)=\sum_L\left[F^+_L(s)+F^-_L(s)\right]P'_L(z)\ .$$

$$F^+_L\quad=(-1)^{L+1}(|\vec{k}||\vec{q}|)^L\sqrt{\chi_i\chi_f}\,\frac{\alpha(L)}{2L{+}1}BW^+_L(s)\ ,$$

$$F^-_L\quad=(-1)^L(|\vec{k}||\vec{q}|)^L\sqrt{\chi_i\chi_f}\,\frac{\alpha(L)}{L}BW^-_L(s)\ .$$

$$\chi_i=m_i+k_{i0}\qquad\qquad \alpha(L)=\prod_{l=1}^{L}\frac{2l-1}{l}=\frac{(2L-1)!!}{L!}\ .$$

γN interaction

Photon has quantum numbers $J^{PC} = 1^{--}$, proton $1/2^+$. Then in S-wave two states can be formed is $1/2^-$ and $3/2^-$.

Then P-wave $1/2^+, 3/2^+$ and $1/2^+, 3/2^+, 5/2^+$.

In general case: $1/2^-$, $1/2^+$ described by two amplitudes and higher states by three amplitudes.

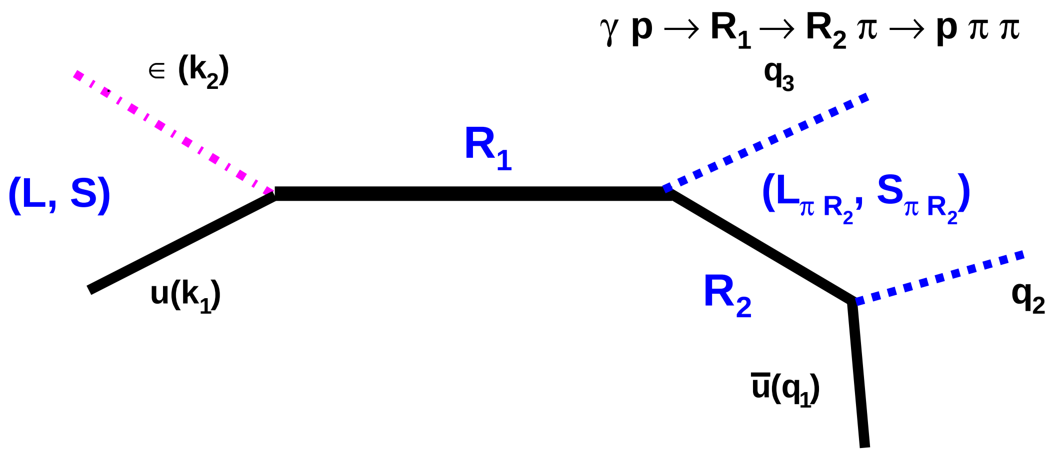
$$
\begin{aligned}
V^{(1+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\mu i\gamma_5 X^{(n)}_{\alpha_1\ldots\alpha_n}\,, & V^{(1-)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\xi\gamma_\mu X^{(n+1)}_{\xi\alpha_1\ldots\alpha_n}\,,\\
V^{(2+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+2)}_{\mu\nu\alpha_1\ldots\alpha_n}\,, & V^{(2-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n+1)}_{\mu\alpha_1\ldots\alpha_n}\,,\\
V^{(3+)\mu}_{\alpha_1\ldots\alpha_n} &= \gamma_\nu i\gamma_5 X^{(n+1)}_{\nu\alpha_1\ldots\alpha_n} g^{\perp}_{\mu\alpha_n}\,, & V^{(3-)\mu}_{\alpha_1\ldots\alpha_n} &= X^{(n-1)}_{\alpha_2\ldots\alpha_n} g^{\perp}_{\alpha_1\mu}\,.
\end{aligned}
$$

Gauge invariance: $\varepsilon_\mu q_{1\mu} = 0$ where q_1-photon momentum.

$$
\varepsilon_\mu V^{(2\pm)\mu}_{\alpha_1\ldots\alpha_n} = C^{\pm}\varepsilon_\mu V^{(3\pm)\mu}_{\alpha_1\ldots\alpha_n}
$$

where $C^{\pm}$ do not depend on angles.

Resonance amplitudes for meson photoproduction



General form of the angular dependent part of the amplitude:

$$\bar{u}(q_1)\tilde{N}_{\alpha_1\ldots\alpha_n}(R_2\to\mu N)F^{\alpha_1\ldots\alpha_n}_{\beta_1\ldots\beta_n}(q_1+q_2)\tilde{N}^{(j)\beta_1\ldots\beta_n}_{\gamma_1\ldots\gamma_m}(R_1\to\mu R_2)$$

$$F^{\gamma_1\ldots\gamma_m}_{\xi_1\ldots\xi_m}(P)V^{(i)\mu}_{\xi_1\ldots\xi_m}(R_1\to\gamma N)u(k_1)\varepsilon_\mu$$

$$F^{\mu_1\ldots\mu_L}_{\nu_1\ldots\nu_L}(p)=(m+\hat{p})O^{\mu_1\ldots\mu_L}_{\alpha_1\ldots\alpha_L}\frac{L+1}{2L+1}\left(g^{\perp}_{\alpha_1\beta_1}-\frac{L}{L+1}\sigma_{\alpha_1\beta_1}\right)\prod_{i=2}^{L}g_{\alpha_i\beta_i}O^{\beta_1\ldots\beta_L}_{\nu_1\ldots\nu_L}$$

$$\sigma_{\alpha_i\alpha_j}=\frac{1}{2}(\gamma_{\alpha_i}\gamma_{\alpha_j}-\gamma_{\alpha_j}\gamma_{\alpha_i})$$